5

Evolution in Lewis Signaling Games

"The emergence of meaning is a moral certainty"

Brian Skyrms, *Evolution of the Social Contract*

"Something is morally certain if its probability comes so close to complete certainty that the difference cannot be perceived."

Jacob Bernoulli, *The Art of Conjecture*

That was the bold claim I made in 1996 about the evolution of signaling systems. Signaling systems had been shown to be the only evolutionarily stable strategies in n-state, n-signal, (and here) n-act signaling games. They were the only attractors in the replicator dynamics. In simple cases, like those discussed in Chapter 1, it was clear why almost every possible starting point was carried to a signaling system. How far do these positive results generalize?

The good news

Consider the two-state, two-signal, two-act, signaling game where nature chooses the states with equal probability. In Chapter 1, we restricted the strategies to those that might be used by those who have signaling in mind. The sender sent a different signal in each state. The receiver picked a different act for each signal. They knew at the onset that states and signals were important, they just hadn't settled on a signaling system. This is making things too easy. Let's put in all possible strategies.

Senders now have two additional strategies: Always send signal 1, always send signal 2. Receivers also have two additional strategies: Always do act 1, always do act 2.

The sender's strategies ignore the state and the receiver's strategies ignore the signal. Why not? We may have a population of senders and a population of receivers. In this case there are four possible strategies represented in each population. Alternatively, there may be a single population where an individual is sometimes in the role of sender and sometimes in the role of receiver. A strategy for an individual specifies what to do when in the role of sender and what to do in the role of receiver. There are 16 possible strategies. What happens?

Everything still works fine. Signaling always evolves, both in one-population and two-population contexts. We can't draw pictures with all the strategies included, but it is still possible to establish that almost every initial point is carried to a signaling system.[1] It can be shown that average payoff increases along every trajectory of the dynamics. Then there can't be cycles like those in rock-scissors-paper. Evolutionary dynamics has to go to an equilibrium. But there are lots of new equilibria when we include all strategies. Notably, there are *pooling equilibria*, in which the sender ignores the state and the receiver ignores the signal. However, it can be shown that all the equilibria other than signaling systems are dynamically unstable. Evolution won't hit them. There are no pictures, but the story is just like that in Chapter 1.

Bad news: states with unequal probabilities

The foregoing is in the context where nature chooses states with equal probability. That is the simplest case, but there is no reason

[1] Huttegger 2007a; Hofbauer and Huttegger 2008.

why nature may not choose states with unequal probability: 60%–40%, 90%–10%, or 99%–1%. Then the pooling equilibria take the form where senders transmit no information and receivers ignore the signal and always do the act suited to the most likely state.

If the more likely state is *very* likely, players in such an equilibrium may do quite well. We can no longer make the case that the mutant signalers will do as well against the natives as the natives do against each other. If both signals are sent at random (but ignored by receivers) in the native population, then mutants pursuing a signaling system strategy will be led to do the wrong act half the time, when they receive a native's signal. They will do perfectly against each other, but most of their interactions are with natives. So they make lots of mistakes, while the natives usually do the right thing. They will do worse than the natives.

For a two-population setting, consider a case where state 1 occurs 90% of the time and state 2 10%. Then a receiver who *always does act 1*, no matter what the signal, gains average payoff of .9. He does the right act for the state 90% of the time and misses 10% of the time. So he does reasonably well without any information transmission. Consider such a population of receivers paired with a polymorphic population of senders, half of whom always send signal 1 and half of whom always send signal 2. Everyone gets an average payoff of .9. Introduce a few senders who discriminate states, and they will do no better and no worse than the natives. But if we introduce a few receivers who discriminate between signals to coordinate with the few senders, they will do very badly against the natives. Against the natives they will get an average payoff of only .5. That was good enough to get a foot in the door when the states were equiprobable and the natives were making .5, but it is not good enough when the states are not equiprobable. Now evolutionary dynamics will sometimes hit signaling systems and sometimes hit pooling equilibria, with the likelihood of the latter increasing with the disparity in probability between the states. The bottom line in both the one- and two-population cases is that *evolution of signaling is no longer guaranteed*. How serious is this problem?

Evolution can lead to pooling equilibria *where no information is transmitted* whenever states have unequal probability. It can also lead to signaling systems. It is more likely that we get pooling the larger the disparity in probabilities of the states, but the impact on the welfare of the players is smaller.

Some good news

Our pooling equilibria, where no information is transferred are characterized by (i) the receivers ignoring the signal and always doing the right thing for the most probable state and senders ignoring the state, either by (a) always sending signal 1 or (b) always sending signal 2. Any mix of senders of types (a) and (b) gives us a pooling equilibrium. Thus there is a *line* of such equilibria, corresponding to the proportion of the two types of sender. The endpoints, representing all one type of sender or all the other type, are unstable. Each endpoint can be destabilized by a few signaling system mutants, of an appropriate kind. But evolution can lead to any of the other points corresponding to a mixed population of different types of senders.

A line of equilibria is *structurally unstable*, like the concentric orbits in the rock-scissors-paper example of the last chapter. A small change in the dynamics can make a big change in the set of equilibria. So far the dynamics have been pure differential reproduction. We can modify the dynamics a little bit by putting in a little natural variation in the form of mutation.

The analysis for two populations has been carried out by Josef Hofbauer and Simon Huttegger. The replicator dynamics is replaced with its natural generalization, the *replicator-mutator* dynamics.[2] Each generation reproduces according to replicator dynamics but ($1-\varepsilon$) of the progeny of each type breed true and ε of the progeny mutate to all types with equal probability. (Self-mutation

[2] Hadeler 1981; Hofbauer 1985.

is allowed.) Taking the continuous time limit gives the replicator-mutator dynamics.

A little uniform mutation (no matter how little) collapses the line of pooling equilibria to a single point. (This is intuitively reasonable. If the receivers are disregarding the signals, there is no selection pressure on the senders. If one type of sender, (a) or (b), is more numerous, more mutate out than mutate in.) The big question concerns the character of this one point. *Is it an attractor* that pulls nearby states to it? *Is it dynamically unstable*, so that for all practical purposes we needn't worry about it?

It depends. For states whose probabilities are not too unequal, this pooling point is unstable. Then our original positive result is restored. Signaling always evolves! That's the good news. But for when one state is much more probable than the other, the pooling point is an attractor. Signaling sometimes evolves, sometimes not. That's the not so good news. For equal and small mutation rates for both senders and receivers, Hofbauer and Huttegger calculate the probability where the switch takes place.[3] It is between .78 and .79.

That's not too bad. Up to probability 3/4, a little mutation assures that almost all initial points evolve to signaling systems. Things are even more favorable, if the receivers have a higher mutation rate than the senders. If receivers experiment twice as often as senders, paradise is regained. The bad equilibrium with no information transfer is always dynamically unstable, for any (positive) state probabilities. But we cannot assume that such favorable mutation rates are always in place.

In addition, we should notice that these are results for payoffs that are all 0 for failures and all 1 for successes. For very infrequent states where the payoffs are much more important—such as the presence of a predator—the disparity in payoffs can balance the disparity in probabilities. Predators may be rare, but it does not pay to disregard them.

This consideration can restore almost sure evolution of signaling for rare events.

[3] Technically, this is called a "bifurcation."

More bad news: partial pooling

What happens when we move to three states, three signals, and three acts? We go back to the favorable assumption that all states are chosen with equal probability. Nevertheless, a whole new class of equilibria appears. Suppose that a sender sends signal 1 in both states 1 and 2, and in state 3 sends either signal 2 or 3 with probabilities x and (1–x) respectively. And suppose that the receiver, on getting signals 2 or 3 always does act 3, but on getting signal 1 does either act 1 or act 2 with probabilities y and (1-y) respectively. This is shown in figure 5.1

For any combination of values of x and y as population proportions, including 0 and 1, we have a population state that is a dynamic equilibrium. We thus have an infinite set of equilibrium components. Considering x going from 0 to 1 and y going from 0 to 1, we can visualize this set as a square of equilibria. These equilibria pool states 1 and 2 together, but do not pool all states together—so they are called *partial pooling* equilibria.[4] Because information is imperfectly transmitted, sender and receiver succeed 2/3 of the time. In comparison, total pooling would give a payoff of only 1/3, and perfect signaling would give a payoff of 1.

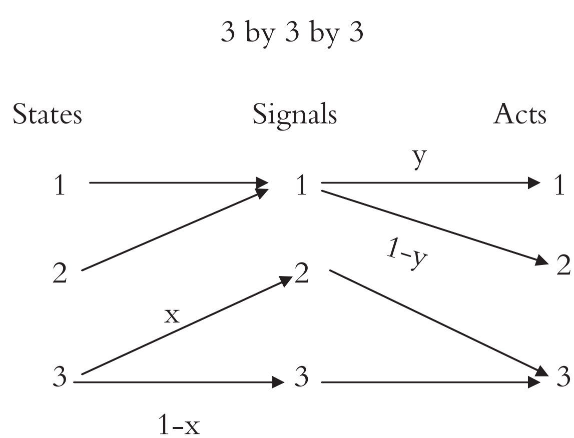

Figure 5.1: Partial pooling equilibria.

[4] There is likewise a square of partial pooling equilibria that lumps states 2 and 3 together, and one that pools states 1 and 3.

In total pooling equilibria, where all states are lumped together, no information is transmitted. In partial pooling equilibria, some information is transmitted, but not as much as would be in a signaling system.

If we run simulations of evolutionary dynamics in 3 state, 3 signal, 3 act Lewis signaling games with equiprobable states, we never observe total pooling equilibria, but we do see partial pooling between 4% and 5% of the time.[5] How is this possible? Are these simulations to be trusted?

There are four possible pairs of pure populations corresponding to values of 0 or 1 for x and y. Each of these population states is a dynamically unstable equilibrium.[6] But mixed populations, corresponding to non-extreme values of x and y, are all stable equilibria. Notice that in any of these states, signaling-system invaders would do worse against the natives than the natives do against themselves. Likewise for any other invaders. You can go through all of the other possible other sender and receiver strategies, and none of them do as well against a mixed pooling population as the poolers do against themselves. If you are close enough to the interior of the plane of partial pooling equilibria, the dynamics will lead you right into it. The simulations were a reliable guide. A non-trivial set of population proportions evolves by replicator dynamics to partial pooling rather than signaling systems.[7] In a perfectly ordinary Lewis signaling game, evolution can sometimes spontaneously create the synonyms and information bottlenecks that we artificially postulated in Chapter 1![8]

[5] Simulations using discrete time replicator dynamics by Kevin Zollman led to partial pooling in 4.7% of the trials, and to signaling systems the rest of the time.

[6] The instability stems from the fact that if a small number of senders and receivers that form the right signaling system were added they would out-compete the natives. They would do equally well against the natives, but better against each other. But each of these partial-pooling type populations requires a different signaling system to destabilize it, and each of these signaling systems does badly against the other type of partial-pooling.

[7] There are proofs of this in Huttegger 2007a and in Pawlowitsch 2008.

[8] Signals 2 and 3 function as synonyms, leaving only one signal for the remaining two states and two acts.

Mutation one more time

The set of partial pooling equilibria in the foregoing discussion is again an indication of *structural instability*. As before, let us try a little mutation. It is hard to do a full analysis of this game, but indications are that a little mutation destroys partial pooling and always gets us signaling. Partial pooling squares collapse to single points and move a little bit inward to accommodate a few mutants of other types. Although these equilibria of partial information transfer survive, they are dynamically unstable. Perturbed signaling systems, in contrast, are asymptotically stable attractors. Simulations using discrete-time replicator-mutator dynamics with both 1% and 0.1% mutation rates found that the system *always* converged to a (perturbed) signaling system equilibrium.

Correlation

In the last chapter, assortment of encounters made a cameo appearance. Assortment of encounters—that is, positive correlation of types in encounters—plays the major role in explanations of the evolution of altruism. Altruism, modeled as cooperation in the Prisoner's Dilemma, cannot evolve with random pairing. But it can when there is sufficient positive correlation of types, so that cooperators tend to meet cooperators and defectors tend to meet defectors.[9] Mechanisms exist in nature to promote an assortment of encounters. There is no reason to believe that they should operate only in Prisoner's Dilemma situations.

They can make a difference in evolution of signaling. Let us go back to a Lewis signaling game with two states, two signals, and two acts, where nature chooses state 1 with probability .2 and state 2 with

[9] See Hamilton 1964; Skyrms 1996; Bergstrom 2002.

probability .8. Here we consider a one-population model, in which nature assigns roles of sender or receiver on flip of a fair coin. We focus on four strategies, written as a vector whose components are: signal sent in state 1, signal sent in state 2, act done after signal 1, act done after signal 2.

$$s1 = \langle 1, 2, 1, 2 \rangle$$
$$s2 = \langle 2, 1, 2, 1 \rangle$$
$$s3 = \langle 1, 1, 2, 2 \rangle$$
$$s4 = \langle 2, 2, 2, 2 \rangle$$

The first two strategies are signaling systems, the others are pooling strategies. (Other strategies neglected here are losers that rapidly go extinct.)

Consider the following model of assortment (due originally to Sewall Wright):

$$\text{Probability}(s_i \text{ meets } s_i) = p(s_i) + e[1 - p(s_i)]$$

$$\text{Probability}(s_i \text{ meets different } s_j) = p(s_j) - e\ p(s_j)$$

where p denotes population proportion. The probability of encountering your own type is augmented and that of encountering a different type is decremented. If e=1, assortment is perfect; if e=0 encounters are random.

Now consider the point, z, in the line of pooling equilibria where p(s3)=p(s4)=.5.

This point is stable. (It is, in fact, the point on the line with strongest resistance to invasion by signalers.) We feed in assortment. Between e=.4 and e=.5, z changes from being stable to unstable. This happens at about e=.45. If probabilities of states are more unequal, it takes greater correlation to destabilize pooling and guarantee the evolution of signaling. This is shown in figure 5.2. But if neither state is certain, there is always some degree of correlation that will do the trick.

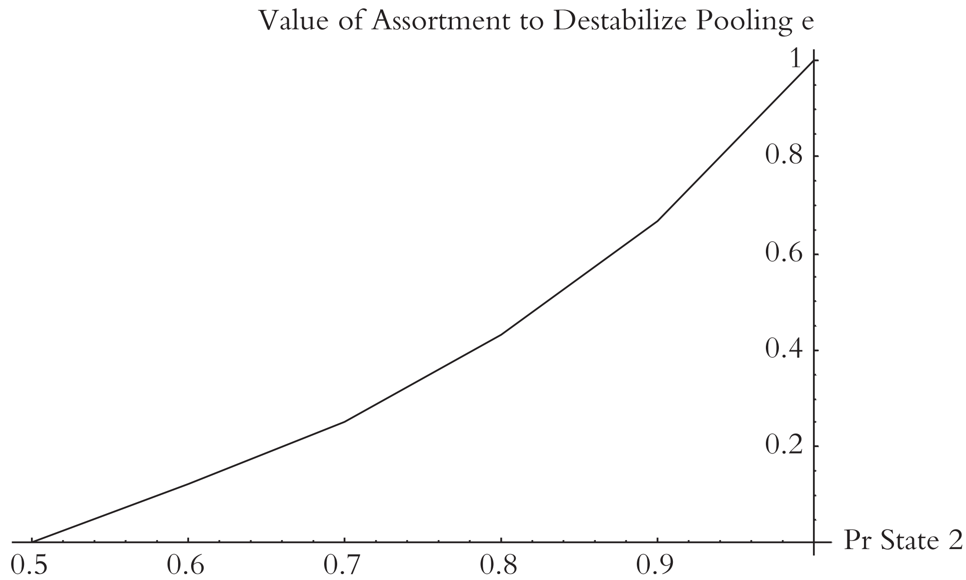

Figure 5.2: Assortment destabilizes pooling.

This shows the power of correlation in the abstract. It remains to investigate the effect of specific correlation devices on the evolution of signaling.[10]

Eating crow

Even after all the good news is in, there remains a real possibility of evolution falling short of a signaling system. The emergence of a signaling system is not always a moral certainty. I was wrong. But signaling can still often emerge spontaneously, even though perfect signaling is not guaranteed to always emerge. Democritus is still right, but we can begin to see the nuance in how he is right.

[10] One correlation mechanism found widely in nature is local interaction in space, or in some social network structure. Wagner 2009 shows how network topology influences evolution of signaling systems.