

Phil 5312
Fall 2024

Assignment 6:

Read MacFarlane chapter 4 sections 4.1 - 4.3. Also, read Thomson, Edgington, and Stalnaker. Then do the exercises below. Answers should be uploaded into Blackboard by Monday, Nov 20th.

Part 1: The material conditional

Say whether the sentence is a logical truth, a contradiction, or neither.

1. $A \supset \neg A$
2. $(A \supset B) \supset \neg(A \supset \neg B)$
3. $\neg(A \supset B) \supset (A \supset \neg B)$
4. $(A \supset B) \wedge (A \supset \neg B)$
5. $(A \supset B) \vee (A \supset \neg B)$
6. $(A \supset B) \vee (\neg A \supset B)$
7. $(A \supset B) \vee \neg(A \supset B)$
8. $A \supset (B \supset A)$
9. $(A \supset B) \supset A$
10. $\neg(A \supset B) \supset A$

For each of the following, say whether the argument is valid or not.

11. $(A \supset B)$ and $(C \supset A)$ so therefore $C \supset B$
12. $(A \supset B)$ and $(A \supset C)$ so therefore $C \supset B$
13. $(A \supset B)$ and $(B \supset C)$ so therefore $A \supset C$
14. $(A \supset B)$ and $(C \supset B)$ so therefore $A \supset C$
15. $A \supset (B \supset C)$ and $A \supset \sim C$ so therefore $A \supset \sim B$
16. $A \supset (B \supset C)$ and B so therefore $A \supset C$
17. $A \supset (B \supset C)$ and $B \supset D$ so therefore $A \supset (D \supset C)$
18. $A \supset (B \supset C)$ and $D \supset B$ so therefore $A \supset (D \supset C)$
19. $(A \supset B) \vee (A \supset C)$ so therefore $A \supset (B \vee C)$
20. $(A \supset C) \vee (B \supset C)$ so therefore $(A \vee B) \supset C$
21. $A \supset B$ therefore $(A \wedge C) \supset B$
22. $A \supset B$ therefore $A \supset (B \wedge C)$
23. $(A \wedge C) \supset B$ therefore $A \supset B$
24. $A \supset (B \wedge C)$ therefore $A \supset B$
25. $A \supset B$ therefore $(A \vee C) \supset B$
26. $A \supset B$ therefore $A \supset (B \vee C)$
27. $(A \vee C) \supset B$ therefore $A \supset B$
28. $A \supset (B \vee C)$ therefore $A \supset B$

Please do Part I first (this week) and when you have completed it, check your answers against the answer key in Blackboard. If you got any wrong, spend the time to make sure you understand the particular problem. Build a truth table or try to do a proof until you can convince yourself of the right answer.

Part 2: Material Conditional semantics

After completing Part I and reading the answer guide, do the following problems:

1. Produce a material conditional sentence (main connective ' \supset ') which is a contradiction. Explain why it is a contradiction. What particular features does this sentence have? (I believe if you produce one you will know what features I mean. If you have an example and don't understand the question, come talk to me). Is it possible for a material conditional to be a contradiction without possessing those features? Either give an example or give an argument there can't be one.
2. Prove that if $Ag \models Ap$ and $Cp \models Cg$ then $Ap \supset Cp \models Ag \supset Cg$

Note that Ap etc. are sentence variables and not individual sentences themselves (so not necessarily atomic). You may assume these are propositional sentences if you wish so you can work entirely with truth value assignments. To answer this question, you will need to manipulate two definitions:

- Def a) $p \models q$ if there is no TVA (ν) where p is true and q is false.
Def b) $\nu(p \supset q)$ is true iff $\nu(p)$ is false or $\nu(q)$ is true.

Part 3: Reasoning with conditionals

There are three defendants – A, B, and C – and the following facts are known:

1. If A is innocent, then both B and C are guilty.
2. If either A or B is guilty, then C is also guilty.
3. If B is guilty, then both A and C are innocent.

Note that (before examining the problem) you do not know how many of these defendants are guilty; it may be 0, 1, 2, or all 3. Who is innocent and who is guilty? Explain your answer (reasoning in English). Also prove that your answer is correct with a natural deduction.

Part 4: Gibbardian stand-offs

Imagine that Alice is playing Bob in the last round of a chess tournament. Neither Charlie nor Diane knows whether Alice won this last game. However, Charlie heard from a reliable source that the player with the black pieces won the game and so Charlie says, "If Alice was black, she won." Diane didn't hear that the black player won the game, however, she heard from a different reliable source that Alice won a few games as white during the tournament, however, every time she was black, she lost. So Diane responds to Charlie and says, "No, if Alice was black, she lost." This is an example of what Jonathan Bennett calls "A Gibbardian stand-off." [[The name

comes from a famous case of Alan Gibbard's known as "Sly Pete."]]

The basic logic of the situation seems to indicate that either both sentences are true, both are false, or one is true and one is false (which one??). But another possibility is that the conditionals don't have truth values at all.

What do you think we should say in this case? Which of these answers is correct and why? Depending on what you say, you should address at least one of these two issues:

1) It is basically universally agreed that unless A is itself a contradiction, $(A \rightarrow B) \wedge (A \rightarrow \neg B)$ is a contradiction. How could a contingent sentence like A possibly imply both B and $\neg B$? The material conditional makes this possibly true, but no other view does.

2) If you say anything *other* than that both sentences are true, then how is it that a listener is able to properly infer from hearing both statements that Alice was not playing Black?

Part 5:

Do the problems from exercise 4.2 on page 114 of MacFarlane.

Part 6:

For each of these arguments, if they are invalid on Stalnaker's semantics for the conditional, give a countermodel. If they are valid, give an argument (informal in English/logic) that they are valid.

1. $A \rightarrow B, A \rightarrow C$ therefore $A \rightarrow (B \wedge C)$
2. $A \rightarrow C$ therefore $(A \wedge B) \rightarrow C$ [antecedent strengthening]
3. $A \equiv B, A \rightarrow C$ therefore $B \rightarrow C$
4. $A \rightarrow B, B \rightarrow A$ therefore $(A \rightarrow C) \equiv (B \rightarrow C)$
5. $(A \vee B) \rightarrow C$ therefore $(A \rightarrow C) \vee (B \rightarrow C)$
6. $(A \wedge B) \rightarrow C, A \rightarrow \neg C$ therefore $A \rightarrow \neg B$