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A PURELY SYNTACTICAL DEFINITION OF CONFIRMATION[1]

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1. Objective of the paper. The concept of confirmation occupies a central position in the methodology of empirical science. For it is the distinctive characteristic of an empirical hypothesis to be amenable, at least in principle, to a test based on suitable observations or experiments; the empirical data obtained in a test—or, as we shall prefer to say, the observation sentences describing those data—may then either confirm or disconfirm the given hypothesis, or they may be neutral with respect to it. To say that certain observation sentences confirm or disconfirm a hypothesis, does not, of course, generally mean that those observation sentences suffice strictly to prove or to refute the hypothesis in question, but rather that they constitute favorable, or unfavorable, evidence for it; and the term "neutral" is to indicate that the observation sentences are either entirely irrelevant to the hypothesis, or at least insufficient to strengthen or weaken it.

A precise definition of the concepts of confirmation, disconfirmation, and neutrality which have just been loosely characterized appears to be indispensable for an adequate treatment of several fundamental problems in the logic and methodology of empirical science; these problems include the elucidation of the logical structure of scientific tests, explanations, and predictions, the logical analysis of the so-called inductive method, and a rigorous statement and development of the operationalist and empiricist conceptions of meaning and knowledge.[2] However, no general definition of the concept of confirmation and of its correlates seems to have been developed so far; and the few rudimentary definitions implicit in recent methodological writings prove unsatisfactory in various respects (cf. section 3, end).

It is the objective of this study to develop a general definition of confirmation in purely logical terms. Confirmation will be construed as a certain logical relationship which may be considered as an ideal model or a "rational reconstruction" of the concept of confirmation as it is used in the methodology of empirical science. Apart from its relevance for the theory of science, the definition of confirmation here to be developed appears to be of interest also from the viewpoint of formal logic; for it is of a purely syntactical character,[3] and it establishes a relation analogous in various ways to the relation of consequence in its syntactical interpretation. In fact, confirmation will prove to be, in a certain

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[1] A modified and expanded version of a paper of the same title of which an abstract appeared in this JOURNAL, vol. 8 (1943), p. 39. The earlier version was scheduled to be read at the meeting of the Association for Symbolic Logic that was to have taken place at Yale University in December 1942.

[2] A detailed discussion of the various methodological aspects of the problem of confirmation will be given in a separate paper, to be published elsewhere, by the present author and Dr. Paul Oppenheim. It is the study, with Mr. Oppenheim, of these broader issues which suggested the problem of defining confirmation in logical terms.

[3] Cf. R. Carnap, ***The logical syntax of language,*** 1937, §1.

sense, a generalization of the converse of the syntactical consequence relation (cf. 3.3 below). The development of a logical theory of confirmation might therefore be regarded as a contribution to a field of study sometimes called inductive logic; a field in which research so far has almost exclusively been concerned with probabilities (in the sense of the "logical" theories[4]) or *degrees* of confirmation[5] rather than with the more elementary non-quantitative relation of confirmation which forms the object of this essay.

2. The model language L. Notational conventions. The present study is restricted to the case where the language of science has a comparatively simple logical structure which will now be characterized; it is essentially that of the lower functional calculus without identity sign. While many different languages satisfy the following stipulations, it will be convenient to formulate the latter as referring to "the" language L.

Throughout this paper, quotation marks to be used for forming designations of expressions will be omitted when the expressions are listed in a separate line.

2.1 The following kinds of signs are permitted in L:

2.11 *Four statement connectives:*

$$\sim \quad \vee \quad . \quad \supset$$

2.12 *Individual constants:* As such we use the small letters 'a', 'b', 'c', 'd', 'e' with and without positive integer subscripts. It will prove convenient, for further reference, to lay down a certain order for these constants, to be called their alphabetical order, namely:

$$a, a_1, a_2, \cdots, b, b_1, b_2, \cdots, c, c_1, c_2, \cdots, d, d_1, d_2, \cdots, e, e_1, e_2, \cdots .$$

2.13 *Individual variables:*

$$x, x_1, x_2, \cdots, y, y_1, y_2, \cdots, z, z_1, z_2, \cdots .$$

2.14 *Predicate constants* of any finite degree; especially:

a) of degree 1,

$$P, P_1, P_2, \cdots, Q, Q_1, Q_2, \cdots ;$$

b) of degree 2,

$$R, R_1, R_2, \cdots .$$

Also, additional predicate symbols—mostly abbreviations of English words—will be used when necessary; example: '*Sw*' for 'Swan.'

2.15 *Universal and existential quantifier signs,* as in:

$$(x)(Ey)R(x, y).$$

2.16 *Auxiliary signs:* parentheses, comma.

2.2 The *sentences of L* or *L-sentences* are formed out of the above kinds of

[4] Such as those of J. M. Keynes, J. Nicod, H. Jeffreys, B. O. Koopman, St. Mazurkiewicz, and others.

[5] Cf. particularly Janina Hosiasson-Lindenbaum, *On confirmation*, this JOURNAL, vol. 5 (1940), pp. 133–148.

signs according to the rules of the lower functional calculus. To indicate grouping, parentheses rather than dots are used. Universality is always expressed by means of a universal quantifier, never by the use of free variables. A class of L-sentences is also called a *sentential class*.

2.3 *The rules of inference for L* are those of the lower predicate calculus without identity sign. As neither sentential variables nor predicate variables are provided for in L, we have to assume that primitive sentential schemata rather than primitive sentences have been laid down, each schema characterizing a certain class of L-sentences as primitive sentences. Furthermore, a set of rules of inference is assumed to be given. The particular way in which the primitive sentential schemata and the rules of inference are chosen is not of importance for the subsequent discussion; we therefore refrain from giving an explicit statement of these stipulations.[6]

2.31 L will be assumed to contain no defined individual or predicate signs; all the terms listed in 2.12 and 2.14 will be considered as primitives. (In effect, the following considerations are also applicable to languages providing for explicit definitions; the above restriction then amounts to the assumption that in sentences studied as to confirmation, all defined non-logical signs have been eliminated by virtue of their definitions.)

A sentence S will be said to be a *consequence* of a class K of sentences if S can be deduced, by means of a finite number of applications of the rules of inference, from the class obtained by adding to K the primitive sentences of L. A sentence S_2 is said to be a consequence of a sentence S_1 if S_2 is a consequence of $\{S_1\}$, i.e., of the class containing S_1 as its only element.

We note the following theorem for later reference:

2.32 *Theorem:* If K is an infinite sentential class, and S is a consequence of K, then there exists a finite subclass K' of K such that S is a consequence of K'.

This follows from the definition of "consequence" in connection with the fact that every rule of inference of the narrower predicate calculus presupposes only a finite number of premises—the entire deduction of S from K can therefore contain only a finite number of sentences.

A class K (or a sentence S_1) will be said to *entail* a sentence S_2 if S_2 is a consequence of K (or of S_1, respectively).

A sentence S is called *analytic* if it is a consequence of the null class of sentences. S is called *contradictory* if its denial is analytic (in this case, every sentence is a consequence of S). S is called *consistent* if it is not contradictory.

A sentential class K is called *inconsistent* if every sentence is a consequence of it; otherwise, it is called *consistent*. Two sentential classes will be said to *contradict* each other, or to be *incompatible* with each other, if their sum is inconsistent; otherwise they are called *compatible*. A sentence S_1 will be said to be incompatible with, or to contradict a sentence S_2 (or a sentential class K) if $\{S_1\} + \{S_2\}$ (or $\{S_1\} + K$, respectively) is inconsistent; otherwise, S_1 will be said to be *compatible* with S_2 (or with K, respectively).

[6] For detailed statements of suitable sets of rules see, for example, R. Carnap, ***Formalization of logic,*** Cambridge, Mass., 1943, p. 135 ff.; and especially D. Hilbert and P. Bernays, ***Grundlagen der Mathematik,*** Vol. I, Berlin, 1934, §4.—Carnap's system involves reference to primitive sentential schemata.

A sentence of atomic form, or an *atomic sentence*, is a sentence consisting of a predicate, followed by a parenthesized expression which consists of as many individual constants—separated by commas—as the degree of the predicate requires. Examples: '$P(a)$', '$R(c, d)$'.

A sentence of basic form, or a *basic sentence*, is a sentence which is either atomic, or the denial of an atomic sentence.—A sentence is said to be a *basic conjunction* if it is either a basic sentence, or a conjunction of basic sentences.

A sentence of molecular form, or a *molecular sentence*, or simply a *molecule*, is a sentence which is either atomic or consists of atomic sentences and statement connectives.

A *generalized sentence* is a sentence containing at least one quantifier.

An expression obtainable from a sentence by replacing at least one individual constant by a variable not occurring in that sentence will be called a (sentential) *matrix*, and in particular a *molecular matrix* if it is obtainable from a molecular sentence.

A sentence consisting of a molecular matrix preceded exclusively by universal quantifiers, or exclusively by existential quantifiers, or by at least one universal and at least one existential quantifier will be called a *universal sentence*, or an *existential sentence*, or a *mixed generalization*, respectively. A sentence will be called *completely generalized* if it contains no individual constants.

A full sentence of a given predicate is an atomic sentence beginning with that predicate. A *full sentence of a given sentential matrix* is one obtainable from the matrix by substituting some individual constant for every variable which is not bound by a quantifier, different occurrences of the same variable being replaced by the same constant.

We now turn to a brief summary of the devices used in this paper to formulate statements about L; some of them have already been employed in the preceding text.

As meta-language for L, we use English, enriched by certain abbreviating symbols and by variables of certain kinds. In particular we use:

Variables for names of sentences of L:

$$M \text{ (reserved for molecules)},$$

$$S, S_1, S_2, \cdots, T, T_1, T_2, \cdots.$$

Variables for names of sentential classes:

$$K, K_1, K_2, \cdots, K^*, K^{**}, \cdots.$$

Unless otherwise stated, the designation of an expression in L is formed by including the expression in quotes.

To form a designation of a finite class of sentences, we shall place their names, separated by commas, between braces. The same device will be used in connection with sentential variables. Thus, '$\{S_1, S_2, S_3\}$' will be short for 'the class whose elements are S_1, S_2, S_3.'

'$\sim S_1$' will designate the denial of S_1, '$S_1 . S_2$' the conjunction of S_1 and S_2, and '$S_1 \vee S_2$', '$S_1 \supset S_2$' will be used analogously. The same stipulation applies to all other sentence name variables and will also be used in connection with names of particular sentences when no misunderstanding seems possible.

The arrow symbol, '$\rightarrow$', will be used as an abbreviation of "has as a consequence" or "entails," as in '$S_1 \rightarrow (S_1 \mathbf{v} S_2)$', '$\{S_1, S_2\} \rightarrow S_1$'. Analogously, the double arrow, '$\leftrightarrow$', will be used to designate equivalence (mutual consequence).

Occasionally the statement connective symbols '$\sim$', '$\mathbf{v}$', '$.$', '$\supset$', '$\equiv$' will be used also in the meta-language; especially in the formulation of theorems, or of definitions such as 3.01, 3.02 below.

Some further notational devices will be explained upon their first occurrence.

Finally, we have to characterize the intended interpretation of the expressions occurring in L. While the solution of our problem will proceed along purely syntactical lines, semantical and pragmatic references are indispensable for an appraisal of the significance of the problem, and for an evaluation of the adequacy of the proposed solution.

The individual constants of L are thought of as names of particular things which form the object of some scientific inquiry. The predicates are to designate certain attributes, i.e., properties or relations, of those things; and we conceive of them as having been chosen in such a way that the observational or experimental methods used in the inquiry make it possible to ascertain the absence or presence of those properties or relations in a given individual or group of individuals. In other words, we assume that any test report, i.e., any complex of "data" obtainable by the testing procedures in question, can be formulated by means of full sentences of predicates of L. To state this idea in a precise manner, one might stipulate that each single datum obtainable in those tests is expressible in a basic sentence, and that every test report has to have the form of a basic conjunction; or one may, as we shall do here, adopt the more liberal stipulation that every possible test report is of molecular form. Now, the scientific test of any hypothesis is based on a confrontation of the hypothesis with empirical data, i.e., with a test report. We shall therefore aim at defining confirmation as a relation which obtains, under certain conditions, between a molecular sentence M (representing the observation report) and a sentence S which may be of any form (representing the hypothesis). This corresponds to the idea that the crude data obtained by the testing procedure are always particular in character; but that they may serve to corroborate or weaken hypotheses of general as well as of particular form.

3. Restatement of the problem. Criteria of adequacy for its solution. In view of the preceding considerations, our problem assumes a purely syntactical character. In its original form it presented also a semantical and a pragmatical aspect; for it amounted to this question: Under what conditions shall we say that a report on the outcome of a certain test confirms a given hypothesis? Thus, reference to actual testing procedures was involved. But clearly, it is immaterial for the definition of a logical relation of confirmation whether the given report represents actual or hypothetical data. In fact, our definition will be satisfactory only if it enables us to discuss questions of the type: "Supposing that such and such data were obtained, would they confirm such and such a hypothesis?"; "What empirical findings would constitute disconfirming evidence for this hypothesis?"; etc. Our problem, then, will be to lay down purely formal conditions under which a molecule M confirms a sentence S.

Any proposed definition of a relation Cf of confirmation will automatically determine definitions of the two associated relations of disconfirmation and of neutrality, according to the following schemata:

3.01 $\mathrm{Dscf}(M, S) \equiv \mathrm{Cf}(M, \sim S)$.

3.02 $\mathrm{Ntl}(M, S) \equiv (\sim \mathrm{Cf}(M, S) \,.\, \sim \mathrm{Dscf}(M, S))$.

While the indications given so far in this essay as to the intended meaning of "confirmation" do not determine this concept unambiguously and precisely, it is clear that any acceptable definition of confirmation will have to satisfy certain criteria of adequacy. These may be divided into two classes, logical and material conditions of adequacy, which will now be discussed in turn.

Logical conditions of adequacy. The logical requirements are stated in three groups. In each group, the fulfillment of the first condition entails that of all others. Those other conditions are mentioned for two reasons; first, because most of them represent important characteristics which would generally be sought in an adequately defined concept of confirmation; and secondly, because some apparently reasonable alternative definitions which we shall examine, turn out to satisfy some of those weaker conditions, but not the strongest of each group. Confrontation with the different requirements explicitly stated will thus provide a yardstick for the appraisal of what might be termed the degree of adequacy of a proposed definition of confirmation.

First group of logical requirements.

3.1 *General consequence condition:* If M confirms every sentence of a class K, then it also confirms every consequence of K.

3.11 *Special consequence condition:* If M confirms S, then it also confirms every consequence of S.

3.12 *Equivalence condition:* If M confirms S, then it also confirms every equivalent sentence. (Intuitively, this means that whether a sentence is confirmed by a given molecule or not, depends only on its content, not on its formulation.)

3.13 *Conjunction condition:* If M confirms every one of a finite number of sentences, then it confirms also their conjunction.

Second group of logical requirements.

3.2 *General consistency condition:* Every consistent molecule is compatible with the class of all the sentences which it confirms.

3.21 *Special consistency condition:* The class of all the sentences confirmed by a consistent molecule is consistent.

3.211 No consistent molecule confirms and disconfirms the same sentence.

3.22 No consistent molecule confirms a contradictory sentence.

Third group of logical requirements.

3.3 *Entailment condition:* If $M \rightarrow S$, then M confirms S. (This means that the relation of entailment in L, with its domain restricted to molecular sentences, is contained in the relation of confirmation, and that, in this sense, the concept of confirmation is a generalization of that of entailment.)

3.31 Any molecule confirms any analytic sentence.

3.32 A contradictory molecule confirms every sentence.

It might be thought plausible that 3.3 should be supplemented by stipulating: If $S \rightarrow M$, then M confirms S. Indeed, if this condition is satisfied—as, for instance, in the case where M = '$P(a) . P(b) . P(c)$' and S = '$(x)P(x)$'—then, it might be argued, all the information conveyed by M bears out the hypothesis S. However, the stipulation under consideration is far too comprehensive: together with 3.11, it would make any molecule confirm every sentence whatsoever. For, let S be an arbitrary sentence, then, by the contemplated stipulation, M would confirm $M.S$, and consequently, by 3.11, also S.

Let us note that as a consequence of 3.01, 3.02, and 3.211, every consistent molecule M determines a division of all sentences of L into three mutually exclusive classes: those which M confirms, those which it disconfirms, and those with respect to which it is neutral.—If M is inconsistent, then it entails every sentence and thus, because of 3.3, both confirms and disconfirms every sentence.

The material condition of adequacy amounts, briefly, to the requirement that the proposed definition of confirmation should be in sufficiently close agreement with the customary meaning of the concept of confirmation as it is used in the methodology of empirical science. Naturally, this requirement is vague, and it refers to certain standards which contain a fairly strong subjective element: what appears as a materially quite satisfactory definition of confirmation to one person, may be judged artificial and too comprehensive or too narrow by another; also, opinion as to the adequacy of a proposed definition may vary with one and the same person in the course of his occupation with the problem.

Consider, for example, the hypothesis 'All swans are white,' which we symbolize by '$(x)(Sw(x) \supset Wh(x))$', and the molecules '$Sw(a) . Wh(a)$' and '$\sim Sw(b) \sim Wh(b)$'. It might be felt—and would probably be argued by many—that under a materially adequate definition of confirmation, the first molecule should be confirming, the second neutral with respect to the hypothesis. However, according to the same intuitive standards, the second molecule would constitute confirming evidence with respect to the hypothesis '$(x)(\sim Wh(x) \supset \sim Sw(x))$'. But the two hypotheses here mentioned are equivalent, and adherence to the intuitive standards exemplified in this illustration would therefore make the confirmation of a hypothesis by a certain body of evidence a matter not only of the content of the hypothesis, but also of the form in which it is expressed. This feature appears intuitively intolerable and was, in fact, explicitly ruled out by the equivalence condition 3.12. As this case shows, some aspects of the intuitive idea of confirmation—each of which separately may appear quite reasonable—are incompatible with each other. It is therefore to be expected that certain apparently very natural intuitive requirements will have to be sacrificed in the interest of others which appear to have greater systematic importance.

In addition to being vague in the sense just illustrated, the material condition of adequacy is also indefinite in the specific sense that the material adequacy of any proposed criterion of confirmation can be ascertained only inductively, as it were, namely by applying the criterion to certain concrete cases which, on intuitive grounds, appear as unquestionable instances of confirmation (or of disconfirmation or neutrality, as the case may be), and by checking whether the formal criterion in question yields the desired result. But clearly, even if a given definition of confirmation passes any number of tests of this type, we can never be quite certain that it does not possess some features which, if only recognized, would stamp it materially inadequate or at least awkward. Despite its indefiniteness, the question of material adequacy cannot be disregarded, and it will be taken into consideration in every step of the subsequent development of a definition of confirmation.

Certain conceptions of confirmation which seem to be reflected in recent methodological discussions would have to be ruled out on the basis of the above requirements of adequacy.

Thus, one customary criterion of confirmation, if stated explicitly, would lead to the following definition: A confirming instance for a hypothesis of universal conditional form—say, '$(x)(y)(R_1(x, y) \supset R_2(x, y))$'—consists in the conjunction of two full sentences obtained by replacing each individual variable in the molecular matrix which constitutes the antecedent by some individual constant, and by performing the same substitution in the consequent.[7] In the above illustration, '$R_1(a, b) \,.\, R_2(a, b)$' would be a confirming instance.—However, this criterion is open to various objections: (a) It violates the formal requirements 3.1, 3.11, 3.12.[8] This is readily seen from the earlier discussion referring to the hypothesis '$(x)(Sw(x) \supset Wh(x))$'. (b) An interesting aspect of the violation of 3.12 is the following: Certain universal conditionals, such as '$(x)((P(x) \,.\, {\sim}Q(x)) \supset (Q(x) \,.\, {\sim}Q(x)))$', where the consequent is contradictory (but the antecedent not analytic), clearly cannot have any confirming instances under the criterion in question; and yet these universal conditionals are not contradictory; the one just mentioned, for example, is equivalent with the hypothesis '$(x)(P(x) \supset Q(x))$'. (c) The criterion under consideration is restricted to hypotheses of universal conditional form, whereas an adequate definition of confirmation should be applicable to hypotheses of other forms as well.

An alternative approach might be suggested by the consideration that the test of a scientific hypothesis consists in examining its predictive power, i.e., in deducing from it and certain observation sentences certain new observation sentences, and in seeing whether these predictions come true. This idea could be reflected in a syntactical definition of the following type: A class K of molecules (which, incidentally, is equivalent to one molecule M) confirms a given sentence S if K can be exhaustively divided into two mutually exclusive classes K_1, K_2 such that K_2 is non-null, and every sentence in K_2 is a consequence of

[7] A criterion to essentially this effect was formulated by Jean Nicod in ***Foundations of geometry and induction,*** London 1930, p. 219.

[8] With respect to the equivalence condition, this difficulty was exhibited already in the author's *Le problème de la vérité*, Theoria (Göteborg), vol. 3 (1937), pp. 206–246, esp. p. 222.

$K_1 + \{S\}$ but not of K_1 alone.—But this type of definition clearly violates the requirements 3.1 and 3.11; also, it is materially inadequate in that, for instance, the molecule '$P(a)$' would not by this criterion confirm the hypothesis '$(Ex)P(x)$' (this feature also violates 3.3); and more generally, it would not provide any possibility of confirmation at all for existential sentences because these cannot satisfy the above condition.—Finally, the consistency condition would be violated; thus, e.g., the class $\{$'$P_1(a)$', '$P_2(a)$'$\}$ would confirm $(x)(P_1(x) \supset (P_2(x) \,.\, P_3(x)))$' as well as, say '$(x)(P_1(x) \supset (P_2(x) \,.\, \sim P_3(x)))$'.

4. The *C*-development of a sentence. The basic idea of the definition of confirmation here to be developed may be illustrated by the following example:

If S and M are the sentences '$(x)(P(x) \supset Q(x))$' and '$P(a) \,.\, Q(a) \,.\, P(b) \,.\, Q(b) \,.\, \sim Q(c) \,.\, \sim P(c)$', respectively, then M can be said to confirm S in the following sense: S asserts that the extension of P is included in the extension of Q (and, consequently, the extension of $\sim Q$ in that of $\sim P$); and for the objects referred to in M, namely, a, b, and c, it is indeed the case that all those reported in M as belonging to the extension of P are also reported as belonging to the extension of Q, and those reported as belonging to the extension of $\sim Q$ are also reported as belonging to the extension of $\sim P$.[9]

The problem now presents itself of developing an explicit criterion which embodies this idea, and which is applicable also to hypotheses that cannot be interpreted as asserting a relation of inclusion, as is the case, in particular, with existential sentences and mixed generalizations. We therefore state the inclusion criterion in a modified version which lends itself more readily to the necessary expansion.

The above molecule M determines a certain class of individuals $\{a, b, c\}$; and we may say that M confirms S because from the information contained in M it can be inferred that in $\{a, b, c\}$ S is completely satisfied; or, to use a metaphor: in a world containing exclusively the individuals a, b, and c, the sentence S would be true, according to the information contained in M. Now, for a world of that finite kind, the content of S could be expressed in terms of a certain molecular sentence, namely '$(P(a) \supset Q(a)) \,.\, (P(b) \supset Q(b)) \,.\, (P(c) \supset Q(c))$'; and the assertion that S would be true in that world, according to the information contained in M, can be expressed more precisely by saying that the above molecular sentence is a consequence of M. We shall call that molecular sentence the *IM-development of S* (i.e., the development of S for the individual constants contained in M). The idea of *IM*-development can readily be defined in a general manner, so as to be applicable to sentences of any form; thus, for example, the development of the sentence '$(x)(Ey)R(x, y)$' for the domain determined by the molecule '$R(a, a) \,.\, \sim R(a, b) \,.\, R(c, b) \,.\, R(b, a)$' is the following:

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[9] This idea of defining confirmation in terms of an "inclusion criterion" was suggested to me by Dr. Nelson Goodman as offering considerable advantages over an attempt which I had made before to define confirmation in terms of "inductive attainability" of S from B—an approach which is not discussed in the present paper. Dr. Goodman's idea proved an invaluable help for the present investigation; in fact, it initiated all the following considerations.

'$(R(a, a) \vee R(a, b) \vee R(a, c)) \,.\, (R(b, a) \vee R(b, b) \vee R(b, c)) \,.\, (R(c, a) \vee R(c, b) \vee R(c, c))$'. In terms of this concept, whose general definition will be given presently, the central idea of the definition of confirmation here to be developed assumes this form: A sentence S is confirmed by a molecule M if M entails the IM-development of S. As will be seen below, this idea still requires certain modifications if it is to yield an acceptable concept of confirmation.

In the above illustrations of the concept of the development of S we referred to a domain of objects, and to their satisfying certain conditions. Thus we left the sphere of purely syntactical analysis and made use of semantical relations. However, it proves possible to express the intended idea in exclusively syntactical terms. This is done in the following definition:

4.1 *Df.* Let C be a finite class of individual constants (not individuals) and S a sentence. Then the *C-development of* S is a sentence $\mathrm{D}_C(S)$, which is determined by the following recursive definition:

A) If C is empty, then $\mathrm{D}_C(S) = S$.

B) If C is not empty, then:

I a. $\mathrm{D}_C(\sim S) = \sim \mathrm{D}_C(S)$.

b. $\mathrm{D}_C(S_1 \vee S_2) = \mathrm{D}_C(S_1) \vee \mathrm{D}_C(S_2)$.

c. $\mathrm{D}_C(S_1 \,.\, S_2) = \mathrm{D}_C(S_1) \,.\, \mathrm{D}_C(S_2)$.

d. $\mathrm{D}_C(S_1 \supset S_2) = \mathrm{D}_C(S_1) \supset \mathrm{D}_C(S_2)$.

II a. If S is atomic, then $\mathrm{D}_C(S) = S$.

b. If S is of the form $(\xi)\Phi$ where ξ is an individual variable and Φ a sentence or a matrix, then:

b1. If Φ contains no free occurrence of ξ, then $\mathrm{D}_C(S) = \mathrm{D}_C(\Phi)$.

b2. If Φ contains free occurrences of ξ, then let

$$\prod_{\gamma \epsilon C} \Phi_\xi^\gamma$$

be the conjunction of the following sentences, in the order here described: The sentence obtained from Φ by replacing all free occurrences of ξ by the alphabetically first (cf. 2.12) individual constant in C; the sentence obtained analogously by means of the alphabetically second constant in C; and so on. Then

$$\mathrm{D}_C(S) = \mathrm{D}_C(\prod_{\gamma \epsilon C} \Phi_\xi^\gamma).$$

c. If S is of the form $(E\xi)\Phi$, then:

c1. If Φ contains no free occurrence of ξ, then $\mathrm{D}_C(S) = \mathrm{D}_C(\Phi)$.

c2. If Φ contains free occurrences of ξ, let

$$\sum_{\gamma \epsilon C} \Phi_\xi^\gamma$$

be the disjunction of the sentences described under 2.2, taken in the same order. Then

$$\mathrm{D}_C(S) = \mathrm{D}_C(\sum_{\gamma \epsilon C} \Phi_\xi^\gamma).$$

Notational conventions: (a) When names of particular sentences occur as arguments of 'D_C', the name-forming quotes will be absorbed by the frame '$D_C(\ )$'. (b) Also, in sentences of the form $D_C(\cdots) = ---$, where on the right hand side a quoted sentential name would have to appear, we shall have the quotes absorbed by the identity sign.

Thus, e.g., if $C = \{\text{'}a\text{'}, \text{'}b\text{'}\}$, we shall write

$$D_C((x)P(x)) = P(a) \,.\, P(b)$$

instead of

$$D_C(\text{'}(x)P(x)\text{'}) = \text{'}P(a) \,.\, P(b)\text{'}.$$

The left-hand side is simplified according to part (a), the right-hand side according to part (b) of the notational conventions.

Illustrations of the above definition.

4.11 Let $S = \text{'}(x)(Ey)R(x, y)\text{'}$; $C = \{\text{'}a\text{'}, \text{'}b\text{'}, \text{'}c\text{'}\}$; then:

$$\begin{aligned} D_C(S) &= D_C((Ey)R(a, y) \,.\, (Ey)R(b, y) \,.\, (Ey)R(c, y)) && \text{(by B II b2)} \\ &= D_C((Ey)R(a, y)) \,.\, D_C((Ey)R(b, y)) \,.\, D_C((Ey)R(c, y)) && \text{(by B I c)} \\ &= D_C(R(a, a) \vee R(a, b) \vee R(a, c)) \,.\, D_C(R(b, a) \vee R(b, b) \vee R(b, c)) \\ &\qquad .\, D_C(R(c, a) \vee R(c, b) \vee R(c, c)) && \text{(by B II c2)} \\ &= (R(a, a) \vee R(a, b) \vee R(a, c)) \,.\, (R(b, a) \vee R(b, b) \vee R(b, c)) \\ &\qquad .\, (R(c, a) \vee R(c, b) \vee R(c, c)) && \text{(by B I b and B II a).} \end{aligned}$$

This, incidentally, is the sentence which was given above as the IM-development of S for $M = \text{'}R(a, a) \,.\, {\sim}R(a, b) \,.\, R(c, b) \,.\, R(b, a)\text{'}$.

4.12 Let $C = \{\text{'}a\text{'}, \text{'}b\text{'}\}$, $S = \text{'}(x)(P(x) \vee Q(c))\text{'}$. Then, as is readily verified: $D_C(S) = (P(a) \vee Q(c) \,.\, (P(b) \vee Q(c))$.

4.13 For every molecular sentence M and every finite class C of individual constants, $D_C(M) = M$.

Subsequently, the expressions 'c.g. sentence' and 'c.g. class' will serve as abbreviations of 'completely generalized sentence' and 'class of completely generalized sentences,' respectively.

We note the following theorems on C-development:

4.2 *Theorem.* If C is a finite class of individual constants and S_1, S_2 are c.g. sentences such that $S_1 \rightarrow S_2$, then $D_C(S_1) \rightarrow D_C(S_2)$.

Proof. If $S_1 \rightarrow S_2$, then $S_1 \supset S_2$ is analytic,[10] and hence identically true for any finite domain;[11] i.e., $D_C(S_1 \supset S_2)$ is an analytic sentence for every finite class C of individual constants. Now $D_C(S_1 \supset S_2) = D_C(S_1) \supset D_C(S_2)$ (4.1 B I d); therefore, the latter sentence is analytic, and by virtue of this fact and the modus ponens rule, $D_C(S_1) \rightarrow D_C(S_2)$.

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[10] Cf. Hilbert and Bernays, loc. cit., p. 155, or Carnap, loc. cit. (see footnote 6), p. 142, T 28-11.

[11] Cf. Hilbert and Bernays, loc. cit., p. 121, Theorem 1.

4.21 *Theorem.* If S_1, S_2 are equivalent c.g. sentences, then $D_C(S_1)$, $D_C(S_2)$ are equivalent for every finite class C of individual constants. (From 4.2.)

4.3 *Theorem.* Let C be a finite class of individual constants, K a finite class of c.g. sentences, $D_C(K)$ the class of the C-developments of the elements of K, and S a c.g. sentence such that $K \rightarrow S$; then $D_C(K) \rightarrow D_C(S)$.

Proof. Let

$$\prod_{T \epsilon K} T$$

be the conjunction of the elements of K taken in any one order, then

$$\prod_{T \epsilon K} T \rightarrow S;$$

hence,. by 4.2,

$$D_C(\prod_{T \epsilon K} T) \rightarrow D_C(S);$$

furthermore, by 4.1,

$$D_C(\prod_{T \epsilon K} T)$$

is a conjunction of the C-developments of the elements of K, and therefore follows from $D_C(K)$; thus we have

$$D_C(K) \rightarrow D_C(\prod_{T \epsilon K} T) \rightarrow D_C(S),$$

which proves the theorem.

4.4 *Theorem.* If a class K of c.g. sentences is inconsistent, then, for every finite C, the class of the C-developments of the sentences of K is also inconsistent.

Proof. K must contain at least one sentence to be inconsistent. Let $S \epsilon K$. Then, since K is inconsistent, $K \rightarrow \sim S$. Hence, by 4.3, $D_C(K) \rightarrow D_C(\sim S)$, or, by virtue of 4.1, $D_C(K) \rightarrow \sim D_C(S)$. This shows that $D_C(K)$ is inconsistent, since also $D_C(S) \epsilon D_C(K)$.

4.41 *Theorem.* For every finite C, a contradictory c.g. sentence has a contradictory C-development. (From 4.4)

4.42 *Theorem.* For every finite C, an analytic c.g. sentence has an analytic C-development.

Proof. If S is an analytic c.g. sentence, then $\sim S$ is a contradictory c.g. sentence; now, by 4.1, $D_C(S)$ is equivalent with $\sim D_C(\sim S)$; and since, by 4.41, $D_C(\sim S)$ is contradictory, $D_C(S)$ is analytic.

4.51 *Note.* The converse of theorem 4.4 and its corollaries does not hold: it may happen that for a certain class C of individual constants a c.g. sentence S has an analytic or a contradictory C-development without being analytic or contradictory itself. For example, let S_1 = '$(x)P(x) \vee (x)\sim P(x)$', and S_2 = $(Ex)P(x) \,.\, (Ex)\sim P(x)$'. Then, if C contains exactly one element, $D_C(S_1)$ will be analytic. Thus, if $C = \{$'a'$\}$, $D_C(S_1)$ is '$P(a) \vee \sim P(a)$'. On the other hand, S_2 can be true only in a domain of at least two individuals; for a class C containing only one element its C-development is contradictory; thus, if $C = \{$'a'$\}$, then $D_C(S_2) = P(a) \,.\, \sim P(a)$.

By developing these illustrations a little further, it can also readily be seen that for a given C and two completely generalized sentences S_1, S_2, it may happen that $D_C(S_1) \rightarrow D_C(S_2)$, while it is not the case that $S_1 \rightarrow S_2$. This shows that the converse of 4.2 does not hold.

4.52 *Note.* None of the theorems 4.2 through 4.42 holds for all generalized sentences, i.e., including those which contain individual constants. For let S_1 = '$(x)P(x)$', S_2 = '$(x)(P(x) \,.\, P(a))$', $C = \{$'b'$\}$; then $S_1 \leftrightarrow S_2$; but $D_C(S_1)$ = '$P(b)$'; $D_C(S_2)$ = '$P(b) \,.\, P(a)$', and thus neither $D_C(S_1) \leftrightarrow D_C(S_2)$ nor even $D_C(S_1) \rightarrow D_C(S_2)$. This provides counter-examples for 4.2—and thus for 4.3—as well as for 4.21; to obtain a counter-example for 4.41—and thus for 4.4—let S = '$(x)P(x).\sim P(a)$', $C = \{$'b'$\}$, so that $D_C(S)$ = '$P(b) \,.\, \sim P(a)$', which is non-contradictory; finally, as counter-example for 4.42, choose S = '$P(a) \supset (Ex)P(x)$', $C = \{$'b'$\}$, which yields the non-analytic $D_C(S)$ = '$P(a) \supset P(b)$'.

5. Preliminary remarks on the subsequent definitions of confirmation. We now turn to the systematic construction of a syntactical concept of confirmation. We shall begin by formulating a first definition in strict accordance with the idea outlined in the beginning of section 4: M will be said to confirm S if M entails the IM-development of S. Closer examination of the concept thus determined will reveal certain inadequacies which will then be removed by constructing a second, revised definition; the latter, in turn, will be replaced by a modified and more satisfactory final version. Lest the reader be alarmed by the propsect of being needlessly led astray by a study of certain tentative definitions which will later be abandoned, it may be well to emphasize that it is not intended to present here all the various attempts at defining confirmation which were made in connection with this study. The few variants that will be considered here have been selected for systematic reasons, namely because they represent, as it were, successive approximations of the definition finally to be proposed; in fact, every definition subsequently to be considered presupposes the preceding ones or certain theorems proved in connection with them.

For reasons which will be exhibited in the following section, the relation of confirmation, Cf, will, in each of the successive stages of our discussion, be defined in terms of a narrower relation of *direct confirmation*, Cfd. The manner in which Cf is defined in terms of Cfd will remain the same throughout, and the gradual modifications referred to will concern the definition of Cfd.

6. A first approximation: $\mathbf{Cfd_1}$ and $\mathbf{Cf_1}$. The following definition of direct confirmation embodies the idea presented in the beginning of section 4:

6.1 *Df.* $\mathrm{Cfd}_1(M, S)$ if and only if: (a) M is a molecule; (b) S is a c.g. sentence; and (c) $M \rightarrow D_{IM}(S)$, where IM is the class of those individual constants which occur in M.

The following theorems hold for Cfd_1:

6.11 *Theorem.* Within the class of all c.g. sentences, Cfd_1 satisfies the general consequence condition 3.1; i.e., if K is a c.g. class and M a molecule such that $\mathrm{Cfd}_1(M, T)$ for every $T \in K$, and if S is a c.g. sentence such that $K \rightarrow S$, then $\mathrm{Cfd}_1(M, S)$.

Proof. In view of 2.32, K may be assumed to be finite. Let $D_{IM}(K)$ be the class of the *IM*-developments of the elements of K. Then, since $K \rightarrow S$, we have $D_{IM}(K) \rightarrow D_{IM}(S)$, by 4.3; furthermore, by hypothesis, $M \rightarrow T$ for every $T \epsilon D_{IM}(K)$; hence: $M \rightarrow D_{IM}(S)$, and thus $\mathrm{Cfd}_1(M, S)$.

6.12 *Theorem.* Cfd_1 satisfies the general consistency condition 3.2; i.e., if M is a consistent molecule and K^* the class of all S such that $\mathrm{Cfd}_1(M, S)$, then $K^* + \{M\}$ is a consistent class.

Proof. Suppose that $K^* + \{M\}$ is inconsistent. Then there exists a sentence T such that $K^* + \{M\} \rightarrow T$, $K^* + \{M\} \rightarrow \sim T$. In view of 2.32, there exists even a finite subclass K of K^* such that $K + \{M\} \rightarrow T$, $K + \{M\} \rightarrow \sim T$. Let S_K be the conjunction of the elements of K. Then $S_K . M$ would be a contradictory sentence. This will now be shown to be impossible. The core of this proof is the following consideration: If $S_K . M$ is a contradictory sentence, then it cannot be satisfied in any domain. But in the domain consisting exclusively of the individuals mentioned in M, clearly M is satisfiable if, as was presupposed, it is consistent. And if M is satisfied, then so is S_K; for in the finite domain in question, S_K is equivalent with $D_{IM}(S_K)$, and the latter sentence is, by hypothesis, a consequence of M, and thus is satisfied whenever M is.

This idea can be expressed more precisely as follows: Let $IM = \{`a_1', `a_2', \cdots, `a_k'\}$; and let '$m$', '$s_k$', '$ds_k$' be abbreviations of the sentences M, S_K, $D_{IM}(S_K)$ respectively. (Note. 'm' is an *abbreviation* for the sentence *designated* by 'M', etc. These abbreviations cannot occur in L, as no definitions are allowed in that language; but we may introduce them into the meta-language.) The idea that in a world containing only the individuals $a_1, \cdots, a_k$, the sentence S_K holds if $D_{IM}(S_K)$ does, can now be expressed by the statement that the sentence

$$(x)((x=a_1) \vee (x=a_2) \vee \cdots \vee (x=a_k)) \supset (ds_k \supset s_k)$$

is analytic (not in L, which does not contain the identity sign, but in the meta-language, whose logic has, of course, to include the lower predicate calculus with identity sign). Now, since by hypothesis M stands in Cfd_1 to every S in K, we have $M \rightarrow D_{IM}(S_K)$; thus, '$m \supset ds_k$' is analytic; hence, finally,

$$((x)((x=a_1) \vee (x=a_2) \vee \cdots \vee (x=a_k)) . m) \supset (s_k . m)$$

is analytic. Now, if $S_K . M$, and thus '$s_k . m$' were contradictory, then

$$(x)((x=a_1) \vee (x=a_2) \vee \cdots \vee (x=a_k)) . m$$

would be contradictory, and hence could not be satisfied in any finite domain; i.e., taking 'a_1', 'a_2', $\cdots$, 'a_k', and all the extralogical predicates occurring in M as uninterpreted parameters, there would not exist a finite domain in which those individual and predicate constants could be so interpreted as to make the last formula a true sentence.[12] But actually, the sentence is satisfiable in a domain of k individuals. We only have to construe 'a_1', 'a_2', $\cdots$, 'a_k' as names of any k individuals, and to interpret the predicates occurring in M in such a way that M becomes a true sentence under that interpretation. And that is possible because by hypothesis M is consistent.

[12] Cf. Hilbert and Bernays, loc. cit., pp. 185–186.

6.13 Cfd_1 fails to satisfy the entailment condition; thus, e.g., $\text{Cfd}_1(M, M)$ cannot hold for any molecule because of the clause 6.1(b).

For the same reason, Cfd_1 violates the general consequence condition; thus, '$P(a)$' stands in Cfd_1 to '$(x)P(x)$', but not to '$P(a)$'.

These shortcomings, however, can be remedied by defining, in terms of 'Cfd_1', a broader concept of confirmation, 'Cf_1', as follows:

6.2 *Df.* $\text{Cf}_1(M, S)$ if and only if (a) M is a molecule, and (b) S is a consequence of a sentential class K each element T of which satisfies one of the following conditions:

$$1.\ M \rightarrow T; \qquad\qquad 2.\ \text{Cfd}_1(M, T).$$

Clause (b) can be symbolized as follows:

$$(EK)((K \rightarrow S) \,.\, (T)(T \in K \supset ((M \rightarrow T) \vee \text{Cfd}_1(M, T))))$$

6.21 *Theorem.* Cf_1 satisfies the general consequence condition.

6.22 *Theorem.* Cf_1 satisfies the general consistency condition.

6.23 *Theorem.* Cf_1 satisfies the entailment condition.

6.24 *Theorem.* Cfd_1 is a proper subrelation of Cf_1.

Proofs of these theorems follow.

Proof of 6.24. (a) Cfd_1 is a subrelation of Cf_1. Let $\text{Cfd}_1(M, S)$; then there exists a K which satisfies 6.2 (b), namely $\{S\}$; hence $\text{Cf}_1(M, S)$. (b) Cf_1 is not a subrelation of Cfd_1. Thus, e.g., '$P(a)$' stands in Cf_1 (via '$(x)P(x)$') but not in Cfd_1 to '$P(b)$'.

Proof of 6.23. If $M \rightarrow S$, then there exists a K which satisfies 6.2 b, namely $\{M\}$.

Proof of 6.21. Let M be a molecule and K_1 a class such that $\text{Cf}_1(M, T)$ for every $T \in K_1$; let $K_1 \rightarrow S$. We have to prove that $\text{Cf}_1(M, S)$. By hypothesis and 6.2(b), every $T \in K_1$ is a consequence of a class K_T such that M entails or stands in Cfd_1 to every element of K_T. Let $\sum K_T$ be the sum of these classes. Then, clearly, S is a consequence of $\sum K_T$, i.e., of a class such that M entails or stands in Cfd_1 to each of its elements. Hence, by 6.2, $\text{Cf}_1(M, S)$.

Proof of 6.22. Let M be a consistent molecule and K^{**} the class of all sentences to which M stands in Cf_1. Assume $K^{**} + \{M\}$ to be inconsistent. Then K^{**} would be inconsistent, for $M \in K^{**}$ by virtue of 6.23. Now, for every $T \in K^{**}$, we have by hypothesis $\text{Cf}_1(M, T)$; i.e., T is a consequence of a sentential class K_T such that M either entails or stands in Cfd_1 to each of its elements. If, therefore, K^* is the class of all sentences to which M stands in Cfd_1, then every sentence in K^{**} is certainly a consequence of $\{M\} + K^*$; and if K^{**} were inconsistent, then $\{M\} + K^*$ would have to be inconsistent. But that is impossible in view of 6.12.

The concept of confirmation determined by 6.2 thus satisfies all the formal conditions of adequacy set up above. As regards its material adequacy, however, it might be argued that while the basic idea of the definition appears satisfactory, its formalization in 6.2 involves an unnecessary restriction. Thus, if a test report contains a certain amount of definitely favorable evidence for a hypothesis, and in addition some entirely irrelevant statements, then the report might well be considered as confirming the hypothesis. Not generally so, however, according to the above definitions of 'Cfd_1' and 'Cf_1'. Let, for example, S = '$(x)P(x)$'; M_1 = '$P(a) . P(b)$', M_2 = '$P(a) . P(b) . Q(a)$', M_3 = '$P(a) . P(b) . Q(c)$'. Then $\mathrm{D}_{IM_1}(S) = \mathrm{D}_{IM_2}(S)$ = '$P(a) . P(b)$', and thus $\mathrm{Cfd}_1(M_1, S)$ and $\mathrm{Cfd}_1(M_2, S)$; and, by 6.24, $\mathrm{Cf}_1(M_1, S)$ and $\mathrm{Cf}_1(M_2, S)$; but $\mathrm{D}_{IM_3}(S)$ = '$P(a) . P(b) . P(c)$', and thus M_3 does not stand in Cfd_1 to S; nor does it stand in Cf_1 to S either. (This latter statement can be proved by means of the method used in the proof of theorem 6.3 below; we omit the details here.)

One might feel inclined to change this situation by defining 'M confirms S' by: 'M has a consequence of molecular form which stands in Cf_1 to S'. By this standard, M_3 would confirm S. However, the new criterion is much too liberal: According to it, the molecule '$P(a) . \sim P(b)$' would, by virtue of its consequence '$P(a)$', confirm '$(x)P(x)$'; and, by virtue of its consequence '$\sim P(b)$', it would confirm '$(x)\sim P(x)$'; thus, the consistency requirement would be violated. But upon somewhat closer inspection the intuitive difficulty which the contemplated modification was designed to overcome appears anyhow to be of minor significance; the concept defined in 6.2 proves to be somewhat narrower than intuitive usage would require; but clearly it has to be expected that a precise redefinition of a customarily vague concept will to some extent be at variance with the intuitive meaning of the original. Besides, the concept introduced in 6.2 provides sufficient means for stating in what sense M_3 constitutes, "on the whole," as it were, favorable evidence for S; namely thus: M_3 has consequences of molecular form which stand in Cf_1 to S_1 but none which stand in Dscf_1 to S (i.e., in Cf_1 to $\sim S$; cf. 3.01).

But there is another consequence of the definition of 'Cf_1' which requires consideration here: The conditions which M has to satisfy to confirm a generalized, but not completely generalized hypothesis appear to be too rigorous. Let, for example, S = '$(y)R(a,y)$', M_1 = '$R(a,a) . R(a,b)$'. Now it may be argued that by the same token by which '$P(a) . P(b)$' confirms '$(x)P(x)$', M_1 should be designated as confirming S; analogously, M_2 = '$R(a, a) . R(a, b) . R(a, c)$', etc., should represent confirming evidence for S. But, while '$P(a) . P(b)$' does stand in Cf_1 to '$(x)P(x)$', neither M_1 nor M_2 nor any of the analogous longer molecules stands in Cf_1 to S. This will now be proved for M_1; the proof can readily be extended to the other cases.

6.3 *Theorem.* M_1 = '$R(a, a) . R(a, b)$' does not stand in Cf_1 to S = '$(y)R(a, y)$'.

Proof. Note that clause (b) in 6.2 is equivalent with

$$(EK)((T)(T \epsilon K \supset \mathrm{Cfd}_1(M, T)) . (K + \{M\} \rightarrow S)).$$

Hence, if the theorem is false, then there exists a class K—in view of 2.32 it may be assumed finite—such that $K + \{M_1\} \rightarrow S$ and $\mathrm{Cfd}_1(M_1, T)$ for every $T \epsilon K$. Let S_K be the conjunction of the elements of K and let 's_k' be an abbreviation of that conjunction. Then $S_K . M_1 \rightarrow S$; i.e., '$s_k . R(a, a) . R(a, b)$' $\rightarrow$ '$(y)R(a, y)$'. Therefore, '$(s_k . R(a, a) . R(a, b)) \supset (y)R(a, y)$' is an analytic sentence;[13] also

[13] Cf. Hilbert and Bernays, loc. cit., p. 155.

'$s_k \supset ((R(a, a) \,.\, R(a, b)) \supset (y)R(a, y))$' is analytic. But then, in view of the modus ponens rule, $S_K \rightarrow$ '$(R(a, a) \,.\, R(a, b)) \supset (y)R(a, y)$', and thus $S_K \rightarrow$ '$(y)R(a, y) \vee \sim R(a, a) \vee \sim R(a, b)$'. But since S_K, being completely generalized, contains neither 'a' nor 'b', it follows[14] from the last statement that even $S_K \rightarrow$ '$(x)(z)((y)R(x, y) \vee \sim R(x, x) \vee \sim R(x, z))$'; hence $S_K \rightarrow$ '$(x)((y)R(x, y) \vee \sim R(x, y) \vee (z)\sim R(x, z))$', and finally, again replacing the sentence on the right hand side of the arrow by an equivalent one, $S_K \rightarrow$ '$(x)((y)R(x, y) \vee \sim R(x, x))$'. Let S'_K be the sentence on the right hand side of the last formula. Then, as $\mathrm{Cfd}_1(M_1, T)$ for every $T \in K$, we have $M_1 \rightarrow \mathrm{D}_{IM_1}(S_K)$; hence, since $S_K \rightarrow S'_K$, $M_1 \rightarrow \mathrm{D}_{IM_1}(S'_K)$ (by 4.2). But this last statement is false; for $\mathrm{D}_{IM_1}(S'_K)$ is equivalent to

$$((R(a, a) \,.\, R(a, b)) \vee \sim R(a, a)) \,.\, ((R(b, a) \,.\, R(b, b)) \vee \sim R(b, b)).$$

And the second component of this conjunction does not follow from M_1. This concludes the proof, which can be extended without difficulty to M_2 = '$R(a, a) \,.\, R(a, b) \,.\, R(a, c)$' and all analogously built longer molecules.

This narrowness in the definition of 'Cf_1' is obviously due to the fact that, according to 6.2, a sentence which is not completely generalized can be confirmed by a molecule M only by virtue of being a consequence of M and directly confirmed c.g. sentences; the relation of direct confirmation having so far been restricted to c.g. sentences. We shall now proceed to define a relation of direct confirmation, Cfd_2, which contains Cfd_1 as a subrelation, and which is applicable also to not-completely-generalized sentences. In terms of it, a corresponding relation of confirmation, Cf_2, will then be defined.

7. A second approximation: Cfd_2 and Cf_2. A definition of direct confirmation which is applicable to sentences of any form can be obtained from the definition of Cfd_1 (6.1) by simply dropping the requirement that S be a c.g. sentence. This is possible because the concept of C-development, which is crucial for the definition of direct confirmation, is defined for sentences of any form. Thus, we obtain the definition:

7.1 *Df.* $\mathrm{Cfd}_2(M, S)$ if and only if (a) M is a molecule, and (b) $M \rightarrow \mathrm{D}_{IM}(S)$.

In analogy to 6.2, we then define:

7.2 *Df.* $\mathrm{Cf}_2(M, S)$ if and only if:

(a) M is a molecule; and

(b) $(EK)(K \rightarrow S) \,.\, (T)(T \in K \supset ((M \rightarrow T) \vee \mathrm{Cfd}_2(M, T)))$.

The new relation Cf_2 is free from that material inadequacy of Cf_1 which was exhibited in the end of the preceding section. Thus, e.g., each of the molecules '$R(a, a) \,.\, R(a, b)$', '$R(a, a) \,.\, R(a, b) \,.\, R(a, c)$', etc. stands in Cf_2 (and indeed in Cfd_2) to '$(y)R(a, y)$'.

7.11 *Note.* Though the relation Cfd_2 is more comprehensive than Cfd_1—it contains the latter as a proper subrelation—it cannot serve as a general relation of confirmation, for it does not satisfy all the formal requirements of adequacy. Thus, e.g., '$R(a, a) \,.\, R(a, b)$' stands in Cfd_2 to the first, but not to the second

[14] Cf. Hilbert and Bernays, loc. cit., p. 106, schema α.

of the equivalent sentences '$(y)R(a, y)$' and '$(y)(R(a, y) \,.\, R(a, c))$'. This shows that the requirements 3.1, 3.11, 3.12 are violated and gives a preliminary justification for the introduction of the further definition 7.2 above.

We note a few theorems concerning Cfd_2 which will be needed in subsequent proofs.

7.12 *Theorem.* Cfd_2 satisfies the conjunction condition 3.13.

Proof. Let M be a molecule and K a finite sentential class such that $\text{Cfd}_2(M, S)$—and thus $M \rightarrow \text{D}_{IM}(S)$—for every $S \epsilon K$. Let S_K be the conjunction of the elements of K; then, by 4.1 B I c, $\text{D}_{IM}(S_K)$ is the conjunction of all $\text{D}_{IM}(S)$ with $S \epsilon K$; hence $M \rightarrow \text{D}_{IM}(S_K)$ and thus $\text{Cfd}_2(M, S_K)$.

7.13 *Theorem.* $\text{Cfd}_2(M, M)$ for every molecule M.—This follows immediately from 7.1 considering that $\text{D}_{IM}(M) = M$ (cf. 4.13).

7.14 *Theorem.* Cfd_2 satisfies the general consistency condition.

Proof. Let M be a consistent molecule, and K^* the class of all S such that $\text{Cfd}_2(M, S)$. We have to show that $\{M\}+K^*$ is a consistent class. Since $M \epsilon K^*$ (7.12), this reduces to proving K^* consistent.

Now, if K^* were inconsistent, then it would contain, by virtue of 2.32, a finite inconsistent subclass K. Let S_K be the conjunction of its elements; then, by 7.12, $M \rightarrow \text{D}_{IM}(S_K)$, while S_K is contradictory. This will now be shown to be impossible.

IM will contain a finite number of constants, say 'a_1', 'a_2', etc. S_K may contain individual constants; they fall into two classes (either of which may be empty): the class C_1 of those constants which are elements of IM, and the class C_2 of all others. (If, for instance, $M =$ '$R(a, a).R(a, b)$' and $S_K =$ '$(y)(R(a, y) \vee U(a, c, d, y))$', then $IM = \{\text{'}a\text{'}, \text{'}b\text{'}\}$, $C_1 = \{\text{'}a\text{'}\}$, $C_2 = \{\text{'}c\text{'}, \text{'}d\text{'}\}$.) Every element of C_2 will, of course, also occur in $\text{D}_{IM}(S_K)$. Now since none of the elements of C_2 occurs in M, and yet $M \rightarrow \text{D}_{IM}(S_K)$, clearly M must likewise entail $\text{D}_{IM}(S)$ for any S obtainable from S_K by replacing the constants belonging to C_2 by arbitrary other constants.[15] (Thus, in the previous illustration, $M \rightarrow \text{D}_{IM}(S_K)$, but also $M \rightarrow \text{D}_{IM}((y)(R(a, y) \vee U(a, a, e, y)))$, etc.) Now let us replace in S_K every element of C_2 (if any) by some one element of IM, say by 'a_1'. Let S_a be the D_{IM} of the sentence thus obtained. Then clearly $M \rightarrow S_a$.

Now, if the sentence S_K were contradictory, then it would not be satisfiable in any finite domain.[16] But from the preceding considerations it follows that S_K is satisfiable in a domain which has the same number of elements as IM, say m. Indeed, we can first choose a domain of m individuals, $a_1, a_2, \cdots, a_m$ and define each predicate occurring in M in such a way that on that interpretation M becomes true. (This is possible because of the presupposed consistency of M.) Now, the following interpretation of the extra-logical constants in S_K will make S_K a true sentence in the domain under consideration: 'a_1', 'a_2', $\cdots$, 'a_m', in so far as they occur in S_K, are to be names of $a_1, a_2, \cdots, a_m$ respectively. All the constants of C_2 in S_K ('b_1', 'b_2', etc.) are interpreted as names of a_1. For the

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[15] Cf. Hilbert and Bernays, loc. cit., p. 106, schema α.

[16] Cf. Hilbert and Bernays, loc. cit., p. 121, Theorem 1, and the statement on satisfiability, p. 128, second paragraph.

predicates in S_K, choose the interpretation mentioned before, which makes M a true sentence. Under this interpretation, the assertion of S_K for the domain $\{a_1, \cdots, a_m\}$ clearly becomes equivalent with that of S_a. But under the given interpretation, S_a is true since $M \rightarrow S_a$. Therefore, there is at least one finite domain in which S_K is satisfiable; hence S_K cannot be contradictory.

The subsequent theorems concerning Cf_2 can now readily be proved:

7.21 *Theorem.* Cf_2 satisfies the general consequence condition.

7.22 *Theorem.* Cf_2 satisfies the general consistency condition.

7.23 *Theorem.* Cf_2 satisfies the entailment condition.

7.24 *Theorem.* Cfd_2 is a proper subrelation of Cf_2.

The proofs can be omitted here: they are exactly analogous to those of 6.21, 6.22, 6.23, 6.24; the proof of 7.22 makes use of the fact that Cfd_2 satisfies the general consistency condition (cf. 7.14).

7.3 *Theorem.* Cf_1 is a proper subrelation of Cf_2.

Proof. a) Let $Cf_1(M, S)$, i.e.,

$$(EK)((K \rightarrow S) \,.\, (T)(T \in K \supset ((M \rightarrow T) \vee Cfd_1(M, T)))).$$

Then, since Cfd_1 is a subrelation of Cfd_2 (cf. 7.11),

$$(EK)((K \rightarrow S) \,.\, (T)(T \in K \supset ((M \rightarrow T) \vee Cfd_2(M, T))));$$

hence $Cf_2(M, S)$.

b) Cf_2 is not a subrelation of Cf_1: '$R(a, a) \,.\, R(a, b)$' stands in Cf_2 to '$(y)R(a, y)$', but not in Cf_1, as proved in 6.3.

7.4 *Note.* The question might arise whether an even more comprehensive relation of confirmation, say Cf_3, might not be obtained from Cf_2 by the same procedure that led from Cfd_2 to Cf_2, i.e., by means of the following definition:

$Cf_3(M,S)$ if and only if M is a molecule, and

$$(7.41) \quad (EK)((K \rightarrow S) \,.\, (T)(T \in K \supset ((M \rightarrow T) \vee Cf_2(M, T))))$$

And a reiteration of this procedure might promise a further broadening of the relation of confirmation. Actually, however, all the relations thus obtainable are coextensive with Cf_2. It suffices to show this for Cf_3.—If $Cf_2(M, S)$, then certainly $Cf_3(M, S)$, for the class $K = \{S\}$ satisfies the condition 7.41.—For proving the converse, note first that by virtue of 7.23, the clause '$(M \rightarrow T) \vee Cf_2(M, T)$' in 7.41 can be replaced by the equivalent '$Cf_2(M, T)$'. Now let $Cf_3(M, S)$; then S is a consequence of a sentential class K such that for every $T \in K$, $Cf_2(M, T)$; and this, by the definition of 'Cf_2', means that every T in K is a consequence of a class K_T such that M entails or stands in Cfd_2 to each of its elements. Thus, S is a consequence of the sum of those K_T, i.e., of a class such that M entails or stands in Cfd_2 to each of its elements; but this means that $Cf_2(M, S)$.

The following examples are intended to illustrate the character of the relations Cfd_2 and Cf_2.

7.61 '$P(a)$' stands in Cf_2 to the sentence '$(x)P(x)$', and to all its consequences, such as

a) '$P(a)$', '$P(b)$', '$P(c)$', and any other full sentence of 'P';

b) '$(x)((P(x) \vee Q(x))$', '$(x)(Q(x) \supset P(x))$', '$(x)P(x) \vee (Ey)Q(y)$';

c) '$(Ex)P(x)$'.

Also, the molecules '$P(a) \,.\, P(b)$', '$P(a) \,.\, P(b) \,.\, P(c)$', etc. stand in Cf_2 to all of the above sentences.

7.62 '$R(a, a) \,.\, R(a, b) \,.\, R(b, a) \,.\, R(b, b)$' stands in Cf_2 to '$(x)(y)R(x, y)$', and so does '$R(a, a)$' and again '$R(b, b)$'—but no other partial conjunction of the first molecule. (Thus, while a full sentence of a predicate of degree 1 stands in Cf_2 to any full sentence of the same predicate—cf. 7.61 a—an analogous rule does not hold for predicates of higher degree.)

7.63 Among others, each of the following molecules stands in Cf_2 to '$(x)(P(x) \supset Q(x))$': '$\sim P(a)$', '$\sim P(a) \vee Q(a)$', '$Q(a)$', '$\sim P(a) \,.\, Q(b)$'.

7.64 Each of the molecules '$R(a, a)$', '$R(a, a) \,.\, R(b, a)$', '$R(a, b) \,.\, R(b, a) \,.\, R(c, c)$' stands in Cf_2 to '$(x)(Ey)R(x, y)$', but '$R(a, a) \,.\, R(a, b)$' does not.

7.65 Cf_2 does not generally satisfy the rule that if each of two molecules separately confirms a hypothesis, then so does their conjunction; for while intuitively plausible, this rule is incompatible with the special consistency condition 3.21. Thus, e.g., if M_1 = '$P(a) \,.\, P(b)$', M_2 = '$\sim P(c)$', S_1 = '$(x)P(x) \vee (x)\sim P(x)$', then, as is readily verified, $Cf_2(M_1, S_1)$ and $Cf_2(M_2, S_1)$; but not $Cf_2(M_1 \,.\, M_2, S_1)$, because $M_1.M_2$ stands in Cf_2 to S_2 = $(Ex)P(x) \,.\, (Ex)\sim P(x)$', which is incompatible with S_1, and Cf_2 satisfies 3.21.

Consider now the following more liberal definition of confirmation which readily suggests itself: Let $Cfd_3(M, S)$ if either $Cfd_2(M, S)$ or M is a conjunction of molecules each of which stands in Cfd_2 to S; and let Cf_3 be defined in terms of Cfd_3 in exact analogy to 7.2. Our last illustration makes it clear that this intuitively satisfactory, more comprehensive relation would violate the consistency condition; for $M_1.M_2$ would stand in Cfd_3 (and thus in Cf_3) to either of the incompatible sentences S_1 and S_2.

This case illustrates once more that one of the main difficulties in defining confirmation lies in striking a balance between that liberality which seems desirable on intuitive grounds and the formal standards of adequacy, especially the consistency condition.

7.66 There is however one other intuitive inadequacy inherent in the concepts of Cfd_2 and Cf_2 which can be remedied by a slight modification of the definitions 7.1 and 7.2. As was pointed out in 4.51, a generalized sentence may have an analytic or a contradictory C-development without being analytic or contradictory itself. As a consequence of this fact, a molecule M may stand in Cfd_2 (or in $Dscfd_2$, i.e., the corresponding relation of direct disconfirmation; cf. 3.01) to a generalized sentence S simply by virtue of the fact that the cardinal number of IM is so small as to make $D_{IM}(S)$ analytic (or contradictory, respectively), while the content of M, intuitively speaking, neither strengthens nor weakens S. Let, for example, S_1 = '$(x)P(x) \vee (x)\sim P(x)$', S_2 = '$(Ex)P(x) \,.\, (Ex)\sim P(x)$', and M = '$P(a)$'. Then $D_{IM}(S_1) = P(a) \vee \sim P(a)$, $D_{IM}(S_2) = P(a) \,.\, \sim P(a)$; hence $Cfd_2(M, S_1)$ and $Dscfd_2(M, S_2)$, while by intuitive standards M will be considered as neither confirming nor disconfirming any of the two sentences.

Similar cases can be constructed involving hypotheses in several variables. Thus the sentence

$$S_1 = \text{'}(Ex)(Ey)(Ez)(W(x, y, z) \,.\, \sim W(x, z, y) \,.\, \sim W(y, x, z) \,.\, \sim W(z, y, x))\text{'}$$

cannot be satisfied by any interpretation of 'W' in a domain of less than 3 individuals; indeed, if C contains exactly one or exactly two individual constants then $D_C(S_1)$ is readily found to be contradictory. This has the awkward consequence that any molecule M which contains no more than two individual constants—no matter whether it contains 'W' or not—stands in Dscfd_2 to S_1, since $D_{I_M}(S_1)$ is contradictory.

These inadequacies of our definition of direct confirmation can be eliminated by a slight modification, to which we now turn.

8. Final version of the definition of confirmation. The following pair of definitions embodies the modifications in question:

8.1 *Df.* Cfd (M, S) if and only if (a) M is a molecule, (b) $D_{I_M}(S)$ is not analytic or S is analytic, and (c) $M \rightarrow D_{I_M}(S)$.

8.2 *Df.* Cf(M, S) if and only if:

(a) M is a molecule; and

(b) $(EK)((K \rightarrow S) \,.\, (T)(T \in K \supset ((M \rightarrow T) \vee \text{Cfd}(M, T))))$.

The previous illustrations might seem to suggest that the definiens of 'Cfd' ought to contain, in addition to the clause (b) an analogous provision to the effect that $D_{I_M}(S)$ is not contradictory, or S is contradictory. This restriction, however, is unnecessary. For while it may happen—as in the case S = '$(Ex)P(x) \,.\, (Ex){\sim}P(x)$', M = '$P(a)$'—that $D_{I_M}(S)$ is contradictory while S is not, we nevertheless do not have Cfd(M, S) unless M is contradictory itself; and the consequence that a contradictory molecule confirms every sentence appears as quite reasonable and is, anyhow, implied by the entailment condition (cf. 3.32).

As is readily seen, the modified definition is free from those intuitively undesirable features of the previous definitions of confirmation which were pointed out in 7.66; in particular, '$P(a)$' neither stands in Cfd to '$(x)P(x) \vee (x){\sim}P(x)$' nor in Dscfd to '$(Ex)P(x) \,.\, (Ex){\sim}P(x)$'; and, in the case of the above sentence S_1 containing the predicate 'W', neither M_1 nor M_2 stand in Dscfd to S_1 because they do not stand in Cfd to ${\sim}S_1$; and this is so because ${\sim}S_1$ is not analytic, while both $D_{I_{M_1}}({\sim}S_1)$ and $D_{I_{M_2}}({\sim}S_1)$ are.

8.11 *Theorem.* Cfd is a proper subrelation of Cfd_2.

8.12 *Theorem.* Cfd satisfies the general consistency condition.—This follows from the analogous theorem for Cfd_2 (7.14) in view of 8.11.

8.21 *Theorem.* Cf satisfies the general consequence condition.

8.22 *Theorem.* Cf satisfies the general consistency condition.

8.23 *Theorem.* Cf satisfies the entailment condition. The proofs of these three theorems are analogous to those of the corresponding theorems for Cf_2.

8.3 *Theorem.* Cf(M, S) if and only if M is a molecule such that

$$(EK)((K \rightarrow S) \,.\, (T)(T \in K \rightarrow \text{Cfd}(M, T)))$$

Proof. By virtue of 8.13, '$M \rightarrow T$' entails 'Cfd(M, T)'; hence the clause '$(M \rightarrow T) \vee \text{Cfd}(M, T)$' in 8.2 (b) is equivalent with 'Cfd(M, T)'.

8.41 *Theorem.* If M is an analytic molecule, then Cf(M, S) if and only if S is analytic.

Proof. If S is analytic, then $M \rightarrow S$, hence Cf(M, S) by 8.23; and if Cf(M, S), then, by virtue of 8.3, *S* is a consequence of a class *K* such that *M* stands in Cfd to each $T \epsilon K$. Since this implies that $M \rightarrow \mathrm{D}_{IM}(T)$, $\mathrm{D}_{IM}(T)$ must be analytic; and by 8.1(b), every *T* of this kind must be analytic; hence also *S*.

8.42 *Theorem.* A molecule *M* stands in Cf to every *S* if and only if *M* is contradictory. (From 8.23 and 8.22.)

Finally, all the illustrations and general comments concerning Cfd_2 and Cf_2 which are included in 7.61 through 7.65 apply likewise to Cfd and Cf.

Thus, the concept of confirmation determined by the definitions 8.1 and 8.2 satisfies all our formal conditions of adequacy and at least all those tests of material adequacy which have been referred to in the discussion of any one of the definitions of confirmation previously considered in this article.

The present study represents only a first attempt to arrive at a systematic logical theory of confirmation. Its main objectives were to characterize the issue and its significance as clearly as possible, to suggest certain conditions which any adequate solution should satisfy, and to prove that the problem thus determined can be solved in purely syntactical terms.

However, the proof as embodied in the above construction of a syntactical definition of confirmation is restricted to languages of the comparatively simple logical structure of the lower functional calculus without the identity sign. From the viewpoint of formal logic as well as of the logical analysis of science it appears highly desirable to generalize the definition of confirmation in two respects. First, it should be so expanded as to become applicable to more complex language forms, such as the lower functional calculus with identity sign, and even the higher functional calculus; and secondly, it would seem desirable to free the definition of confirmation from the restricting condition that the confirming sentence has to be of molecular form. The generalization of the concept of confirmation in these directions represents perhaps the most important open problem for further research in this field.

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