A QUERY ON CONFIRMATION

Hempel, Carnap, Oppenheim, and Helmer [1] have recently made important contributions towards the precise definition of the concepts of confirmation and degree of confirmation. Yet they seem to me to leave untouched one basic problem that must be solved before we can say that the proposed definitions are intuitively adequate—even in an approximate sense and for very limited languages.

Induction might roughly be described as the projection of characteristics of the past into the future, or more generally of characteristics of one realm of objects into another. But exact expression of this vague principle is exceedingly difficult. Some of the contradictions that result from seemingly straightforward formulations of it were explained and overcome in Hempel's papers. Unfortunately, equally serious difficulties remain.

Suppose we had drawn a marble from a certain bowl on each of the ninety-nine days up to and including VE day, and each marble drawn was red. We would expect that the marble drawn on the following day would also be red. So far all is well. Our evidence may be expressed by the conjunction "$Ra_1 \cdot Ra_2 \cdot \ldots \cdot Ra_{99}$," which well confirms the prediction "Ra_{100}." But increase of credibility, projection, "confirmation" in any intuitive sense, does not occur in the case of every predicate under similar circumstances. Let "S" be the predicate "is drawn by VE day and is red, or is drawn later and is non-red." The evidence of the same drawings above assumed may be expressed by the conjunction "$Sa_1 \cdot Sa_2 \cdot \ldots \cdot Sa_{99}$." By the theories of confirmation in question this well confirms the prediction "Sa_{100}"; but actually we do not expect that the hundredth marble will be non-red. "Sa_{100}" gains no whit of credibility from the evidence offered.

It is clear that "S" and "R" can not both be projected here, for that would mean that we expect that a_{100} will and will not be red. It is equally clear which predicate is actually projected and which is not. But how can the difference between projectible and non-projectible predicates be generally and rigorously defined?

That one predicate used in this example refers explicitly to

[1] In the following papers: C. G. Hempel, "A Purely Syntactical Definition of Confirmation," *Journal of Symbolic Logic,* Vol. 8 (1943), pp. 122–143; "Studies in the Logic of Confirmation," *Mind,* n.s., Vol. 54 (1945), pp. 1–26; Hempel and Paul Oppenheim, "A Definition of 'Degree of Confirmation,'" *Philosophy of Science,* Vol. 12 (1945), pp. 98–115; Oppenheim and Olaf Helmer, "A Syntactical Definition of Probability and of Degree of Confirmation," *Journal of Symbolic Logic,* Vol. 10 (1945), pp. 25–60; Rudolf Carnap, "On Inductive Logic," *Philosophy of Science,* Vol. 12 (1945), pp. 72–97; and "The Two Concepts of Probability," *Philosophy and Phenomenological Research,* Vol. V (1945), pp. 513–532.

temporal order is inessential. The same difficulty can be illustrated without the supposition of any order. Using the same letters as before, we need only suppose that the subscripts are merely for identification, having no ordinal significance, and that "*S*" means "is red and is not a_{100}, or is not red and is a_{100}."

The theories of confirmation in question require the primitive predicates to be logically independent.[2] This is perhaps a dubious stipulation since it places a logical requirement upon the informal, extrasystematic explanation of the predicates. Such doubts aside, the requirement would make it impossible for the predicates "*R*" and "*S*" to belong to the same system. Hence the conflicting confirmations would not occur in any one system. But this is of little help, since the system containing the predicate "*S*" alone is quite as admissible as the one containing "*R*" alone; and in the former system, as we have seen, "Sa_{100}" will be formally confirmed by the very evidence which intuitively disconfirms it. Carnap's concept of the "width" of a predicate does not bear on this point, since all atomic predicates are of the same width.[3]

More complex examples illustrating various phases of the same general question can easily be invented. I give only one more, to show how the theory of degree of confirmation is affected.

Suppose [4] that a certain unfamiliar machine tosses up one ball a minute and that every third one and only every third one is red. We observe ninety-six tosses. How much confidence does this lead us to place in the prediction that the next three tosses will produce a non-red ball, another non-red ball, and then a red ball? Plainly a good deal. But what degree of formal confirmation does the prediction derive from the observations according to the theories under consideration? The answer seems to be that this varies widely with the way the given evidence is described.

(i) If we let "a_1," "a_2," and so on represent in temporal order the individual tosses, our evidence may be expressed by

$$\text{``}-Ra_1\cdot -Ra_2\cdot Ra_3\cdot -Ra_4\cdot -Ra_5\cdot Ra_6\cdot \ldots \cdot -Ra_{94}\cdot -Ra_{95}\cdot Ra_{96}.\text{''}$$

This imparts to the prediction " $-\mathrm{Ra}_{97}\cdot -\mathrm{Ra}_{98}\cdot \mathrm{Ra}_{99}$" the degree [5]

[2] Although this requirement is not explicitly stated in the articles cited, Dr. Hempel tells me that its necessity was recognized by all the authors concerned.

[3] See page 84 of the first article by Carnap listed in footnote 1.

[4] The example in its present form is due to Dr. Hempel. He constructed it as the result of a conversation with Dr. W. V. Quine and the present writer concerning the problems here explained.

[5] The degrees of confirmation given in this paper are computed according to the Hempel-Oppenheim theory. The values under Carnap's theory would differ somewhat, but not in a way that appreciably affects the general question under discussion.

of confirmation $\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3}$, or $\frac{4}{27}$. This figure seems intuitively much too low.

(ii) If we let "b_1" stand for the discontinuous individual consisting of the first three tosses, "b_2" for the individual consisting of the next three tosses, and so on, and let "S" mean "consists of three temporally separated parts ('tosses') of which the earliest and second are non-red and the latest red," our evidence may be expressed by

$$\text{``}Sb_1 \cdot \ldots \cdot Sb_{32}.\text{''}$$

This gives to "Sb_{33}" the degree of confirmation 1. Yet "Sb_{33}" expresses the same thing as "$-Ra_{94} \cdot -Ra_{95} \cdot Ra_{96}$," and we have assumed the same observations. Hence we seem to get different degrees of confirmation for the same prediction on the basis of the same evidence.

Now it may be argued that in (i) we ignored the fact of temporal order in stating our evidence, and that it is thus not surprising that we get a lower degree of confirmation than when we take this fact into account, as in (ii). However, it would be fatal to accept the implied thesis that an intuitively satisfactory degree of confirmation will result only when all the observed facts are expressed as evidence. Suppose the first ninety-six tosses exhibited a wholly irregular distribution of colors; the hypothesis that this distribution would be exactly repeated in the next ninety-six tosses would have the degree of confirmation 1. What is worse, if we are to express *all* the observed data in our statement of evidence, we shall have to include such particularized information—e.g., the unique date of each toss—that repetition in the future will be impossible.

Undoubtedly we do make predictions by projecting the patterns of the past into the future, but in selecting the patterns we project from among all those that the past exhibits, we use practical criteria that so far seem to have escaped discovery and formulation. The problem is not peculiar to the work of the authors I have named; so far as I am aware, no one has as yet offered any satisfactory solution. What we have in the papers cited is an ingenious and valuable logico-mathematical apparatus that we may apply to the sphere of projectible or confirmable predicates whenever we discover what a projectible or confirmable predicate is.

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