# GOODMAN, 'GRUE' AND HEMPEL*

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It is now commonly accepted that N. Goodman's predicate "grue" presents the theory of confirmation of C. G. Hempel (and other such theories) with grave difficulties. The precise nature and status of these "difficulties" has, however, never been made clear. In this paper it is argued that it is very unlikely that "grue" raises any *formal* difficulties for Hempel and appearances to the contrary are examined, rejected and an explanation of their intuitive appeal offered. However "grue" is shown to raise an informal, "over-arching" difficulty of great magnitude for all theories of confirmation, including Hempel's theory.

## I.

The predicate 'grue', together with some further considerations he adduces, first introduced by Goodman [1] in the context of Hempel's theory of confirmation [3], [4],[1] have been accepted as posing a classical difficulty for any putative logic of confirmation, but especially for those that construe the confirmation relation as a purely logical relation. Certainly Hempel's own efforts are regarded as being frustrated in part by this predicate and others of its type. Speaking of this problem whilst discussing Hempel's work, Goodman says:

> Then at time $t$ we have, for each evidence statement asserting that a given emerald is green, a parallel evidence statement asserting that that emerald is grue. And the statements that emerald $a$ is grue, that emerald $b$ is grue, and so on, will each confirm the general hypothesis that all emeralds are grue. Thus according to our present definition (i.e. Hempel's definition of confirmation) the prediction that all emeralds subsequently examined will be green and the prediction that all will be grue are alike confirmed by evidence statements describing the same observations. But if an emerald subsequently examined is grue, it is blue and hence not green. Thus although we are well aware which of the two incompatible predictions is genuinely confirmed, they are equally well confirmed according to our present definition. (Goodman, [2], pp. 74–75.)

Here $x$ is grue if and only if either $x$ is examined before time $t$ and $x$ is green or $x$ is not examined before time $t$ and $x$ is blue. Clearly, what Goodman is suggesting is that there exists some evidence statement, formulable within Hempel's system, and such that it confirms two incompatible hypotheses. Hempel himself acquiesces in this conclusion and remarks:

> But confirmation—whether in its qualitative or in its quantitative form—cannot be adequately defined by syntactical means alone. This has been made clear especially by Goodman, who has shown that some hypotheses of the form '$(x)(Px \supset Qx)$' can obtain no confirmation at all even from evidence sentences of the form '$Pa \cdot Qa$'. To illustrate this, I will adapt Goodman's example to my ornithological paradigm. Let '$x$ is $P$' stand for '$x$ is a raven' and '$x$ is $Q$' for '$x$ is blite', where an object is said to be blite if it has been examined before a certain time $t$ and is black or has not been examined before $t$ and is white. Then any raven observed before $t$ and found to be black affords a formally confirming instance,

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[1] See also Hempel's latest comments in [5], pp. 50 ff.

> in the sense of Nicod's criterion, of the hypothesis 'All ravens are blite'. Yet no matter how many such instances may have been collected, they lend no support or confirmation to the hypothesis; for the latter implies that all ravens not examined before *t*—hence in particular all those that might be examined after *t*—are white, and this consequence must surely count as disconfirmed rather than as confirmed. (Hempel, [5], pp. 50–51.)

These are serious charges to lay against Hempel's system and it is therefore somewhat remarkable that no more formal demonstration of the inconsistency—for that is what Hempel's system is being accused of—than those I have quoted above should have appeared in the literature.[2] Perhaps this is because it has been felt that the charge is so obviously merited that no formal discussion is required. If so, it is to be regretted for I have found that, appearances notwithstanding, the derivation of a formal contradiction is not simple and, indeed, I want, in this paper, to challenge the assumption that Hempel's system does fall victim to Goodman's predicate and others like it. My attack upon this assumption will take the form of considering the likely possibilities for deriving the contradiction mentioned by Goodman and Hempel and showing that none of these succeeds in establishing it. Of course, this method does not amount to a *proof* that such a contradiction cannot be derived within Hempel's system, but it will place upon those who accept Goodman's claim the onus of providing a formal demonstration of the contradiction. I shall also show that there *is* a difficulty for Hempel over predicates like 'grue' but that it is not the difficulty mentioned by Goodman and others. I shall offer an explanation of the intuitive appeal of Goodman's claim which is not unlike Hempel's suggested explanation of the "raven-paradoxes."[3]

Before we can proceed any further it is necessary to know precisely what it is that Hempel's system is accused of. Certainly, one is left with the *impression* from what has been said in the literature that a contradiction can be derived at some point and I have, thus far, uncritically fallen in with this suggestion. Unfortunately, the extant literature leaves the precise nature of this "difficulty" far from clear. Most plausibly, it is being suggested (argued?) that a pair of statements of the form:

(1) (i) $M$ confirms $H$
(ii) It is not the case that $M$ confirms $H$

is derivable within Hempel's system, using the predicate 'grue'. But the closest that the actual grue example ever appears to get to this situation is the suggested triad:

(2) (i)′ $M$ confirms $H$
(ii)′ $M$ confirms $H'$
(iii)′ $H$ and $H'$ are incompatible (in some sense).

And (2) does not by itself yield a contradiction. At most we might conclude that the definition of confirmation which gave rise to (2) was thereby shown to be inadequate. Not even the statements:

(3) (i)″ $M$ confirms $H$
(ii)″ $M$ confirms $-H$

[2] At the least, I have been able to find no formal demonstration of the claims.

[3] See Hempel, [3], section 5 and his later comments in [5], pp. 47–48.

where '$-H$' is read as 'not $H$,' form a contradictory pair unless we also have the assertion:

(4) $M$ confirms $H$ entails $-(M$ confirms $-H)$.

(But in any system where (3) *was* derivable, we might expect that (4) would in any case have been dropped.)

Hempel at least cannot allow (3) because of his *meta-theory*. Specifically, meta-theorem 8.22 of [4] asserts that the confirmation relation, as Hempel defines it, satisfies the "General Consistency Condition," one of whose entailments is the assertion (3.21, p. 124 of [4]) that "The class of all sentences confirmed by a consistent molecule (an evidence statement not containing quantifiers) is consistent." But this latter directly entails (4), which (3) violates. Thus if (2) above could be turned into something like (3), and if the claims made in (2) are valid, then a genuine contradiction *would* be derivable within Hempel's system, viz. a contradiction between what Hempel's meta-theory asserts his system is like and what the system is in fact like. Nevertheless, this still does *not* show that from the definition of confirmation *alone* a contradiction is derivable. The most it could show is that:

(a) Hempel had committed errors in developing his meta-theory, and (b) the definition of confirmation which he gives is, *because it does not satisfy his own desiderata*, inadequate. On the other hand Hempel has taken great care to develop his system with full logical rigour and it would therefore be surprising to find that so crucial an error as is suggested by (3) and (4) in combination should have crept into so straightforward a task. (Hempel's formal language, $L$, is a relatively simple, well-known, one—it is the first order predicate calculus without identity sign.) Moreover, there is no explicit suggestion in the literature that Hempel made a *formal* blunder. Prima facie, it would appear very plausible to regard (2) as not reducible to the form of (3) and that the claim (a) above is best dropped. But then this should make us very suspicious of any claim that $H$ and $H'$ of (2) are incompatible in any normal sense. Later in the paper I hope to make clear just how a sense of incompatibility gets introduced and in what ways this incompatibility, though formally a fraud (i.e. no contradictions are derivable), nevertheless generates an "over-arching" difficulty for Hempel's system of confirmation (and any other—so far as I can see).

## II

Hempel's system is, as I have said, restricted to a language, $L$, which is the first order predicate calculus without identity. We must therefore formulate evidence and hypotheses to suit these requirements.[4]

[4] Strictly, only evidence-*schemas* and hypothesis-*schemas* occur in $L$. Specific claims of confirmation are substantiated by deciding (intuitively) that the evidence and hypothesis sentences are instances of schemas of $L$, schemas standing in the confirmation relation. For simplicity and brevity I shall speak of instances of Hempel's confirmation schemas as belonging also within Hempel's system.

From this it clearly follows that only *statements* (or sentences) stand in confirmation relations to hypotheses (i.e. to *other statements*). Despite this, there is a largely unexamined habit of also regarding *objects* and the like as confirming hypotheses. This habit, well illustrated in the

Let the following predicates in $L$ be formed:

> '$Ex$' to be read '$x$ is examined before time $t$'.
> '$Gx$' to be read '$x$ is green'.
> '$Emx$' to be read '$x$ is an emerald'.
> '$Grx$' to be read '$x$ is grue'.

Let us suppose, temporarily without discussion, that:

(D) '$Grx$' is short for '$Ex \cdot Gx \vee -Ex \cdot -Gx$'.

I shall now set down several relevant hypotheses.

$$H_0: (x)(Emx \supset Grx).$$
$$(x)(Emx \supset (Ex \cdot Gx \vee -Ex \cdot -Gx)).$$

(I shall refer to each of these hypotheses as $H_0$.)

$$H_1: (x)(Emx \supset Gx).$$
$$H_2: (x)(Emx \supset -Gx).$$

It is not hard to show that the following hypotheses are, each of them, consequences of $H_0$.

$$H_0^1: (x)(Emx \cdot Ex \supset Gx); \qquad {}^1H_0: (x)((Emx \cdot -Ex) \supset -Gx);$$
$$H_0^2: (x)(Emx \cdot Gx \supset Ex); \qquad {}^2H_0: (x)((Emx \cdot -Gx) \supset -Ex);$$
$$H_0^3: (x)(Ex \supset (Emx \supset Gx)); \qquad {}^3H_0: (x)(-Ex \supset (Emx \supset -Gx));$$
$$H_0^4: (x)((Emx \supset Gx) \vee -Ex); \qquad {}^4H_0: (x)((Emx \supset -Gx) \vee Ex);$$
$$H_0^5: (x)((Emx \supset Ex) \vee -Ex); \qquad {}^5H_0: (x)((Emx \supset -Ex) \vee Ex).$$

It is also convenient to have certain central features of Hempel's system before us. Hempel first defines a relation which he calls "direct confirmation," thus:

*M directly confirms S* ($M$ and $S$ being sentences of $L$) if and only if (a) $M$ is a molecule and (b) either $S$ is analytic or the development of $S$ for $M$ is not analytic and (c) $M$ entails the development of $S$ for $M$.

literature and probably unjustifiable, has been critically discussed by Mr. D. Stove in [6]. I shall avoid its use here.

There is another impropriety in this context which must also be exposed and removed if any sort of rigour and clarity is to be achieved; this is the conflating of colloquial and formal expressions. In the passages quoted above, for example, such expressions as "All emeralds are blue" occur where what can only be meant is "$(x)$ (Emerald $x \supset$ Blue $x$)," or something akin to it. For it is Hempel's definition of confirmation that is being discussed and no other, but in Hempel's system no colloquial expressions can, or do, appear! However, we should not conclude from this that Hempel's formal language is such that there is no way to render its expressions colloquially, since there is. For example, the above hypothesis is accurately rendered "Everything is either not an emerald or blue." It is just that the correct rendering is not (very often) the intuitively appealing rendering and this creates unnecessary and, indeed, extremely dangerous, confusions (cf. Stove's discussion in [6]). It creates a great deal of the apparent difficulty in the present context too, as I shall attempt to show. Throughout what follows, therefore, I shall consider only questions which can arise *within* Hempel's system, giving what I consider to be the most plausible versions of the colloquially expressed hypotheses and other expressions found in the literature.

I am greatly indebted to Mr. D. Stove for his help and encouragement and I gladly acknowledge his patient guidance and helpful criticism in all matters pertaining to the theory of confirmation.

He then defines confirmation as follows:

*M confirms S* ($M$ and $S$ being sentences of $L$) if and only if (a) $M$ is a molecule and (b) there is a class $K$ of sentences (of $L$) such that $K$ entails $S$ and for every sentence $T$, of $K$, $M$ entails or directly confirms $T$.[5]

The words "molecule" and "development" in these definitions call for explanation.

A molecule is any sentence of $L$ which is "either atomic or consists of atomic sentences and statement connectives."[6] An atomic sentence of $L$ consists of "a predicate, followed by a parenthesised expression which consists of as many individual constants—separated by commas—as the degree of the predicate requires."[7]

The development of a sentence for the (finite) class of individual constants occurring (essentially) in a molecule (in short, the development of a sentence "for" a molecule), is defined as follows: If the sentence contains no quantifier, the development is the sentence itself; if the sentence does contain a quantifier, its development for a given molecule is the sentence that results when any universally quantified matrix that the sentence contains is replaced by the conjunction of its substitution instances for all the individual constants occurring (essentially) in the molecule, and any existentially quantified matrix is replaced by the disjunction of its substitution instances for all the individual constants in the molecule.

Finally, we have that *M disconfirms S* if and only if $M$ confirms $-S$.

To illustrate these ideas let us write down the development of $H_0$ for molecules in which (i) the individual $a$ only occurs (essentially) and (ii) the individuals $a$ and $b$ only occur (essentially). They are:

(1) $$D(H_0) = -Ema \vee Ea \cdot Ga \vee -Ea \cdot -Ga.$$

(2) $$\begin{aligned} D(H_0) = {} & -Ema \cdot -Emb \vee -Ema \cdot Eb \cdot Gb \vee -Ema \cdot -Eb \cdot -Gb \\ & \vee Ea \cdot Ga \cdot -Emb \vee Ea \cdot Ga \cdot Eb \cdot Gb \vee Ea \cdot Ga \cdot -Eb \cdot -Gb \\ & \vee -Ea \cdot -Ga \cdot Eb \cdot Gb \vee -Ea \cdot -Ga \cdot -Emb \\ & \vee -Ea \cdot -Ga \cdot -Eb \cdot -Gb. \end{aligned}$$

There are three further features of Hempel's system which are needed below and which I next state as theorems:[8]

*Theorem 1:* (General Consequence Condition): If a molecule $M$ confirms every sentence of a class $K$, then it also confirms every consequence of $K$.[9]

*Theorem 2:* If the development of an hypothesis for a molecule $M$ is in disjunctive form and if $M$ entails one or more of the disjuncts then $M$ confirms that hypothesis.[10]

[5] See [4], p. 142. In giving these definitions, I have translated Hempel's symbolism back into ordinary English (though without, I hope, abandoning the agreement of footnote 5 to translate accurately). I have also shortened Hempel's "development" idiom.

[6] See [4], p. 125.

[7] [4], p. 125. For simplicity, I shall omit the parentheses.

[8] Strictly, they are meta-theorems, for each asserts properties of Hempel's system.

[9] In his formal statement of his system, [4], Hempel builds up the proof of this theorem from the recursive definition of the development of an hypothesis and the reader is referred there for further information.

[10] Proof: Let the hypothesis and evidence be $H$ and $E$ respectively and let '$C_h(H, E)$' stand for '$E$ confirms $H$'. Let the development of $H$ for $E$ be $A_1 \vee A_2 \vee \cdots \vee A_k$, symbolize this as

*Theorem 3:* If the development of an hypothesis for a molecule $M$ is in disjunctive form then, for every consistent disjunct, the conjunction of that disjunct and molecules of the form '$Pa \cdot Qb \ldots Rf$', where the individuals $a, b, \ldots, f$ occur essentially in $M$ and the resulting molecule is consistent, confirms the hypothesis.[11]

As an immediate application of theorem 1 we take $K$ to be the unit class whose only member is $H_0$ and assert that if evidence $M$ confirms $H_0$ then $M$ also confirms each of the $H_0^j$ ($j = 1, \ldots, 5$) and each of the ${}^jH_0$ ($j = 1, \ldots, 5$) for these hypotheses are all consequences of $H_0$.

Theorem 2 will be appealed to as grounds for the assertion that any evidence which entails any disjunct of (1) or (2) above—so long as only the appropriate individuals occur essentially in it—confirms $H_0$.

Theorem 3 allows the conjoining of *Ema* or *Ema* · *Emb* (as appropriate) to those disjuncts of (1) or (2) above not containing the predicate $-Em\_$ without prejudice to their confirming power with respect to the hypotheses of interest to us.

## III

It is now time to discuss in more detail the predicate "grue" itself. Initially I have assumed that the right-hand side of (D) above adequately captures what is intended by "grue." If this is so, a stronger equivalence than that which I have used would be appropriate in (D). Notice however that whilst (D) is expressible in Hempel's language $L$, any stronger form of the equivalence would not be expressible in $L$. Thus the meaning of "grue" cannot be 'unpacked' explicitly in $L$. On this basis I have heard it objected that no conflict between grue and green can therefore be derived in $L$, there being no way to express in $L$ the connection between the two terms. But, other replies aside, this objection overlooks the fact that the (weaker) *material* equivalence, (D), *is* expressible in $L$ and if any conflict is to arise at all it must surely arise with (D) operating in $L$.

On the other hand there is, in the colloquial rendering of "grue," the suggestion that perhaps a more accurate rendering might be: (P), $Grx = Gx \cdot (\exists y)$(Examined-before-$t\ yx$) $\vee\ -Gx \cdot -(\exists y)$(Examined-before-$t\ yx$) where 'Examined-before-$t\ yx$' stands for '$y$ examines $x$ before time $t$'. However such a suggestion, if accurate, would prove fatal to the derivation of Goodman's contradiction. For if grue is defined as in (P) then no evidence statement whatever in Hempel's system can contain that predicate in virtue of the fact that only molecules, *which contain no quantifiers*, are admissible as evidence statements in Hempel's system. It would seem, therefore, that a formulation of "grue" such as that given in (D) must be chosen if even the appearance of a contradiction is to be sustained.

'$D(H)$'. By hypothesis, there exists, for some $j$, $1 \leqslant j \leqslant k$, an $A_j$ such that $E$ entails $A_j$. Therefore, $E$ entails $A_1 \vee A_2 \vee \cdots \vee A_k$; that is, $E$ entails $D(H)$. But $E$ entails $D(H)$ entails that $C_h(H, E)$. Therefore $C_h(H, E)$.

[11] Proof: Let everything remain as it was for the proof of theorem 2. Let an arbitrary consistent disjunct be $A_j$; then $D(H)$ for the evidence $E \cdot Pa \cdot Qb \ldots Rf$ is the same as $D(H)$ for $E$, by hypothesis. But $A_j \cdot Pa \cdot Qb \ldots Rf$ entails $A_j$; and $A_j$ entails $A_1 \vee A_2 \vee \ldots \vee A_k$, which is $D(H)$. Hence $C_h(H, A_j \cdot Pa \cdot Qb \ldots Rf)$.

There is another ambiguity associated with (D) and it concerns the precise manner in which the phrase '$x$ has not been examined before time $t$' is to be understood. This phrase could be read as 'it is not the case that $x$ has been examined prior to time $t$' or it could be read '$x$ has been examined after time $t$'. For the present I shall choose the former, wider sense for the phrase, as expressed in (D), but the ambiguity is important and I will refer to it again later.

## IV

Before any serious discussion can begin the appropriate evidence statements must be decided upon and introduced so that their confirmation relations to the hypotheses under scrutiny can be examined. Both Goodman and Hempel describe colloquially the evidence involved whereas we require a more formal statement, expressible in $L$. On the other hand, which statements are to be set down must be carefully considered. Thus, suppose we take as our evidence the statement $Ema \cdot Gra$. But from (D) this is equivalent to $Ema \cdot Ga \cdot Ea \vee Ema \cdot -Ga \cdot -Ea$, and this molecule does *not* confirm $H_1$. However evidence such as $Ema \cdot Ga \cdot Ea$ confirms both $H_1$ and $H_0$ (the latter by theorems 2 and 3). Alternatively, the molecule $Ema \cdot Gra \cdot Ea$ could be used but this is only because this molecule is logically equivalent to $Ema \cdot Ga \cdot Ea$. Thus the type of evidence both required and sufficient for our purposes is that consisting of consistent conjunctions of the predicates $Em\_\_$, $G\_\_$, $E\_\_$, or their negations.

However even this latter remark is too general in one respect. We do not wish to consider the case of evidence statements containing the predicate $-Em\_\_$. It is clear, I think, from the passages which I have quoted in section I above that the evidence statements with which we are concerned here contain only positive instances of emeralds. Thus it is reasonable to exclude from consideration evidence statements containing the predicate $-Em\_\_$ which will (trivially) confirm $H_0$ by falsifying the antecedents of the instantiations of the material implication.

With this restriction on the occurrences of $-Em\_\_$ imposed the available disjuncts of (1) and (2) become:

(1′) $Ea \cdot Ga \vee -Ea \cdot -Ga.$

(2′) $Ea \cdot Ga \cdot Eb \cdot Gb \vee Ea \cdot Ga \cdot -Eb \cdot -Gb \vee -Ea \cdot -Ga \cdot Eb \cdot Gb$
$\vee -Ea \cdot -Ga \cdot -Eb \cdot -Gb.$

According to theorems 2 and 3, any of these disjuncts may be conjoined with positive instances of $Em\_\_$, containing the same individuals, and will confirm $H_0$.

The following evidence statements give a representative coverage of the types of evidence statements which are of relevance to the present context:

$E_1^a$: $Ema \cdot Ea \cdot Ga.$
$E_2^a$: $Ema \cdot Ea \cdot Ga \cdot Emb \cdot Eb \cdot Gb.$
$E_3^a$: $Ema \cdot Ea \cdot -Ga.$
$E_4^a$: $Ema \cdot Ea \cdot Ga \cdot Emb \cdot -Eb \cdot Gb.$
$E_5^a$: $Ema \cdot Ea \cdot Ga \cdot Emb \cdot Eb \cdot -Gb.$

$E_1^b$: $Ema \cdot -Ea \cdot -Ga$.
$E_2^b$: $Ema \cdot Ea \cdot Ga \cdot Emb \cdot -Eb \cdot -Gb$.
$E_3^b$: $Ema \cdot -Ea \cdot Ga$.
$E_4^b$: $Ema \cdot -Ea \cdot -Ga \cdot Emb \cdot -Eb \cdot Gb$.
$E_5^b$: $Ema \cdot -Ea \cdot -Ga \cdot Emb \cdot Eb \cdot -Gb$.

There is another version of $E_2^b$ corresponding to permutations of $a$ and $b$, and another version of $E_2^a$ corresponding to the use of two sets of $E_1^b$-type evidence. Similarly, in the $E_4$ and $E_5$ types of evidence the roles of $a$ and $b$ could be interchanged to form new molecules. But in all these cases nothing essentially new is added and I neglect these for the sake of simplicity. In a similar spirit I will not consider any evidence statements containing more than the two individuals $a$ and $b$.

The confirmation relations holding between these evidence statements and a selection of the relevant hypotheses is given in table **M** below:

| | $H_0$ | $cH_0$ | $H_1$ | $H_2$ | $H_0^1$ | ${}^1H_0$ |
|---|---|---|---|---|---|---|
| $E_1^a$ | $C_h$ | $DC_h$ | $C_h$ | $DC_h$ | $C_h$ | $C_h$ |
| $E_1^b$ | $C_h$ | $DC_h$ | $DC_h$ | $C_h$ | $C_h$ | $C_h$ |
| $E_2^a$ | $C_h$ | $DC_h$ | $C_h$ | $DC_h$ | $C_h$ | $C_h$ |
| $E_2^b$ | $C_h$ | $DC_h$ | $DC_h$ | $DC_h$ | $C_h$ | $C_h$ |
| $E_3^a$ | $DC_h$ | $C_h$ | $DC_h$ | $C_h$ | $DC_h$ | $C_h$ |
| $E_3^b$ | $DC_h$ | $C_h$ | $C_h$ | $DC_h$ | $C_h$ | $DC_h$ |
| $E_4^a$ | $DC_h$ | $DC_h$ | $C_h$ | $DC_h$ | $C_h$ | $DC_h$ |
| $E_4^b$ | $DC_h$ | $DC_h$ | $DC_h$ | $DC_h$ | $C_h$ | $DC_h$ |
| $E_5^a$ | $DC_h$ | $DC_h$ | $DC_h$ | $DC_h$ | $DC_h$ | $C_h$ |
| $E_5^b$ | $DC_h$ | $DC_h$ | $DC_h$ | $C_h$ | $DC_h$ | $C_h$ |

Table **M**

where $C_h$ indicates confirmation and $DC_h$ disconfirmation. I have also introduced the would-be contrary to $H_0$, $cH_0$: $(x)(Emx \supset Ex \cdot -Gx \vee -Ex \cdot Gx)$.[12] I have included the hypotheses $H_0^1$ and ${}^1H_0$ for comparison; the hypotheses $H_0^j$ and ${}^jH_0$ ($j = 3, 4$) behave in the same manner as $H_0^1$ and ${}^1H_0$ respectively with regard to confirmation relations to these evidence statements.[13]

What is of substance in this paper can be made clear on the basis of a discussion of the evidence statement $E_1^a$ alone. But the nature of my argument (cf. section I) compels me, for reasons of plausibility, to consider all of a set of evidence statements which might reasonably be held to be representative of all relevant evidence statements. Thus I am forced to discuss the remaining evidence statements, as well as $E_1^a$. For the sake of simplicity I shall first discuss $E_1^a$, presenting the core of the paper in so doing. Then I shall conclude by briefly discussing the remaining

[12] $cH_0$ is not the strict contrary of $H_0$, for one requires also the existential assumption $(\exists y)(Emy)$, which is not given here.

[13] The results of column 1 follow immediately from application of theorems 2 and 3 to (1′) and (2′), as do the positive elements of columns 3 and 4. The negative evaluations of columns 3 and 4 are most easily arrived at by observing that the *negations* of the hypotheses $H_1$ and $H_2$ (i.e. the hypotheses $(\exists y)(Emy \cdot -Gy)$ and $(y)(Emy \cdot Gy)$ respectively), are *confirmed* by the evidence statements concerned, for these two hypotheses have a disjunctive development for any given evidence and then theorems 2 and 3 may be applied.

statements, attempting to show that they add nothing of importance to what has been said under the discussion of $E_1^{a}$. The substance of the paper occurs, therefore, in the next section and I ask the reader to excuse the order of presentation in the name of clarity.

From column 1 we deduce that $E_1^a$ confirms each of the consequences of $H_0$, i.e. $H_0^j$ and ${}^jH_0$ ($j = 1, \ldots, 5$). If Goodman's predicate is to create any *logical* difficulties for Hempel then it must be possible to show that the same evidence confirms conflicting hypotheses.

## V

*Consideration of* $E_1^a$

(i) Certainly $H_1$ and $H_2$ conflict in the presence of $E_1^a$, but then, $E_1^a$ confirms $H_1$ and disconfirms $H_2$. Indeed, this result serves to highlight the general fact that one cannot simply derive a contradiction from the pair $H_0$ and $H_1$ on the grounds that $H_0$ entails $H_2$ and $H_1$ and $H_2$ conflict with one another. For $H_0$ entails $H_2$ *only* in the presence of $(x)(-Ex)$, an assertion that violates a basic assumption of the very setting up of the original problem. An assertion that is, in any case, not admissible as evidence within Hempel's system.

(ii) We also have that $E_1^a$ confirms $H_1$ and also $H_0^1$, but these two hypotheses seem eminently compatible. In fact none of the $H_0^j$ ($j \neq 1, \ldots, 5$) hypotheses can be expected to lead to a conflict with $H_1$.

(iii) But we *do* have that $E_1^a$ confirms $H_1$, disconfirms $H_2$, and yet confirms each of the ${}^jH_0$ ($j = 1, \ldots, 5$) hypotheses! How is this possible? The hypotheses ${}^1H_0$, ${}^3H_0$, and ${}^4H_0$ especially seem in direct conflict with $H_1$ and in agreement with $H_2$. Thus we might read ${}^1H_0$ as asserting that 'All emeralds after time $t$ are not green', in direct conflict with $H_1$: 'All emeralds are green'.

However a closer examination of these hypotheses indicates that there is no logical incompatibility between them. For example, $H_1 \cdot {}^1H_0$ entails $(x)(Emx \supset Gx \cdot Ex)$ and also entails $(x)(Emx \supset Ex)$. Thus $E_1^a$ confirms both of these consequents of $H_1 \cdot {}^1H_0$ (General Consequence Condition), a state of affairs which is at least not inconsistent, nor, on the basis of $E_1^a$ *alone*, can it be regarded as unacceptable on other grounds. Indeed, if one considers that $H_0^2$, ${}^2H_0$, and ${}^3H_0$ are also confirmed by $E_1^a$ the result becomes an expected one. (This situation, and especially that $(x)(Emx \supset Ex)$ is confirmed if $H_1 \cdot {}^1H_0$ is confirmed, is further illuminated by considering the hypotheses ${}^4H_0$ and $H_0^5$ which are also confirmed by $E_1^a$.) Thus $E_1^a$ leads to no contradiction within Hempel's system.

On the other hand, we can point to the hypotheses which generate the intuitive feeling of conflict—they are ${}^1H_0$ and ${}^3H_0$. Thus ${}^3H_0$, read colloquially, says of any object $x$ that if it is not examined prior to time $t$, then if it is an emerald, then it is not green (cf. the colloquial reading of ${}^1H_0$ given above). And this assertion (of ${}^3H_0$), coupled with our reasonable expectation that things which are emeralds *will* be examined after time $t$, leads to the conclusion that these emeralds will not be green. But the reply to the intuitively generated difficulty which we have here must be, in the first place, that there is no room in Hempel's system for 'colloquial' readings of (instances of) sentence-schemas (see fn. 5). In the second place there is no room for introducing external, if reasonable, knowledge claims and, in the third place, there will be no conflict, even of an intuitive nature, so long as the evidence actually introduced is kept carefully in mind.

To amplify the *first and second* points a little. What is obviously at the root of the intuitive conflict is the belief that there exist emeralds which will not be examined before time $t$ and which will be examined after time $t$. In the language of $L$: $(\exists y)(Emy \cdot -Ey \cdot \text{Examined-after-}t\ y)$. This latter assertion entails the contradictory of what is entailed by $H_1 \cdot {}^1H_0$. It is therefore disconfirmed by $E_1^a$. Now it is of no use to say that therefore $E_1^a$ itself must be held in doubt, for we are not debating whether certain evidence statements are, or are not, *true* and hence admissible, but, rather, whether it is the case that if a given evidence statement *is admitted* then a contradiction ensues. In our context there is no sense to the claim that $E_1^a$ is "in doubt." Nor does the above information cast doubt upon *every* hypothesis which $E_1^a$ confirms, for $H_1$ falls into this latter category and $H_1$ is what is, intuitively, being defended. Intuitively, what we want to be able to do is to argue as follows:

$H_1 \cdot {}^1H_0$ entails $(x)(Emx \supset Ex)$.
But $(\exists y)(Emy \cdot -Ey)$, and $H_1$ is true.
Therefore ${}^1H_0$ is false.

The argument is certainly valid. But this is not sufficient in the present context. What is required is that there exist some evidence statement which, when taken into account, will still allow $H_1$ to be confirmed, will confirm $(\exists y)(Emy \cdot -Ey)$ and will disconfirm ${}^1H_0$.[14] But what could this evidence be? If to $E_1^a$ we add either $Emb$ or $Emb \cdot -Eb$ then the new evidence statement becomes *neutral* to both $H_0$ and $H_1$ (i.e. it neither confirms, nor disconfirms, either hypothesis), though it would confirm $H_0^1$ and disconfirm ${}^1H_0$. On the other hand, if to $E_1^a$ we add $Emb \cdot Gb \cdot -Eb$ we obtain $E_4^a$ (considered below) which does do the required job for us, but only at the expense of insisting on evidence about the *colour*, as well as the existence, of emeralds for times later than $t$. (To obtain $Emb \cdot -Eb \cdot -Gb$ to $E_1^a$ would yield $E_2^b$ which disconfirms $H_1$ and is thus unacceptable.) Thus short of actually confirming $H_1$ *beyond time t* no way exists in which to introduce our intuitive expectations into Hempel's system and yet retain the desired results.

Of course, if the assertion $(\exists y)(Emy \cdot -Ey)$ could itself be conjoined with $E_1^a$ then it might do the trick, but this assertion is excluded from Hempel's system—at least, in the "evidence" position. This serves to highlight the fact that the difficulty imagined to have been created for Hempel's system by predicates such as grue arises largely because of intuitive judgments made *externally* to Hempel's system and which we falsely believe either properly belong to the system or could without difficulty be incorporated within it. And now we are in a position to appreciate the sense in which the "incompatibility" of (2) of section I is felt to arise—*it arises only given an extra statement, not contained in (2)*, a statement which, because of long experience, intuition unconsciously introduces.

[14] This is the weakest requirement that could be imposed.

In a sense, the introduction of this 'illicit' evidence and the effect of this upon our intuitive judgments parallels the situation which Hempel[15] claims gives rise to the "raven-paradoxes." At least in part, the ambiguity involved in the interpretation of the predicate $-E\_$, mentioned in section III above, is responsible for creating this temptation to 'smuggle' the extra information in—a temptation which I have deliberately resisted by choosing the widest sense for $-E\_$. Yet the passage from Hempel's writings quoted above in section I suggests that Hempel himself has fallen victim to this very temptation. For what can the evidence be which allows him to conclude: ". . . and this consequence (that all ravens examined after time $t$ are white) must surely count as *disconfirmed* rather than as confirmed"? That evidence cannot be, as we have seen, black ravens examined before time $t$ (to use Hempel's colloquial rendering).[16] But then, the only other likely candidate would be information of the type exemplified by $(\exists y)(\text{raven}\, y \cdot -Ey)$, but this, we now know, cannot be 'legally' included in this present context or, if support for it is legitimately admitted in an $E_4^a$ form, the difficulties over grue-like predicates do not arise.

In an indirect way, this situation is at variance with Goodman's judgment concerning a similar suggestion made in this context. Goodman says:

> Since our definition (i.e. Hempel's definition of confirmation) is insensitive to the bearing upon hypotheses of evidence so related to them, even when the evidence is fully declared, the difficulty about accidental hypotheses cannot be explained away on the ground that such evidence is being surreptitiously taken into account.

But although it is true that all evidence statements *expressible in L*, temporally restricted, and of simple positive forms which I have considered above, would not serve to remove the conflict once it had occurred (but then, they do not suffice, contra Goodman, to get the conflict started), it is *not* true that there is no information at all that we can point to and which explains the intuitive feeling of conflict. However this information is not expressible in $L$, as evidence.

I have pointed to one source of the temptation to smuggle in extra information, viz. the ambiguous character of the predicate $-E\_$, for in its narrower sense (which I have not chosen) it actually does entail that $(\exists y)(Emy \cdot -Ey)$. But there is a second major source of this same temptation and that is the practice of giving colloquial renderings of hypotheses and evidence statements which are instances of the schemas of $L$. Already I have drawn attention to the fact that colloquial renderings of the predicate "grue" have led to uncertainty as to its precise interpretation in $L$ and to the fact that the colloquial reading of certain information leads to the misleading impression that it can easily be incorporated into Hempel's system. But now I want to draw attention to another consequence of this dangerous procedure. I had previously said that the hypothesis ${}^1H_0$ could be read, colloquially, as 'All emeralds examined after time $t$ are not green', but it is not only not correct to allow this reading as a legitimate rendering of that hypothesis, it is fatal to clear thinking about our present conflict over grue to do so. It is incorrect because the

[15] See fn. 4 for references.

[16] Here I have used an obvious transposition, *green–black*, *ravens–emeralds*, so as to embrace Hempel's example.

truth conditions of the colloquial rendering of ${}^1H_0$ and those of ${}^1H_0$ itself are not the same. Thus, for example, a sufficient condition for the truth of ${}^1H_0$ is $(x)(-Emx \vee -Ex)$ whilst neither this, nor any colloquial rendering of it, is a sufficient condition for the truth of 'All emeralds examined after time $t$ are not green'. The acceptance of the colloquial rendering of ${}^1H_0$ is fatal just because of this latter point. For the colloquial version of ${}^1H_0$ carries with it the (implicit) existential claim that emeralds examined after time $t$ exist and this is already to be tacitly committed to the introduction of the extra information we have been discussing, viz. $(\exists y)(Emy \cdot -Ey)$. It is also fatal in a second way, for the colloquial rendering is pushed by these very existential implications towards choosing, quite naturally, the narrower rendering of $-E\_$, thus reinforcing the already existing tendency to smuggle in the 'illicit' information via this channel. After what now appears a strictly unnecessary uproar over the so-called 'paradoxes of the ravens' just because of the neglect of this distinction between formally and colloquially expressed statements[17] we should be especially cautious about simply accepting informally stated statements concerning formal properties of formal systems (in particular, Hempel's system), and the above discussion has, I think, reinforced this conclusion.

With regard to the *third* point concerning the intuitive effect of not keeping the form of the evidence closely in mind, a more careful investigation is rewarding. The evidence $E_1^a$ contains only positive instances of the predicate $E\_$. Thus this evidence refers only to examinations made *before* time $t$. The apparently conflicting hypotheses however all make claims regarding times *after* time $t$. At least, if they are to be regarded as being in conflict at all they must be regarded as making claims concerning times after time $t$. But there is no contradiction in supposing that the law-like behavior of the world should change at time $t$[18] and hence no difficulty in supposing that evidence obtained only *before* time $t$ should confirm two hypotheses which would, under certain conditions not mentioned in that evidence, diverge beyond time $t$. The condition under which they do not diverge beyond time $t$ has already been stated, viz. when $(x)(Emx \cdot Ex)$ is true. The situation here is well illustrated by a consideration of the hypothesis ${}^4H_0$. For there to be a genuine intuitive conflict between $H_1$ and $H_0$ we should require that $E_1^a$ confirm ${}^4H_0$ via the *first* of the two disjuncts of ${}^4H_0$ (the implication relation). (Even here intuitive conflict might be avoided if the evidence contained only instances of $-Em\_$ in it.) But of course this does not happen. Rather, $E_1^a$ confirms ${}^4H_0$ via the harmless second disjunct, thus avoiding even the appearance of intuitive conflict.

Thus although $E_1^a$ confirms hypotheses which we feel intuitively will ultimately be in conflict (simply because we do not expect that the laws of nature will change at any time), there is no ground here for asserting that a formal counterpart to this intuitive feeling can be derived within Hempel's system since in applying that intuition we must pass *outside* of the scope of the evidence involved. The ability of

[17] My authority for this claim is the article by Stove, [6], which gives an excellent and detailed examination of the formal status of the "raven-paradoxes."

[18] At least, this will be true for anyone who, like myself, is not committed to an analysis of laws in terms of "necessary connections."

Hempel's criterion to avoid difficulty over the predicate "grue" at this point may be summarized by saying that, for those hypotheses in the present context which have temporal predicates incorporated into them, and for any relevant, explicitly temporally restricted evidence statements, Hempel's system always holds those hypotheses within the temporal bounds set by the relevant evidence statements. In this regard, Hempel's system does not go beyond the evidence.

But now we are able to raise a genuine difficulty, albeit not a formal one, for Hempel. There is, of course, no complaint to be made merely because a definition of confirmation, such as Hempel's, issues in the result that the same evidence confirms two hypotheses which would be, under certain other, external, assumptions, incompatible with one another. But in this case the conjoining of the two relevant hypotheses issues, not in the typical result that the class to which both hypotheses refer is null,[19] but in the conclusion $(x)(Emx \supset Ex)$. There is no question of attempting to overthrow Hempel from within his system, but we should take stock of precisely what is involved here. Evidence of type $E_1^a$ confirms $(x)(Ex)$ as well as $H_0$ and $H_1$. Since this is so, such evidence also confirms *any and every* hypothesis of the form $(x)(\ldots x \ldots \supset Ex)$, *regardless of what is inserted in the place of* '$\ldots x \ldots$'. Every hypothesis of this form, no matter what it asserts—in particular, no matter what it asserts will happen after time $t$—is confirmed by $E_1^a$-type evidence statements. Indeed these same remarks hold true for every hypothesis of the form $(x)(\ldots x \ldots \supset (\ldots x \ldots \supset Ex))$, where the two space holders may or may not be filled in with distinct expressions, and for more complex hypotheses built upon these general forms. In particular, $H_0$ is of the latter variety and all hypotheses containing grue-type predicates would seem to be able to be cast into one or another of this general class of forms. Under these conditions, *no* set of hypotheses of this general form which we judge will be in conflict after time $t$, if they are consistent before time $t$, will produce a *formal* contradiction or conflict within Hempel's system under $E_1^a$-type evidence. Under that evidence they will all be confirmed!

(Obviously spatial analogues to "grue" could easily be constructed. Then two hypotheses of the appropriate form which conflict *outside* of some spatial region, but which do not conflict *inside* it, will be in exactly the same situation, *vis-à-vis* spatially restricted evidence statements of the $E_1^a$ type, as that encountered above.)

But now comes the blow. The very concept of a logic of confirmation is bound up with an ability (or the desire for the ability) to proceed, rationally, "beyond the evidence." After all, an essential philosophical function of a logic of confirmation is to provide a rational reply to the inductive sceptic, i.e. to one who does not believe that there is any rational way to proceed "beyond the evidence." In this case the sceptic is simply presented with a *fait accompli*—a coherent and reasonable system for judging the acceptability of hypotheses.[20] But now see what havoc our present

[19] The "typical case" is that in which one has both $(x)(Px \supset Qx)$ and $(x)(Px \supset -Qx)$ which together entail that $(x)(-Px)$.

[20] I do not intend to imply that I think that there is necessarily any intuitively clear and simple connection between confirmation as Hempel defines it and our practical assessments. There certainly is a temptation to take confirmation in Hempel's system as being the same as the colloquial term of the same sound and spelling, but I believe that, on the contrary, such formal definitions of confirmation as Hempel provides us with present a technically complex and quite

findings wreak upon this corner of the philosophical field. For the sceptic can point out that the very system of confirmation which was raised with the intention of making his sceptical problem seem unnecessary and unreasonable has itself regenerated that very same problem. We need only take the fatal time $t$ referred to above a fraction of a day ahead of the present time and Hempel's system will afford no way of judging among hypotheses that assert things (no matter how wild!) about tomorrow—so long, that is, as the hypotheses are carefully framed using grue-type predicates and *it is insisted that the evidence statements incorporate the times of their observations, etc.* (Analogous remarks hold for the case of spatially restricted hypotheses: Unless we set eyes on every region of space concerned, Hempel's system will confirm hypotheses making all kinds of wild predictions about other spatial regions.) For most of us this situation will reintroduce the sceptical problem about induction in a fairly acute form, though, of course, *hypotheses and corresponding evidence statements not mentioning specific spatio-temporal locations will still go through Hempel's system as usual.* This last remark is of some importance, for it points up the "asymmetry" here between "normal" predicates, such as ravenhood and blackness, and those predicates attributing spatio-temporal location. Provided that these latter predicates are prevented from entering the situation, no trouble seems to arise and the sceptic, insofar as he is a sceptic about the rationality of our expectation that ravenhood will, in the future, be accompanied by blackness and like cases, has his answer. There is an important division of opinion among philosophers concerning whether having a certain spatio-temporal location is a quality or property of objects, events, etc., in just the way that being black is, for example. Some think this so, others that there are reasons for treating assertions about spatio-temporal location as in some way special and different. I do not propose to enter that debate here. I only point out that the present dilemma may provide yet one more reason for excluding from the ranks of "normal" predicates those concerned with spatio-temporal location. But these speculations aside, let me return to the major theme of the paper by reminding ourselves that at least this is clear: Whatever "over-arching" difficulties there may be, there is no fear of a *contradiction* here for Hempel over the grue-type predicates and for $E_2^a$-type evidence.

## VI

The discussion of the evidence statement $E_1^a$ is now completed. Since almost all features of note have been reported within this present discussion the remaining evidence statements can be dismissed rather summarily and I shall argue that these remaining evidence statements introduce no new features of interest into the present situation.

*Consideration of $E_1^b$*

Obviously this evidence statement operates as the dual of $E_1^a$ with everything

distinct term from the colloquial forebear, whose merit and relations with our practice in science and elsewhere must be carefully assessed by study of the behaviour of hypotheses within the formal systems involved. Such relations may, and do, I believe, turn out to be much more subtle and complex than might be supposed at first sight.

said under the discussion of $E_1^a$ remaining true for $E_1^b$ if we apply the transformation $H_1 \rightarrow H_2$, $H_0^j \rightarrow {}^jH_0$, $E_1^a \rightarrow E_1^b$, and nothing more need be added.

*Consideration of $E_2^a$*

This evidence statement behaves in precisely the same manner as $E_1^a$. Its dual, $Ema \cdot -Ea \cdot Ga \cdot Emb \cdot Eb \cdot -Gb$, will behave in the same way as $E_1^b$.

*Consideration of $E_2^b$*

This evidence statement includes instances of both $E\_\_$ and $-E\_\_$, colloquially, of both emeralds examined before time $t$ and emeralds not so examined. If we yielded to intuitive pressure it would be said to include instances of emeralds examined both before and after time $t$. Thus $E_2^b$ might be expected, under one of these two senses, to close the "time gap" which occurs in the other evidence statements considered above.

But $E_2^b$ is precisely analogous in its behavior to $E_1^a$ and $E_1^b$, and this is not surprising since $E_2^b$ is simply $E_1^a \cdot E_1^b$ in so far as form is concerned. Thus the $E_1^a$-type part of $E_2^b$ confirms $H_0$ and $H_1$ but disconfirms $H_2$ whilst the $E_1^b$-type part of $E_2^b$ confirms $H_0$ and $H_2$ but disconfirms $H_1$.[21] The net effect of the two in combination is that $E_2^b$ disconfirms $H_1$ and $H_2$ but confirms $H_0$. But this result rules out any prospect of the evidence $E_2^b$ introducing a conflict between $H_1$ and $H_2$, or between either $H_1$ or $H_2$ and $H_0$ or any of its consequences. The only likely candidates for intuitive conflict would be clashes between the two families of consequences of $H_0$, for example between ${}^1H_0$ and $H_0^1$. But only colloquial readings of these two hypotheses (and total neglect of their formal origins) combined with the introduction of 'background' knowledge such as "Mere temporal position can make no difference to the law-like connection, if any, between such predicates as '__ is green' and '__ is an emerald'" could persuade anyone to forget the fact that both of the hypotheses in question are consequences of the same hypothesis, $H_0$, and hence could not possibly be in conflict with one another, since $H_0$ is consistent. Moreover, in the case of the hypotheses ${}^4H_0$ and $H_0^4$ either of their two disjuncts can, in each case, be regarded as that via which the evidence confirms the hypotheses, thus removing even the intuitive difficulty associated with the other evidence statements of this group. (On the other hand there is no conflict between this finding and the previous conclusions. All that has been shown here is that, as each of the set times $t$ is passed, our difficulties over a grue-type predicate defined for that particular time disappear. But this is the minimum that we could expect from any definition of confirmation and does not remove the intuitive difficulty which I have stated.)

Because it is the $H_0^j$ hypotheses which may often present to intuition the appearance of agreeing with $H_1$, and the ${}^jH_0$ hypotheses which may often present the appearance to intuition of agreeing with $H_2$, no intuitive difficulties arise over rows 5, 6, 7, and 10 of table **M** above. The (apparent) conflict arises for row 8 where $E_4^b$ disconfirms $H_1$ but confirms $H_0^1$ and for row 9 where $E_5^a$ disconfirms $H_2$ but confirms ${}^1H_0$. But in these cases the conflict is not at all about the color of

[21] I speak loosely here. Strictly, an evidence statement cannot be broken up and the effects of its parts separately considered in Hempel's system.

emeralds so much as about the restriction to being examined before time $t$. By considering $H_0^3$ we see that this case is the now familiar case in which $H_0^3$ is (trivially) confirmed by "negative evidence," viz. $-Ea \cdot -Eb$, and is therefore of little interest here. There is, of course, no formal conflict so far as Hempel's system is concerned. (Essentially similar remarks apply to row 6 where $E_3^b$ confirms $H_0$ and also confirms $H_1$. The result depends upon $E_3^b$ containing $-Ea$.) The case of $H_2$ and ${}^1H_0$ is essentially the same, with the "negative evidence" being in this case $Ea \cdot Eb$ (i.e. it falsifies the antecedent of the conditional of ${}^3H_0$). This now completes my consideration of the evidence statements listed above.

## VII

In conclusion, it seems unlikely that formal difficulties can be derived for Hempel's system from Goodman's predicate 'grue', or predicates of like structure. Moreover the examination of why this is so leads to valuable insights into Hempel's system. In particular, we are again reminded of the dangers of reading expressions of formal languages colloquially and of importing "illicit" information into the confirmation context.

Finally, there remains an external, over-arching difficulty over the performance of Hempel's system under the introduction of explicitly spatio-temporally restricted evidence statements. I am inclined to agree that this situation does raise further problems related to that vexed question of the nature and content of the relations between a purely logically based system of confirmation such as Hempel's and our actual right to rational expectation and its attendant behavior. But this is a far cry from a claim that Hempel's system is inconsistent.

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