Philosophy 4310
Homework 3
Supplemental Help sheet

Here I am going to give full, perfect answers to questions just like those in Part I #3-7. There are multiple ways to do these problems. I am doing them algebraically – this is just one way to solve the problem.

Probability Logic: [if you need help, you may want to start reading Bennett, Ch 9 though you don't need anything in that chapter for these problems]
For the material conditional $A \supset C$, call $P(C|A)$ 'the corresponding conditional probability'

For each of arguments 3-7, say whether they are deductively valid. Now replace any material conditionals with the corresponding conditional probability. Now assume that the probability of each of the premises is 1. What is the possible range of the probability of the conclusion? Next, make the probability of the premises each .9. Now what is the possible range of the probability of the conclusion?

3) $A \supset C, A \vdash C$
4*) $A \supset C \vdash C \supset A$
6*) $(A \vee B) \supset C \vdash A \supset C$
7*) $A \supset C \vdash A \supset (B \vee C)$

--NOTE: #3 is the same. The others are similar or harder versions of problems on the actual homework.

3) First part: This argument is deductively valid.
Second: If we assume that $P(C|A) = 1$ and $P(A) = 1$, then what is $P(C)$?

Answer: If $P(C|A) = 1$, then $P(A\&C)/P(A) = 1$. So $a1/(a1+a2) = 1$ so $a2=0$. Now since $P(A) = 1$, $a1+a2 = 1$ so $a1 = 1$ making a2, a3, a4 all $=0$. Now $P(C) = a1+a3$ and since $a1=1$ and $a3=0$, $a1+a3=P(C) = 1$.

Third: If we assume that $P(C|A) = .9$ and $P(A) = .9$, then what is $P(C)$?

Answer: $P(C|A) = .9$, then $P(A\&C)/P(A) =.9$. So $a1/(a1+a2) = .9$. Now since $P(A) = .9$, $a1+a2 = .9$ Now plugging this in to our previous equation, $a1/.9=.9$ thus $a1=.81$ and so since $P(A) = a1+a2 = .9$, $a2 = .09$.

Now $P(C) = a1+a3 = .81 + a3$. However, we know that $P(\sim A) = a3+a4 = .1$ and so a3 has a maximum of .1 so since $a1 = .81$ and $0 \leq a3 \leq .1$, $.81 \leq P(C) \leq .91$.

Note that this means that the argument is probabilistically valid

4*) First part: This argument is deductively **IN**valid.
Second: If we assume that P(C|A) = 1, then what is P(A|C)?

Answer: If P(C|A) = 1, then P(A&C)/P(A) = 1. So a1/(a1+a2) = 1 so a2=0. Now the since P(A|C) = P(A&C)/P(C), P(A|C) = a1/(a1+a3). Since a3 is (nearly) completely unconstrained by the premise, This has a minimum when a1 = 0 and a maximum when a1 = 1 thus 0 ≤ P(A|C) ≤ 1. The only issue here is that if a3 = 1, then a1+a2 = P(A) = 0 and then our premise, P(C|A) would be undefined and not equal to 1. So technically, a3 can't be =1, but it can be arbitrarily close. So the correct answer is 0 ≤ P(A|C) < 1.

Third: If we assume that P(C|A) = 1, then what is P(A|C)?

Answer: If P(C|A) = .9, then P(A&C)/P(A) = .9. So a1/(a1+a2) = .9. Again, P(A|C) = P(A&C)/P(C), P(A|C) = a1/(a1+a3). And here again, since a3 is (nearly) completely unconstrained by the premise, 0 ≤ P(A|C) < 1.

Note that this means that the argument is probabilistically invalid

6*) First part: This argument is deductively valid.
Second: If we assume that P(C|A∨B) = 1, then what is P(C|A)?

Answer: If P(C|A∨B) = 1, then (a1+a3)/(a1+a2+a3+a4+a5+a6) = 1. Thus a3, a4, a5, and a6 must all =0. Now P(C|A) = a1/(a1+a3). Since a3 = 0, this = a1/a1 = 1.

Third: If we assume that P(C|A∨B) = .9, then what is P(C|A)?

Answer: If P(C|A∨B) = 1, then (a1+a3)/(a1+a2+a3+a4+a5+a6) = .9. Thus 10*(a1+a3) = 9*(a1+a2+a3+a4+a5+a6) and so (a1+a3) = 9*(a2+a4+a5+a6). Now P(C|A) = a1/(a1+a3) but a1 and a3 are (nearly) unconstrained by the premise (they can't both be 0 otherwise the premise would be =0 or undefined) and so a1/(a1+a3) is also unconstrained. It has a maximum =1 when a3 = 0 and a minimum of 0 when a1=1 (as long as a3 doesn't also =1). So 0 ≤ P(C|A) ≤ 1

Note that this means that the argument is probabilistically invalid

7*) First part: This argument is deductively valid.
Second: If we assume that P(C|A) = 1, then what is P(B∨C|A)?

Answer: If P(C|A) = 1, then P(A&C)/P(A) = 1. So (a1+a3)/(a1+a2+a3+a4) = 1 so a2 and a4 =0. Now P(B∨C|A) = P((B∨C)&A)/P(A) = (a1+a2+a3)/(a1+a2+a3+a4). Since a4 = 0, this =1.

Third: If we assume that P(C|A) = .9, then what is P(B∨C|A)?

Answer: If $P(C|A) = .9$, then $P(A\&C)/P(A) = .9$. So $(a1+a3)/(a1+a2+a3+a4) = .9$ so $10*(a1+a3) = 9*(a1+a2+a3+a4)$ and thus $(a1+a3) = 9*(a2+a4)$. Now $P(B\vee C|A) = P((B\vee C)\&A)/P(A) = (a1+a2+a3)/(a1+a2+a3+a4)$. Now this has a maximum =1 when a4=0, and a minimum when a4 is as high as it can be. But because of earlier constraints in the problem, a1+a3 has a maximum of .9 [--if it was higher, then a2+a4 would have to be lower than .1 and so we couldn't get $(a1+a3) = 9*(a2+a4)$ --]. So a2+a4 a maximum of .1 and so a4 a maximum of .1. When a4 = .1, a1+a3 = .9 and so a2 = 0 and so $P((B\vee C)\&A)/P(A) = (a1+a2+a3)/(a1+a2+a3+a4) = .9/1 = .9$. So $.9 \leq P(B\vee C|A) \leq 1$.

Note that this means that the argument is probabilistically invalid

-- As a further note, this problem is much easier if you notice that $A\&C \vdash (B\vee C)\&A$ and so $P(A\&C) \leq P((B\vee C)\&A)$ and so $P(A\&C)/P(A) \leq P((B\vee C)\&A)/P(A)$ and so $P(C|A) \leq P(B\vee C|A)$