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The White Shoe is a Red Herring

HEMPEL's paradox of confirmation can be worded thus 'A case of a hypothesis supports the hypothesis. Now the hypothesis that all crows are black is logically equivalent to the contrapositive that all non-black things are non-crows, and this is supported by the observation of a white shoe.'

The literature of the paradox is large and I have myself contributed to it twice.[1] The first contribution contained an error, but I think the second one gave a complete resolution. The main conclusion was that it is simply not true that a 'case of a hypothesis' necessarily supports the hypothesis; and an explanation was also given for why it seems to be true.

In the present note we show in six sentences, and even without reference to the contrapositive, that a case of a hypothesis does not necessarily support it.

Suppose that we know we are in one or other of two worlds, and the hypothesis, H, under consideration is that all the crows in our world are black. We know in advance that in one world there are a hundred black crows, no crows that are not black, and a million other birds; and that in the other world there are a thousand black crows, one white one, and a million other birds. A bird is selected equiprobably at random from all the birds in our world. It turns out to be a black crow. This is strong evidence (a Bayes-Jeffreys-Turing factor [2] of about 10) that we are in the second world, wherein not all crows are black. Thus the observation of a black crow, in the circumstances described, undermines the hypothesis that all the crows in our world are black. Thus the initial premise of the paradox of confirmation is false, and no reference to the contrapositive is required.

In order to understand why it is that a case of a hypothesis seems to support it, note that

$$W(H : \text{Black}|\text{Crow}) > 0,$$

where $W(H : E|G)$, the weight of evidence, support, or log-factor, for H provided by E given G, is the logarithm of the Bayes-Jeffreys-Turing factor, $P(E|G \text{ and } H)/P(E|G \text{ and not } H)$. The above inequality is clear from the fact that $P(\text{Black}|H \text{ and Crow}) = 1$, and a similar inequality will follow for all other explicata of corroboration.[3] On the other hand $W(H : \text{Crow}|\text{Black})$ can be negative.

It is formally interesting to recall that

$$W(H : \text{Black Crow}) = W(H : \text{Crow}) + W(H : \text{Black}|\text{Crow}),$$

and that only the last of these three terms needs to be positive. The first two terms can both be negative.

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[1] This *Journal*, 1960, **11**, 145-149; and **12**, 63-64

[2] See either the first reference in note 1, or my *Probability and the Weighing of Evidence* (London, Griffin, 1950), chap. 6

[3] *Journal Roy. Statist. Soc.* **B, 22** (1960), 319-331; and K. Popper, *The Logic of Scientific Discovery* (London, Hutchinson, 1959), Appendix ix