VOLUME LIX, No. 24 NOVEMBER 22, 1962

THE JOURNAL OF PHILOSOPHY

TASKS, SUPER-TASKS, AND THE MODERN ELEATICS *

MANY years ago Zeno of Elea raised some questions concerning the possibility of motion. He presented arguments designed to show that motion was impossible: that any claim that motion had really taken place was self-contradictory. I don't believe that anyone holds this view today—which proves that some things eventually become evident, even to philosophers. So I won't try to show that motion is really possible.

However, the difficulties Zeno raised were far from silly. They were grounded in legitimate problems concerning space and time, and, although what he claimed to have shown seems to be false, there is far from universal agreement on just what was wrong with his arguments. The debate has lasted these several thousand years. Most likely, it will last several thousand more—which proves that some things don't eventually become evident, even to philosophers. I am not entering the arena to do battle on this issue. My purpose is much more modest: I wish to discuss another question, which Zeno may or may not have raised, but which his recent commentators have certainly raised in their analyses of his arguments. I shall limit my discussion to issues raised by J. F. Thomson in his remarkable and stimulating paper "Tasks and Super-tasks,"[1] basing my own remarks on his treatment of these issues and letting him represent what I take to be a number of widely held views. In this way, I shall manage to be unfair both to Thomson (by scrutinizing his remarks more closely than perhaps they were meant to be) and to those who are in substantial agreement with him (by slighting their individual views where it might conceivably make some difference to the general position).

The Zenonian argument that Thomson discusses goes as follows:

* Read to the Philosophy Department Seminar at Princeton University, April 12, 1961. I would like particularly to thank Gregory Vlastos and Ian Hacking for having stimulated my interest in these questions and for their many valuable suggestions and criticisms during the writing of this paper.

[1] *Analysis*, **15**, 1 (October, 1954):1–10. Subsequent references to this article will appear in the text and will be by page number.

> "To complete any journey you must complete an infinite number of journeys. For to arrive from A to B you must first go from A to A′, the mid-point of A and B, and thence to A″, the mid-point of A′ and B, and so on. But it is logically absurd that someone should have completed all of an infinite number of journeys, just as it is logically absurd that someone should have completed all of an infinite number of tasks. Therefore it is absurd that anyone has ever completed any journey" (1).

Thomson argues that, whereas previous disputants concerning the proper analysis of this argument divide into two schools: those who deny the first premise and those who deny the second, both schools granting the validity of the argument, the proper analysis shows that the argument is invalid, that it commits the fallacy of equivocation. The expression 'completing an infinite number of journeys' can be taken in two ways. If it is taken in one way, the first premise is false and the second true; in the other, the first premise is true and the second false. No way of interpreting it renders *both* premises true.[2]

Very briefly, the two ways are these. If we have made a continuous uninterrupted journey from A to B we can be said to have covered all the stretches described in the first premise; that is, our motion *can be analyzed as* covering in turn AA′, A′A″, etc. If there is at least one of these stretches that we haven't covered, then we haven't completed our journey. In this sense, the first premise is true; but in this sense also, the *second* premise is false: completing infinitely many journeys takes no more effort than completing one; to say of someone that he has completed an infinite number of journeys (in *this* sense) is just to describe in a different (and possibly somewhat peculiar) way the act he performed in completing the single continuous journey from A to B. No absurdity is involved with the feat. If, however, we think of "completing an infinite number of journeys" as completing an infinite number of physically distinct acts, each with a beginning and an end, and with, say, a pause of finite duration between any two, then according to Thomson the second premise is true: it *is* logically absurd that one should have completed an infinite number of journeys. But, of course, under this interpretation the first premise is obviously false: one need not complete an infinite number of journeys of this kind in order to complete a single one.

Thomson does two things. First, he considers the second

[2] This fact alone, of course, does not establish the formal invalidity of the argument. At best it shows that the argument could not be used to establish its conclusion. If the phrase in question is indeed ambiguous, then one not averse to arguing sophistically could well employ the argument in a debate and then, if pushed, admit that one of his premises was false. He would thereby maintain the purity of his logic, if not of his soul.

premise under this last interpretation and argues in two stages that it must be true—that it is logically impossible to complete an infinite number of journeys or to perform an infinite number of tasks (a "super-task"). He argues that some reasons why people might have thought this possible are bad reasons, and then offers arguments to prove that the concept of "having performed an infinite number of tasks or acts" is a self-contradictory one. The second part of his paper is devoted to showing why, under the first interpretation of 'completing an infinite number of journeys', the second premise turns out to be false, but might appear to some to be true. It should now be obvious why I have called Thomson and Thomsonites "modern Eleatics." Whereas Zeno tried to show that performing a single task was impossible, his twentieth-century emendators are content to retreat to the position that, although single tasks might be all right, we mustn't have too many of them. As I have already stated, I shall not discuss the correctness of this analysis of the Zenonian argument. But I shall discuss Thomson's arguments to the effect that it is logically impossible to perform an infinite number of tasks.[3] In a final section I shall make some general remarks concerning the possibility of proving or disproving the logical impossibility of performing infinitely many acts or tasks.

I

Thomson's first argument, concerning the lamp, is short, imaginative, and compelling. It appears to demonstrate that "completing a super-task" is a self-contradictory concept. Let me reproduce it here:

> There are certain reading-lamps that have a button in the base. If the lamp is off and you press the button the lamp goes on, and if the lamp is on and you press the button, the lamp goes off. So if the lamp was originally off and you pressed the button an odd number of times, the lamp is on, and if you pressed the button an even number of times the lamp is off. Suppose now that the lamp is off, and I succeed in pressing the button an infinite number of times, perhaps making one jab in one minute, another jab in the next half minute, and so on. . . . After I have completed the whole infinite

[3] In fairness to Thomson, I must add that he is not univocal in the conclusions that he draws from his arguments. He alternates between concluding that super-tasks are a logical impossibility and that "the concept of super-task has not been *explained*" (6), that "talk of super-tasks is *senseless*" (9). (In both cases the italics are Thomson's.) Perhaps he does not distinguish between the two, or perhaps he thinks that to show that a contradiction arises from supposing that a super-task has been performed establishes the senselessness of such talk. So far as I can tell, he is not explicit on this point. I shall try to say something about this distinction in the last part of this paper.

sequence of jabs, i.e. at the end of the two minutes, is the lamp on or off? . . . It cannot be on, because I did not ever turn it on without at once turning it off. It cannot be off, because I did in the first place turn it on, and thereafter I never turned it off without at once turning it on. But the lamp must be either on or off. This is a contradiction (5).

Rarely are we presented with an argument so neat and convincing. This one has only one flaw. It is invalid. Let us see why. Consider the following two descriptions:

A. Aladdin starts at t_0 and performs the super-task in question just as Thomson does. Let t_1 be the first instant after he has completed the whole infinite sequence of jabs—the instant about which Thomson asks "Is the lamp on or off?"—and let the lamp be *on* at t_1.

B. Bernard starts at t_0 and performs the super-task in question (on another lamp) just as Aladdin does, and let Bernard's lamp be *off* at t_1.

I submit that *neither* description is self-contradictory, or, more cautiously, that Thomson's argument shows neither description to be self-contradictory (although possibly some other argument might).

According to Thomson, Aladdin's lamp cannot be on at t_1 because Aladdin turned it off after each time he turned it on. *But this is true only of instants before* t_1*!* From this it follows only that there is no time *between* t_0 *and* t_1 at which the lamp was on and which was not followed by a time *also before* t_1 at which it was off. Nothing whatever has been said about the lamp *at* t_1 *or later*. And similarly with Bernard's lamp. The only reasons Thomson gives for supposing that *his* lamp will not be off at t_1 are ones which hold only for times *before* t_1. The explanation is quite simply that Thomson's instructions do not cover the state of the lamp at t_1, although they *do* tell us what will be its state at every instant *between* t_0 and t_1 (including t_0).[4] Certainly, the lamp must be on or off at t_1 (provided that it hasn't gone up in a metaphysical puff of smoke in the interval), but nothing we are told implies which it is to be. The arguments to the effect that it can't be either just have no bearing on the case. To suppose that they *do* is to suppose that a description of the physical state of the lamp at t_1 (with respect to the property of being on or off) is a *logical* consequence of a description of its state (with respect to the same property) at times prior to t_1. I don't know whether this is true or not, and in section II I shall briefly investigate some matters that bear on this issue. But, true or not, the argument is invalid without the addition of a premise to that effect. This

[4] Provided, of course, that we have been diligent in following them and that nothing happens to the lamp between jabs.

will emerge even more clearly if we consider a parallel argument. Imagine someone telling us:

> There are two kinds of numbers, fair and foul, and every number <1 is one or the other. Consider the infinite converging sequence 1/2, 1/4, 1/8, 1/16, 1/32, ... Its first member is foul, its second member fair, its third member foul, its fourth fair, etc., alternating in such a way that $1/2^n$ is foul if n is odd and fair if n is even, for all positive integers n. What about the limit of the sequence? It is, of course, not in the sequence; but is it foul or fair? It can't be foul, because after every foul there was a fair; and it can't be fair, because we started with a foul and thereafter there was not a fair that was not immediately followed by a foul. But it must be one or the other, for every number >1 is either fair or foul. This is a contradiction.

Of course not. The answer is simply that we haven't been told how to classify the limit number. The instructions cover the sequence and the sequence only. Nothing was said about any number not in the sequence. The same is true in the case of the lamp. Thomson tells us nothing about the state of the lamp at t_1. Consequently Aladdin's and Bernard's results were perfectly possible outcomes—at least insofar as the argument under discussion is concerned. No magic was necessary to overcome it. And indeed, Thomson himself was not far from the truth, for in a sentence that I omitted (between "Is the lamp on or off?" and "It cannot be on") he says: "It seems impossible to answer this question" (5). Quite so. It *is* impossible, on the information given. But a contradiction can be shown to arise only by assuming that the instructions given are complete in the sense that the statement that they have been followed entails either that the lamp is on at t_1 or that it is off at t_1. They are not.

We can now see that what is correct and convincing about Thomson's argument can be put in this way:

> There are certain reading lamps that have a button in the base. If the lamp is off and you press the button the lamp goes on, and if the lamp is on and you press the button the lamp goes off. So if the lamp was originally off and you pressed the button an odd number of times the lamp is on, and if you pressed the button an even number of times the lamp is off. Suppose now that the lamp is off and I succeed in pressing the button an infinite number of times, perhaps making one jab in one minute, another jab in the next half minute, and so on. Does having followed these instructions entail either that at the end of the two minutes the lamp is on or that at the end of the two minutes the lamp is off? It doesn't entail that it is on because in following them I did not ever turn it on without at once turning it off. It doesn't entail that it is off because in following them I did in the first place turn it on, and thereafter I never turned it off without at once turning it on.

But if we now continue the argument with "But it must entail one or the other," we are struck with the obvious falsity of the remark; whereas the continuation "But the lamp must be either on or off," is striking by its obvious irrelevance.

Thomson has a further, parallel argument to the effect that a machine could not exist that would write down in two minutes the sequence of integers that constitutes the decimal expansion of π. Briefly reconstructed, it is this. Call one such machine "Albert." If Albert could exist, so, presumably, could Bosco, a machine which records whether the integer written down by Albert is odd or even (by showing either '0' or '1' on its dial: '0' if the integer in question is even, '1' if it is odd). Then "What appears on the (Bosco's) dial after the first machine has run through all the integers in the decimal expansion of π?" is the rhetorical question. Poor Bosco. Is he supposed to be confused? Or just tired? Bosco has only been told what to do in case Albert presents him with an integer. He has no will of his own, no initiative. Had he been told: "After you are through with Albert's integers start on Cuthbert's" (Cuthbert works out the decimal expansion of $\sqrt{2}$ in two minutes), Bosco's dial would show '1' as soon as he started on Cuthbert's problem. And had *we* been told that these were his instructions, *we* should know that he shows '1' as soon as he starts. Thomson's argument "proves too much." It "proves" the nonexistence of any function f satisfying the following conditions:

1. If x is the yth integer in the decimal expansion of π and x is odd, then $f(x,y)=1$.

and

2. If x is the yth integer in the decimal expansion of π and x is even, then $f(x,y)=0$.

It "proves" it by arguing that the question "What is the value of $f(x,\omega)$?" has no answer. Of course it has no answer. f is defined only over finite ordinals in its second-argument place. But that hardly proves that f is self-contradictory or nonexistent. There is no ωth integer in the decimal expansion of π.

But Thomson has a third argument, which we must meet. In effect it is his own analysis of *why* the "contradiction" arises in the case of the lamp (and in the other parallel cases). If his explanation is correct, then indeed there is something wrong with our argument that a contradiction does not arise. Let me quote him again:

> Now what exactly do these arguments come to? Say that the reading lamp has either of two light values, 0 ("off") and 1 ("on"). To switch the lamp on is then to add 1 to its value and to switch it off is to subtract 1 from its value. Then the question whether the lamp is on or off after the infinite number of switchings have been performed is a question about the

value of the lamp after an infinite number of alternating additions and subtractions of 1 to and from its value, i.e. is the question: What is the sum of the infinite divergent sequence $+1$, -1, $+1$, . . . ? Now mathematicians do say that this sequence has a sum; they say that its sum is 1/2. And this answer does not help us, since we attach no sense here to saying that the lamp is half-on (6).

This too is convincing. It appears that Thomson has found the mathematical analogue of these cases, that *it* is perfectly self-consistent, and that the "contradiction" in the case of the physical example carries into the mathematical example—not *as* a contradiction—but as something that has no physical analogue (for *logical* reasons: it makes no sense here to speak of the lamp as being half on). And indeed this appears to work. If the initial value was 0, then the value of the lamp after n switchings is the sum of the first n terms of the series. Consequently the value of the lamp after *all* the switchings is the sum of *all* the terms, or 1/2. But if the value of the lamp at t_1 is *always* the value of the lamp at t_0 plus the sum of the values corresponding to the sequence of switchings, this represents a contradiction, since there are only two possible values for the lamp: 0 and 1. This is a different "proof" of the self-contradictoriness of the concept of "super-task." Thomson doesn't explicitly present it as such (he presents it as his analysis of what "went wrong"), but he accepts every one of its assumptions (as he must if his explanation is to make sense).

Thomson then argues that this shows that there is no established method for deciding what is done when a super-task is done. But the argument shows no such thing. [For the sake of brevity and convenience, I shall call the sum of the first n terms of a given series (for positive finite n) its "partial$_n$ sum."] True, for each n the value of the lamp after n switchings is accurately represented by its partial$_n$ sum; but what reason is there to believe that its value after all the switchings will be accurately represented by the sum of all the terms, i.e., by the limit of the partial$_n$ sums? None.

To make this clear, let me say a bit more about fair and foul numbers. So far we know that $1/2^n$ is foul if n is odd and fair if n is even. In fact, the following is also true: if k is odd and $<2^n$, then $k/2^n$ is foul if n is odd and fair if n is even. But now it is easily verified that, although each member of the infinite sequence of partial$_n$ sums corresponding to the convergent sequence 1/2, 1/4, 1/8, . . . is either fair or foul, the *limit* of this sequence of sums cannot be said to be either. Someone might say:

You see, although I said nothing about it when this example first came up, 0 is really fair. Adding the first term in the series to it yields a foul: 1/2.

Adding the next term yields a fair: 3/4, and the next, a foul: 7/8. In general, adding the nth term changes the value of the partial$_n$ sum from fair to foul or foul to fair. We might even say that the value of the series at the nth term is the value of the partial$_n$ sum, for every n. Surely it must follow that the value of the series after *all* the terms must be the value of the sum of the whole series: the limit of the series of partial$_n$ sums. But the sum of the whole series is 1. Is 1 fair or foul? It makes no sense to say of 1 either that it is or that it isn't. I take this to mean that there is no established method for deciding *what* it would be for there to exist an infinite series of rationals.

Of course not. There is no reason to expect that, just because all the *partial*$_n$ sums have a given property ("representing the value of the series at n"), the sum of the *whole* series must have that property. The concept of the value of the series *has not been defined* for series whose sum $\geqslant 1$.

For the lamp, there is, similarly, no reason to expect the sum of the infinite series $+1, -1, +1, -1, \ldots$ to represent the "value" of the lamp after the hypothesized infinite series of switchings. To be sure, every partial$_n$ sum represents its value after n switchings, just as in the fair-foul case every partial$_n$ sum represents the "value" of the series after n terms. But just as the fact that 1 cannot be said to represent a (fair-foul) value does not show that there cannot exist an infinite series of rationals, so the fact that 1/2 cannot be said to represent a (lamp) value does not show that there cannot be an infinite series of lamp switchings. All that *either* shows is that defining the concept of "value" for partial$_n$ sums does not *ipso facto* define it for sums of *infinite* series.

So far as the first two of these attempts to prove the (logical) impossibility of super-tasks are concerned, I think it is clear what went amiss. In each case a super-task was defined. But during the course of the argument a question was asked about what could be described as the result of performing a *super-duper-task*. [If a super-task is a task sequence of order type ω, then a *super-duper task* is the result of tacking an extra (ωth) task at the end of a super-task.] Since the definition of the super-task specifies nothing about such an ωth task, it is no wonder that the question goes begging for an answer. Thomson apparently believes that to describe a super-task would *ipso facto* be to describe some corresponding super-duper-task, that there could be no such thing as a super-task *tout court:* if there were anything so big as a super-task, it would have to be at least one task bigger. This would account for the fact that, in each one of his arguments, although what he describes is only a super-task, the question he asks presupposes that the description given is determinate with respect to what would be the outcome of some corresponding super-duper-task. This is a suggestion worth considering.

What would make such a view appear plausible is the picture we have of running the racecourse. Thomson defines the set Z of points on the course as the following series: 0, 1/2, 3/4, 7/8, ... He argues that, if we consider 0 the starting point and 1 the end point, it is impossible to run through the entire Z-series without reaching a point outside of Z, namely 1. In this case to go from 0 to 1 is, of course, not to perform a super-task. It is to perform the ordinary task of making a continuous run from one point to another. The argument goes:

> . . . suppose someone could have occupied every Z-point without having occupied any point external to Z. Where would he be? Not at any Z-point, for then there would be an unoccupied Z-point to the right. Not, for the same reason, between Z-points. And, ex hypothesi, not at any point external to Z. But these possibilities are exhaustive (10).

If ordered to run so as to occupy every point in the Z-series, we cannot obey without also occupying 1. It is logically impossible to do otherwise. Let us assume that this is right, as it appears to be. But Thomson continues:

> The absurdity of having occupied all the Z-points without having occupied any point external to Z is exactly like the absurdity of having pressed the lamp-switch an infinite number of times (10).

Even supposing that the argument of the racecourse is valid (and we shall return to this), this last point *appears* to be mistaken. If the analogy is exact, then his sentence cries out for completion by a corresponding "without . . ." clause. As I shall try to show, that Thomson thinks there is an exact analogy between these two cases explains what misled him in the lamp argument.

If we complete a super-task in a finite time interval (how else?), there must come a time at which we are no longer performing any task belonging to the super-task. To expire beforehand is to leave some member of the set of tasks undone. Similarly, if we run from 0 to 1, there must come a time at which we are no longer occupying any point in the Z-series. In the latter case, we have two parallel sequences of order type $\omega+1$: the sequence of members of the Z-series, plus 1 at the end, on the one hand, and the corresponding moments of time on the other. In the case of the lamp, we have a sequence of order type ω, the lamp switchings, and a sequence of order type $\omega+1$, the moments at which they take place *plus* the first moment after we're through, which must inexorably come. The passage under discussion indicates that Thomson must believe that, just as we cannot go through all the Z-points without reaching a point outside of Z, the description of the lamp super-task is self-contradictory because it fails to provide an answer to his question about the state of the lamp at the ωth moment, about the outcome of an ωth act had there been one. *But there need not be an ωth act of the relevant kind!* We can, if we please, light up a cigarette or heave a sigh or quietly expire or what have you at the ωth moment. The analogy apparently fails. And the reason why is that, whereas the members of the Z-series are *abstracted* from a presupposed existing set of points (the line 0 to 1 inclusive), the tasks that constitute the super-task are, as it were, generated serially as we need them; there is not even an apparent logical necessity connected with the existence of a task of the relevant kind to fill the ωth spot in the parallel time series, although there *might* seem to be such a necessity concerning the points on the line.

In the racecourse, covering all the Z-points at least partially determines where you are: you cannot cover them all and remain in the Z-set. If the absurdity of performing a super-task is to be exactly like the absurdity of running through all the Z-points without reaching 1, it must be because there must be an ωth task whose performance is *logically* entailed by the performance of the tasks that properly belong to the super-task.[5] The arguments all aim at showing that such a task could not have been performed and, hence, that the super-task whose existence would have entailed its performance could not itself have been performed. But now, let us return to the racecourse argument, for this too is invalid although, admittedly, much less clearly so.

Imagine that the runner has run through all the members of Z. Now Thomson asks: "Where would he be?" Suppose that we answered "Nowhere." Suppose that in fact the runner was none other than Aladdin's genie, that he had been told to occupy all the Z-points and then vanish (without having occupied 1). Would this strain his magical powers to the breaking point? Let us see.

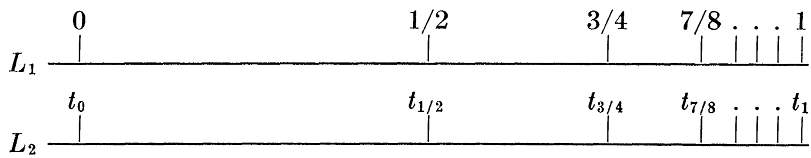
Of course, we cannot refute the view *just* by pointing out that the genie could cease to exist after having occupied all the points in Z, that he needn't be *anywhere*. To be sure, that shows something wrong with *this* argument, but it is open to the quick retort:

[5] I am not attributing to Thomson the explicit belief that if a super-task has been performed then a super-duper-task has been performed: that the performer cannot as a matter of logic put on the brakes in time. I use this merely as a rhetorical device to point up what Thomson *does* appear to believe but fails to argue for, namely, that if super-tasks were impossible then the statement that one had been performed together with a description of the task would logically imply a statement describing the state of the system at the ωth moment with respect to the relevant property. I suppose that another way to put this assumption is to say that to achieve the implied result a super-duper-task *need not* be performed—the super-task will do.

> All right then, at what point does he vanish? For if he vanishes he must vanish *at some point;* there must be some point that is the last point be occupied before vanishing. Call this p. Now, p lies between two members of Z, or else it lies to the right of every member of Z (we shall disregard points to the left of 0 as well as points to the right of 1). If the first, then there is a member of Z to the right of p, which has, therefore, not been occupied. If the second, then $p=1$ and the genie reached 1. But these possibilities are exhaustive (granted our restrictions). Therefore, even if the genie vanished, he either failed to cover all the Z-points or he occupied 1.

After such a crushing rejoinder, we might well be expected to give up, but let us be stubborn. Consider the following case.

Let t_0 be the time at which the genie started from 0, and where applicable, for each i let t_i be the time at which he is at i. The question then becomes: Does this imply that at t_1 he occupies 1? I argue no; my imaginary adversary argues yes. If the genie has carried out my instructions, at t_1 he cannot be at 1, because at t_1 he is no more. To be sure, he vanishes *at* a point: 1. But what does this mean? In particular, does this mean that 1 is *the last point he occupied?* Of course not. There need not be any last point he occupied—any more than there need be a *first* point he *didn't* occupy (although there *must* be one or the other). To disappear at a point is neutral with respect to the question of "having occupied" that point. There is no necessity either way. 'He disappeared at 1' could mean either that 1 is the last point he occupied *or* that 1 is the first point he didn't occupy, just as to have disappeared at t_1 could involve *either* that t_1 was his last moment on earth *or* that t_1 was earth's first moment without him. *Which* we say is a function of how we choose to regard trajectories and time intervals.



To illustrate, we draw two lines L_1 and L_2. These two lines represent, respectively, the racecourse and the corresponding time scale defined above. We may view each line in two different ways, corresponding to the ways in which each point may be seen as dividing its line into two disjoint and jointly exhaustive sets of points: any point may be seen as dividing its line either into (a) the set of points to the right of and including it, and the set of points to the left of it; or into (b) the set of points to the right of it and the set of points to the left of and including it. That is, we may assimilate each point to its right-hand segment (a) or to its left-hand segment (b). Which we choose is entirely arbitrary, but (*modulo* the assumption that any run covers a line segment) it determines how we answer the question: "Given that he disappeared at 1, did he occupy 1?" Those who assimilate each point to its left-hand segment (method b) will say that he did and that t_1 was his last moment on earth. On this account earth had no first moment without him, and there was no first point he didn't occupy. Similarly, if we choose method a, then the genie is said not to have occupied 1, although he disappeared at 1, and there is a first point he didn't occupy and a first moment earth was without him: t_1. What we *cannot* do, given the correspondence we have established between the points on L_1 and L_2, is to regard these lines each in a different way. This would lead us to say, for example, that there was a last point he covered on his trajectory (namely 1), but that he was not on earth at t_1. Since t_1 was defined as the time at which he occupied 1 (if he occupies 1, otherwise the time at which he would have occupied 1, had he done so), this is a contradiction. We are inclined to regard the racecourse (L_1) according to method b, whereas, for each of the two methods, there are circumstances where it is clearly more natural to regard the time series in *that* way rather than in the other. But we are at liberty to view it as we like. Normally it makes no difference, but in *this* case *how* we view it makes the *only* difference. My imaginary opponent assumes that if the genie vanished at a point there must be a *last* point he occupied. This holds only if method b of viewing the line is *mandatory*. We have seen that it is not. But wait:

> You agree that it is natural to regard the race course as I have regarded it, but you claim that it is possible to regard it otherwise. But is it really? Can you describe a case that it would be reasonable to regard as one in which all the points to the left of 1 have been covered, but not 1 itself. If not, then all your argument comes to naught, for a possibility that has no conceivable description is not a real possibility.

I accept the challenge (though not necessarily the implied view). Let me tell you more about our genie:

Ours is a reluctant genie. He shrinks from the thought of reaching 1. In fact, being a rational genie, he shows his repugnance against reaching 1 by shrinking so that the ratio of his height at any point to his height at the beginning of the race is always equal to the ratio of the unrun portion of the course to the whole course. He is full grown at 0, half-shrunk at 1/2; only 1/8 of him is left at 7/8, etc. His instructions are to continue in this way *and* to disappear at 1. Clearly, now, he occupied every point to the left of 1 (I can tell you exactly when and how tall he was at that point), but he did not occupy 1 (if he followed instructions, there was nothing left of him at 1). Of course, if we *must* say that he vanished at a point, it must be at 1 that we must say that he vanished, but in this case, there is no temptation whatever to say that he occupied 1. He couldn't have. There wasn't enough left of him. Note that it does not follow from the description of the shrinking alone that he never occupies 1. We could describe his shrinking by saying quite generally (this is not quite general since it gives his height only at the rational points) that he is $(1/2^n)$th his original size when he is at $(2^n-1)/2^n$. From this nothing follows about his size *at* 1. This is perfectly consistent with his appearing at 1 full blown. If furthermore, he is instructed to *vanish* at 1, then indeed he will, for he is obedient (and also very reluctant to reach 1). The difference between this case and the vanishing genie as originally described (before we said anything about his shrinking) is that, in the former case, it would appear to take some effort on his part to vanish at 1, and we might be reluctant to think that he can do it, whereas here, having got started downhill, as it were, it would take quite an effort to reappear full blown at 1. But, of course, although this difference explains our reluctance in each case to use a particular description, in neither case does it make it impossible to use the "less natural" description.

So, even if he vanished at 1, he need not have occupied 1. Therefore, he could have occupied every Z-point without occupying any point external to Z.

To recapitulate, then. I have argued in this section that Thomson's arguments fail to establish the logical impossibility of super-tasks and that what misleads him in each case is what he takes to be an analogy between performing a super-task and running the racecourse. He feels that, in the latter case, you cannot occupy every Z-point without occupying some ωth point, not in Z —that this is a logical impossibility. Similarly, every putative proof of the impossibility of super-tasks takes the following form:

1. Assume that a super-task has been performed.
2. Consider what happens at the ωth moment:
   a. In the case of the lamp: Is it on or off?
      By one argument, it can't be either, but it *must* be one or the other; by the other argument, it would have to be half-on and half-off, which presumably it can't be.
   b. In poor Bosco's case: Does '0' or '1' appear on his dial?
      Supposedly *neither* can, but one or the other must.

It has been my contention that the analogy Thomson claims to find does not exist, but that another one *does;* he has failed to establish the impossibility in question even in the case of the racecourse.

The reason why his argument fails in the *latter* case is that he does not show that to occupy all the points in an infinite convergent series of points *logically* entails occupying the limit point. *For all that he has said,* it is perfectly possible to cross over into another dimension or pass into Genieland or simply cease to exist. It has not been shown that the existential compulsion we feel which drags us from one moment to the next is a *logical* one—but *this too* must be shown in proving that to occupy all the Z-points, the runner must occupy a point outside of Z.

The reason why the analogy between super-tasks and the racecourse might seem not to hold is this: even if we viewed the race in a manner more appropriate (though I insist, not mandatory) to the nonshrinking Genie, i.e., even if we said that in disappearing at 1 he had to occupy 1, there would still be no corresponding necessity concerning the state of the lamp or the state of Bosco. In the racecourse we are dealing with a sequence of points abstracted from a continuum of points. The limit point exists, and we have a choice of saying that the genie occupied it or that he didn't. In the case of super-tasks, there is no assumption of an underlying continuum. The sequence of *points* has a limit: 1. But what reason is there to suppose that the sequence of *tasks* "has a limit," that a task of the corresponding kind is performed at the ωth moment, turning our ordinary super-task into a super-duper-task? None. So, it does not follow that "the absurdity of having occupied all the Z-points without having occupied any point external to Z is exactly like the absurdity of having pressed the lamp-switch an infinite number of times," except possibly vacuously.

II

What conclusions are we to draw from this rather heady mixture of genies, machines, lamps, and fair and foul numbers? In particular, has it been shown that super-tasks are really possible —that, in Russell's words, they are at most *medically* and not logically impossible? Of course not. In a part of his paper that I did not discuss, Thomson does a nice job of destroying the arguments of those who claim to prove that super-tasks are logically possible; had there been time I should have examined them. In the preceding section I tried to do the same for Thomson's own neo-Eleatic arguments. I think it should be clear that, just as Thomson did not establish the impossibility of super-tasks by destroying the arguments of their defenders, I did not establish their *possibility* by destroying his (supposing that I did destroy them).

I am not quite sure what constitutes proving that something is logically possible. I think I *do* know, at least in part, what it is to prove that something is logically *im*possible. It is this: if we call an *explicit contradiction* the conjunction of some statement with its negation, then to prove that some statement S is self-contradictory—that what it asserts to be the case is logically impossible—is to prove that S logically implies an explicit contradiction. This is, if you like, a simple-minded but roughly accurate account of disproof by *reductio ad absurdum.* Even a cursory examination of Thomson's arguments will show that this is precisely what he sets out to do for the statement that a super-task has been performed. We have seen that in each case the arguments were invalid, that they required for their validation the addition of a premise connecting the state of the machine or lamp or what have you at the ωth moment with its state at some previous instant or set of instants. The clearest example is that of the lamp, where we can derive a contradiction only by explicitly assuming as an additional premise that a statement describing the state of the lamp (with respect to being on or off) *after* all the switchings is a *logical* consequence of the statements describing its state during the performance of the super-task.

But consider to the following dialogue:

He: Well, if that's all we need, why not add it as a new premise? Then we'll have a valid argument with S (the statement we're trying to prove contradictory) as premise and an explicit contradiction as conclusion. Isn't that what we want?

I: No, it isn't. We need more. That would show only that the conjunction of S with our new premise is self-contradictory. That's not much help, since the fault may lie with the new premise or, for that matter, with neither in particular but only with their (possibly illicit) logical relation to each other.

He: But suppose the new premise is *true*. We've validly derived a contradiction from S by assuming only true auxiliary premises. Surely *that* proves S to be self-contradictory!

I: No. Let S be 'The hat is on the cat,' and suppose that in fact the cat is hatless. Then S in conjunction with the true statement, 'The hat isn't on the cat', implies a contradiction.

He: All right, then. Suppose that this additional premise is not only true, but *obviously* true. Isn't S contradictory if in conjunction with obvious truths it logically implies an explicit contradiction? And it's obviously true that the state of the lamp at t is a consequence of its state before t.

I: No. At best this would show that it was obviously impossible to perform the particular super-task in question. But I grant at the outset that it's obviously impossible. What right thinking man would not? For one thing, the parts would soon be worn to a frazzle—as would Aladdin and Bernard.

He: So its being obviously true won't do. Suppose I add the requirement that the additional premise be necessary. Wouldn't something that implies an explicit contradiction when conjoined with necessary premises *have* to be self-contradictory?

I: Again no. It would have to be self-contradictory only if the necessity of the premise involved were *logical* necessity, only if the negation of the auxiliary premise were itself self-contradictory. (This is of course not a vicious circle, nor even a circle at all, since it is possible to establish some statements as self-contradictory without appeal to any auxiliary assumptions whatever.) The only auxiliary premises permitted in a demonstration that a statement is self-contradictory are *analytic* ones—ones true by virtue of the meanings of their constituent parts.

He: If the additional premise concerning the state of the lamp is analytic, then Thomson has in fact succeeded in showing that at least *this* super-task is a logical impossibility. All your counters have gone to naught.

I: No, if the suppressed premise is analytic, then indeed the statement that the lamp has been switched on and off an infinite number of times is self-contradictory—*but Thomson hasn't shown it.* To achieve his Eleatic purpose, he must not only derive an explicit contradiction from the statement that a super-task has been performed, using only analytic auxiliary assumptions along the way, but he must also *show* that his assumptions *are* analytic. Were this not so, and were he right about the self-contradictoriness of the concept of super-task, then he could make a mockery of the proof by presenting the following one:

a. A super-task has been performed.
b. No super-task has been performed.

Therefore: *c.* $p \cdot {\sim} p$

Ex hypothesi (for us) *a* is self-contradictory. *b* is therefore analytic, and *c* follows from the conjunction of *a* and *b*. This would be too easy.

In the above dialogue I try to give a rough idea of what I think is missing from Thomson's "proofs." A "swindle" has taken place, and we have been the victims. Somehow, all was going along swimmingly, and suddenly we find ourselves drowning in contradiction with no idea of how we got there. We are told that the concept of a super-task is to blame, but we are not told *what* about it has such dire consequences. We are sufficiently sophisticated mathematically to know that the concept of infinity is not at fault (or if it is, a lot more than the future of super-tasks is at stake). But what then? What *could* he do that he has failed to do?

I said that he had to *show* that the auxiliary premises were analytic. But how does one show that? How does one *show* that a statement owes its truth solely to meanings (if that is the same question)? Obviously there are problems. The key issues in the philosophy of language are involved in the discussion of this point. Perhaps there is no answer to my question. Perhaps the concepts involved are too confused. If so, then so much the worse for Mr. Thomson and his co-neo-Eleatics, for I think that their only hope lies here. If analyticity construed as truth by virtue of meanings collapses, then so does the enterprise of showing that the concept of super-task is self-contradictory—for that is merely the other side of the same coin. So I will assume that something sensible can be said about this.

To show that the concept of super-task is self-contradictory, it must be shown that there is something self-contradictory in the concept of a completed infinite series of tasks. There are three possibilities. The first, which I discard at the outset in *this* discussion, although it is an interesting question in its own right, is that the concept of the infinite is itself self-contradictory. The second, which I also discard, for obvious reasons, is that the concept of a completed sequence of tasks is by itself self-contradictory. The last is that somehow the conjunction of the two has this property, whereas neither has it separately. This is the most promising. In order to show this, it would suffice, for example, to show that it is part of the meaning of 'task' that nothing can be called a task that does not take some time to perform *and* that there is a lower bound on the length of time allowable for the performance of a single task.

Similarly, to show that super-tasks are not logically impossible, it would suffice to show that a correct analysis of each of the concepts involved permits their conjunction without explicit contradiction. In defense of Thomson, his arguments would have sufficed for his purposes had they been valid. But they were not. In each case, they needed supplementation with an additional premise. This should make us suspect that there exists no such easy proof that super-tasks are a logical impossibility—just as there is no easy proof that they are logically possible. In fact, I strongly suspect that whatever conditions a proper analysis would associate with the concept of a completed series of tasks would fail to preclude the series' being infinite. And furthermore, there is probably no set of conditions that we can (nontrivially) state and show to be includable in a correct statement of the meaning of the expressions in question whose satisfaction would lead us to conclude that a super-task had been performed. I don't mean that we don't understand the meaning of the expression 'completed infinite sequence of tasks'. We obviously do, provided that we grasp its syntactic structure and that we understand the meanings of its component parts. I mean only that there is no circumstance that we could imagine and describe in which we would be justified in saying that an infinite sequence of tasks had been completed. It is probably this fact that accounts for Thomson's vacillation (cf. my footnote 3) between the conclusion that super-tasks are logically impossible and that "the concept of super-task has not been *explained*" (6), "that talk of super-tasks is *senseless*" (9). Indeed this is a peculiar state of affairs, but similar cases are not hard to find. A thinking robot is such a case. We know much about what it is to think, and we know much about what robots are, but we are not able to describe something that we should be justified in calling a thinking robot. Similarly, there is much we can say about tasks and about infinite sequences, but there is nothing we can describe that it would be reasonable to call a completed infinite sequence of tasks. Possibly someday someone will find such a description. I am not arguing that it could never be done. I am only pointing here to what I take to be a difference between a concept like that of a super-task and that of a man eight feet tall. We may never have seen an instance of the latter, just as presumably we have never witnessed an instance of the former. But the one strains our imagination to the breaking point, whereas the other does not. This is an important difference, but it does not show either that the concept in question is self-contradictory or that it is no concept at all, that the expression is without meaning.

I want to insist here that logic has not stepped in just because our imaginations fail us; something is not logically impossible just because we cannot imagine what it would be like for it to be the case. Our shrinking genie is very much to the point. If he is of any clarificatory value at all, it is insofar as we *recognize* that he covers all the Z-points but fails to cover 1, although, before he was described, *we might not have been able to think of anything that would meet that condition.*[6] To describe such a set of circumstances for the first time is what some have misleadingly called "giving an expression a sense" or "giving it a meaning" or "giving it a use," where the implication is

[6] The example is, to be sure, far-fetched, imaginary, what you like. But we are talking about language, and it is important to remember than language is an instrument rich enough to describe the far-fetched and fanciful.

that it had no sense or meaning or use before this one was conferred upon it. This is mistaken. The expression has a meaning, a sense, a use. Possibly it didn't have *this* sense or (what is not the same) was not put to *this* use before. Possibly, even, its meaning has changed to some degree—although that would be more drastic. But the picture of a meaningless, senseless, useless phrase that we now endow with a use (and, therefore, possibly a sense and meaning) is a mistaken one. We see that it is mistaken when we see that we have recognized that the shrinking genie covers all the Z-points but fails to occupy 1. For how could we recognize this if (*a*) Thomson were right and this was a contradictory notion, or (*b*) my hypothetical opponents (and Thomson's *alter ego*) were right and the expression 'covers all the Z-points but fails to occupy 1' had no sense, meaning, or use? I submit that we have not given it something it had little or none of before: meaning. Rather we have exploited the meaning that it had. I did not *stipulate* that the genie should count as a case in point. I merely described him and then argued that he *does* so count—and my arguments took the form of linking the features of the case described with the conditions associated with 'covering all the Z-points and failing to occupy 1'. It is by virtue of the meaning of that expression that the shrinking genie can be said to have covered all the Z-points but not occupied 1. Had I described him differently, e.g., had I said that his height at any rational) point $(2^n-1)/2^n$ on the course is $(1/2^n)+\epsilon$ times his original height (where ϵ is some arbitrarily small quantity), that he should continue shrinking at the same rate after he shrinks to a height of ϵ, and that he should vanish when he reaches a height of $(1/2)\epsilon$, then it would have been clear that he occupied 1—no matter how small one chose ϵ (his run is a continuous one, so he cannot skip 1).

I suspect that, by and large, it is principally compound expressions that suffer the fate I attribute to 'completed infinite sequence of tasks' and 'thinking robot'. What seems most notable about such compounds is the fact that one component (e.g., 'infinite sequence') draws the conditions connected with its applicability from an area so disparate from that associated with the other components that the criteria normally employed fail to apply. We have what appears to be a conceptual mismatch. Sequences of *tasks* do not exhibit the characteristics of sequences that lend themselves to proofs of infinity. And since there seems to be an upper bound on our ability to discriminate (intervals, say) and none on how finely we cut the task, it appears that we should never be in a position to claim that a super-task had been performed. But even if this is true, it only takes account of one kind of super-task, and, as I argue above, it hardly establishes that even this kind constitutes a logical impossibility.

To look at the matter diachronically and therefore, I think, a little more soundly, we can see our present situation as akin to that of speakers of English long before electronic computers of the degree of complexity presently commonplace when confronted with the question of thinking robots (or, for that matter, just plain thoughtless robots, I suspect). They were as unthinkable as thinking stones. Now they are much less so. I am not sure that even then they constituted a logical contradiction. However, I would not resist as violently an account which implied that the expression 'thinking robot' had changed in meaning to some degree in the interim. Viewed as I suggest we view them, questions of meaning are very much questions of degree—in the sense that although relative to one statement of meaning there may be a more or less sharp boundary established, no statement of meaning (viewing things synchronically now) is uniquely correct. Other hypotheses, and therefore other lines may be just as reasonable in the light of the evidence. The statement of the meaning of a word is a hypothesis designed to explain a welter of linguistic facts—and it is a commonplace that where hypotheses are in question many are always possible.

Therefore, I see two obstacles in the way of showing that super-tasks are logically impossible. The first is that relevant conditions associated with the words and the syntactic structure involved must be found to have been deviated from; *and* it must be argued that these conditions are sufficiently central to be included in any reasonable account of the meaning of the expression. The second is simply my empirical conjecture that there are no such conditions: that in fact the concept of super-task is of the kind I have been describing above, one suffering from the infirmity of *mismatched* conditions. If this is right it would go a long way toward explaining why Thomson is so successful in showing that arguments *for* the performability of super-tasks are invalid and why nevertheless his own arguments against their possibility suffer the same fate. The modern Eleatics, although faced with an easier task than that which faced Zeno (people aren't performing super-tasks right and left), have yet to perform it.

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