Philosophy 4310 –- Assignment #3

This assignment is to be turned in at the beginning of class on Thursday, March 9th.

Part I: The Equation
$P(C|A) = P(A \rightarrow C)$

Prove that each of the following holds for any probability function P and any propositions A, C where $P(A) > 0$ (so that $P(C|A)$ is defined)

1) $P(C|A) \leq P(A \supset C)$.
2a) if $P(A) = 1$, then $P(C|A) = P(A \supset C) = P(C)$
2b) $P(C|A) = 1$ if and only if $P(A \supset C) = 1$
2c) $P(C|A) = P(A \supset C)$ entails that one of these two cases obtains (that is, entails that either $P(A) = 1$ or $P(A \supset C) = 1$)

Probability Logic: [if you need help, you may want to start reading Bennett, Ch 9 though you don't need anything in that chapter for these problems]
For the material conditional $A \supset C$, call $P(C|A)$ 'the corresponding conditional probability'

For each of arguments 3-7, say whether they are deductively valid. Now replace any material conditionals with the corresponding conditional probability. Now assume that the probability of each of the premises is 1. What is the possible range of the probability of the conclusion? Next, make the probability of the premises each .9. Now what is the possible range of the probability of the conclusion?

3) $A \supset C, A \vdash C$
4) $A \supset C, C \vdash A$
5) $C \vdash A \supset C$
6) $A \supset C \vdash (A \& B) \supset C$
7) $A \supset (C \& B) \vdash A \supset C$

COMMENT: Arguments that preserve high probability are called "probabilistically valid" (technically, the uncertainty of the conclusion cannot exceed the uncertainty of the premises). As a special case of this, probability 1 works like deductive validity so that arguments that preserve truth also preserve probability 1. Arguments are deductively valid if and only if they are probabilistically valid. All probabilistically invalid arguments are such that the probability of the premises can get arbitrarily close to 1 while the probability of the conclusion can get arbitrarily close to 0.

However, if we replace the probability of a material conditionals with the corresponding conditional probability, while the 'probability 1' property is preserved, probabilistic validity in general is not. Some authors claim that these are precisely the arguments that show that the material conditional is problematic.

Part II: Triviality Proofs involving The Equation

David Lewis proved that modulo a weak assumption, The Equation entails that A and C are independent. Two propositions A and C are said to be probabilistically independent if P(A&C) = P(A) * P(C). [You may wish to look at Titelbaum, *FBE* section 3.2 for a discussion of this]. Notice that independence is symmetric, meaning that if A is independent of C then C is independent of A.

FACT: Probabilistic independence could just as well be defined as A and C are independent if P(C|A) = P(C) or also as P(C|A) = P(C|~A). Prove that these three definitions are equivalent by proving each of these biconditionals:

1. Prove that P(A&C) = P(A) * P(C) if and only if P(C|A) = P(C).
2. Prove that P(A&C) = P(A) * P(C) if and only if P(C|A) = P(C|~A).
3. Now prove that if A and C are independent, then A and ~C are also independent.

For the rest of Part II, assume that P(A) = 1/3 and P(C) = 1/4 and that A and C are independent.

4. Now build the full stochastic truth-table for this probability distribution (fill in the probabilities for the four rows of the table).

5. Calculate each of the following and for each conditional probability, find a proposition that has that same unconditional probability. For example, if one of these conditional probabilities is 1/3, then it equals P(A). If one of these is 2/3, it equals P(~A).

P(A|C)
P(C|A)
P(C|~A)
P(~C|A)
P(A|A)
P(A|~A)

While this is a very special case (all of the atomic propositions are independent from each other), Alan Hájek proved that this example is not consistent with The Equation holding for *all* propositions nor is *any* example with a finite number of possible worlds or state-descriptions (rows on a truth-table).

Hájek's proof starts but putting all of the 'possible world' probabilities in ascending order. The possible worlds are the state-descriptions in the stochastic truth-table (so there are four of them in this case). Look at Bennett's description of Hájek's proof starting on page 74 of Bennett.

6. In this example, the p1, p2, p3, p4 referred to by Bennett are the probabilities of the four rows of the stochastic truth table (remember to put them in ascending order). List all of the possible unconditional probabilities for any proposition that can be stated in this

language. HINT: every proposition is either true or false on a given row and so all of the probabilities are sums of the probabilities of the rows (a sum of either 0, 1, 2, 3, or 4 of the rows). There are 16 non-equivalent propositions with two atoms and so a maximum of 16 possible probabilities here though in this case, some of these 16 might be the same.

7. Call A1 the proposition (state-description) which has probability p1, A2 for p2, etc. Calculate:

P(A1|~A2)
P(A1|~A3)
P(A1|~A4)

COMMENT: Now we know that P(~A2) = 1-P(A2) = 1-p2. Thus P(A1|~A2) = p1/(1-p2) and the other two are = p1/(1-p3) and p1/(1-p4) respectively which is how Bennett describes the counterexamples. If you did this problem correctly, none of these conditional probabilities will be anything on your list of possible unconditional probabilities and so you will have proved that P(~A2→A1) ≠ P(A1|~A2), P(~A3→A1) ≠ P(A1|~A3), and P(~A4→A1) ≠ P(A1|~A4). Thus The Equation is false.

Part III: Gibbardian stand-offs

Imagine that Alice is playing Bob in the last round of a chess tournament. Neither Charlie nor Diane knows whether Alice won this last game. However, Charlie heard from a reliable source that the player with the black pieces won the game and so Charlie says, "If Alice was black, she won." Diane didn't hear that the black player won the game, however, she heard from a reliable source that Alice won a few games as white during the tournament, however, every time she was black, she lost. So Diane responds to Charlie and says, "No, if Alice was black, she lost." This is an example of what Bennett calls "A Gibbardian stand-off."

The basic logic of the situation seems to indicate that either both sentences are true, both are false, or one is true and one is false. On page 94, Bennett very briefly recaps why he thinks that none of these three possibilities could be right.

1) For each of these three possibilities, explain in more detail (a paragraph or two each) why each of these three cases is problematic.

2) Choose one of the three cases to defend against Bennett's (and your) attack. Alternatively, you may defend the view that both conditionals are neither true nor false against the following objection:

Objection: Imagine that Edward knows both Charlie and Diane and trusts that neither one of them would assert anything without a good reason. After hearing Charlie and Diane, Edward is able to properly infer that Alice must not have been black. But how could he infer that on the basis of the two conditionals he heard unless they were both true?