Philosophy 4300: Decision Theory
Spring 2019

Homework 6 – due in class on Monday, May 6th (the last day of class).

You should feel free to work with others on this homework and to talk to me about it. However, any work you produce must be your own.

1) Take the following version of the prisoner's dilemma where you are Row:

	C	D
C	3, 3	0, 4
D	4, 0	1, 1

1a) Assume that your choice is independent of column's choice. Prove that the EU(D) > EU(C) no matter what P(they play C) is.

1b) Assume that your choice is correlated with column's choice such that P(they play C | you play C) = P(they play D | you play D) = x. How high does x have to be before EU(C) > EU(D)? [HINT: to calculate expected utilities in the general case, you use conditional probabilities instead of unconditional probabilities. And remember that P(C|C)+P(D|C) as well as P(D|D) + P(C|D) must add to 1.]

1c) Assume that there are two kinds of players in this game. There are selfish individuals who will play D no matter what you do. Then there are reciprocal cooperators who want to cooperate with cooperators, but want to defect against defectors. They will initially be leaning toward playing C and so if you are going to play C, they will too for sure. But if you try to cheat then and play D, they might detect this and if they do, they will also play D. They are successful at detecting cheating 50% of the time. [Totally unrealistically, we will assume that they never mistakenly think you might cheat if you don't]. You get paired randomly with someone from the population. What percentage of the population has to be reciprocal cooperators before it is actually beneficial to be a cooperator? That is, when EU(C) > EU(D)? [HINT: As in 1b) you will want to use conditional probabilities. Here P(C|C) will just be the percentage of reciprocators in the population but P(C|D) and P(D|D) are not quite as straightforward.]

2) Imagine the following bargaining game: A seller is trying to sell a house. The house is worth $200,000 to the seller and worth $300,000 to the buyer. Both the buyer and the seller value money exactly the same – namely, the utility of each dollar is worth 1. If they agree on a sale price $x, the buyer pays out $x and ends up with $300,000 - $x plus the house. The seller has to pay a 5% commission on the sale price to a realtor and so then will end up with $.95x. If they cannot agree upon a

sale price, the seller keeps the house and the buyer keeps the money. What is the Nash bargaining solution to this game?

3) For this question, you get credit just for answering it. Imagine that our class is going to play a multi-person public goods game. There are the ten students in the class and there is a banker financing the game. The banker gives you each $10 to start. Now you can return any amount of it to the banker. If you do, for each dollar you return, he will give each player (including you) $.50. So for example, if everyone keeps all their money, everyone gets $10. If everyone gives $10 back to the banker, everyone will end up with $50. If everyone keeps $5 and gives back $5, everyone will end up with 5+2.50x10 = $30. If you give $10 back and no one else gives any back, you will end up with $5 and everyone else will end up with $15.

3a) Imagine that you are playing with your classmates but everything is totally anonymous. You will know how many people put money back in, but you will never find out who put how much back in. How much would you give back to the banker? What do you expect that others would do?

3b) Imagine that you are playing with your classmates but it is not anonymous. You get to keep the money, but everyone will see how much you chose to give back and how much you got to keep. Would you do anything differently? Do you expect that others would do anything differently? [I will reveal the anonymous results in 3a, but I promise not to reveal your individual choices as in 3b!]

3c) If you were on a game show playing this game with people you just met 30 minutes before but it would be recorded and put on youtube, would you do anything differently?

4) In ranked choice voting (sometimes called instant runoff voting), each voter gets to rank each of the candidates from 1-n. Then we look at the first place votes. If any candidate has more than 50% of the 1st place votes, they win. If not, the candidate with the least number of first place votes is removed and each ballot's preferences are reordered with this candidate removed. In other words, the people who put that removed candidate first will now have their second choice counted. We again see if anyone is at more than 50% of the 1st place votes and if not, again remove the new lowest vote getter. This process is repeated until someone has a majority of the votes. [If this is unclear, there are lots discussions of it online. For example, here: https://www.youtube.com/watch?v=oHRPMJmzBBw]

There are various ways of dealing with ties at the bottom, but we will assume that if two candidates are tied for the least first place votes you get rid of both of them simultaneously.

4a) Imagine we have seven voters (v1-v7) and five candidates (a-e). Here are individual votes with the top candidate on the left:

v1: aebcd
v2: aecdb
v3: aecbd
v4: ceadb
v5: debac
v6: cebda
v7: becda

Who wins the election if we use ranked choice voting? Explain your answer.

4b) A different method of voting is a Borda count. Here each candidate receives points on each ballot proportional to the rank on that ballot. So for example, we might give each first place winner 5 points, second plate winner 4 points, third place 3, fourth place 2, and fifth place 1. So for example, v1's vote of aebdc would give 5 points to a, 4 points to e, 3 points to b, 2 points to d, and 1 point to c.

Using the exact same voter preferences as in 4a but using this 5, 4, 3, 2, 1 borda count election method, who wins the election now? [explain how you arrive at all of the point totals].

4c) If you did the problem right, the answers are different in 4a) and 4b). What is the key feature of this example that makes the answers come apart? On this basis, which of these two election methods do you think is better?