Handout for Phil 4300

The Dutch Book Argument for Probablism concludes that your degrees of belief (= subjective probabilities) should be probabilities in the sense that they satisfy the axioms of probability.

Here I will show how there is a similar (though slightly more involved) Dutch Book Argument to show that when you learn some proposition E, you should update your degrees of belief by conditionalizing on E.

Imagine that your degree of belief in A, $DoB(A) = a$, your degree of belief in B, $DoB(B) = b$, and your degree of belief in A&B, $DoB(A\&B) = c$. Then your conditional degree of belief $DoB(A|B) = d = DoB(A\&B)/DoB(B) = c/b$. [Note that Resnik treats this not as a definition but as additional axiom. On a previous handout we showed how you could be Dutch Booked if you did not satisfy it].

The principle of conditionalization says that if between times t_0 and t_1 all you learn is B, then for any A, your new degree of belief, $DoB_1(A)$ should equal your old conditional degree of belief in A given B – i.e. $DoB_0(A|B)$. For example, imagine that your degree of belief in the Vikings winning the super bowl is .03, but that your conditional degree of belief in the Vikings winning the super bowl given that they get into the super bowl is .5. Then if you learn only that they in fact did get into the super bowl, then you should update your degree of belief that they will win from .03 to .5.

The Dutch Book Argument for conditionalization says that you should do this because if you do have a specific strategy for changing your degree of belief and it is not conditionalization, then a diachronic Dutch Book can be made against you.

Example: Imagine that the Vikings are in the NFC championship game and a bookie posts a betting quotient of .5 that the Vikings will win the NFC championship game (VN), .3 that the Vikings will win the super bowl (VS) and posts a conditional betting quotient of .6 that they Vikings will win the super bowl if they win the NFC championship game [note that $P(VS|VN) = .3 = P(VN\&VS)/P(VN)$ since $P(VN\&VS)=P(VS)$]. However, they also advertise the fact ahead of time that if the Vikings do get to the super bowl, they will then post a betting quotient of .5 for the Vikings winning that game. Now a clever bettor can win money for sure with the following strategy:

1) The bookie's conditional betting quotient now is 'too high' so make him pay for a bet to win $1 on VS given VN which he will pay .6 for. Now wait to see what happens.

2) If the Vikings do win the NFC game, then you buy the ticket for the Vikings to win the super bowl. Like the last handout, for ease of understanding, we will make the bookie pay .5 for a ticket that pays $1 if the Vikings lose. Now the bookie has paid a

total of $1.10 and he has two tickets – one for the Vikings winning and one for the Vikings losing so he will win $1 for sure and lose $.10 overall.

3) Now imagine that the Vikings do not win the NFC championship game. Then the bookie would get his original money back. So if you wanted to win money in that case for sure, simply make a side bet to win a small amount of money in that case – that is, bet against the Vikings winning the NFC game (or make the bookie pay some money for a bet that pays out $.06 if the Vikings win the NFC championship game. The bookie will pay $.03 for this bet and so now if they do win, he gains this $.03 but still loses the $.10 so still loses $.07 overall). But now notice that if the Vikings lose the NFC game, the bookie is now out the $.03. Thus the bookie now loses money in either case no matter what.

4) – Note that of course if the advertised new betting odds were too high – say if the Vikings win the new betting quotient tomorrow would be .7 that the Vikings would win the super bowl, then reverse all the bets. That is, you buy the conditional ticket for VS given VN and you buy a side ticket against VN. Then if the Vikings win the NFC game, sell the ticket for VN. You will win money for sure in either case.

SO WHAT?
First, it is worth noting that our strategy utilized conditional bets for ease of understanding the situation. But this is not necessary. The previous handout showed how to simulate a conditional bet with two unconditional bets. So no conditional bets are necessary for this argument.

But what is necessary is that you know the new betting odds ahead of time so that you know if it will be too high or too low so you know which side of the bet to buy or sell. So here are some situations where no Dutch Book can be made:

1) The bookie has a strategy for updating his betting quotients – he conditionalizes.
2) The bookie has a strategy, but doesn't advertise it.
3) The bookie advertises a strategy but then doesn't necessarily follow it.
4) The bookie has no set strategy – for example, each morning he sets his new betting quotients by rolling dice.

You might think that the first strategy is rational but that strategies 3 and 4 are not; however, in cases 3 and 4 there is no way to exploit this irrationality to Dutch Book the bookie.