of known accuracy place a limit upon the *maximum* radius of the earth's orbit. If the size of the earth's orbit is known (and it can, in theory, be determined by Aristarchus' technique for measuring the earth-sun distance), then observations of known accuracy place a limit upon the *minimum* size of the sphere of the stars. For example, if the distance between the earth and sun is, as indicated by Aristarchus' measurement described in the Technical Appendix, equal to 764 earth diameters (1528 earth radii) and if observations are known to be accurate within 0.1°, then the radius of the sphere of the stars must be at least 1000 times the radius of the earth's orbit or at least 1,528,000 earth radii.

Our example is a useful one, because, though Copernicus' observations were not quite this accurate, those made by his immediate successor, Brahe, were if anything slightly more accurate than 0.1°. Ours is a representative estimate of the minimum size of the sphere of the stars by a sixteenth-century Copernican. In principle, there is nothing absurd about the result, for in the sixteenth and seventeenth centuries there was no direct way of determining the distance to the sphere of the stars. Its radius might have been more than 1,500,000 earth radii. But if it were that large – and Copernicanism demanded that it should be – then a real break with traditional cosmology must be admitted. Al Fargani, for example, had estimated the radius of the sphere as 20,110 earth radii, more than seventy-five times smaller than the Copernican estimate. The Copernican universe must be vastly larger than that of traditional cosmology. Its volume is *at least* 400,000 times as great. There is an immense amount of space between the sphere of Saturn and the sphere of the stars. The neat functional coherence of the nesting spheres of the traditional universe has been violated, though Copernicus seems to remain sublimely unaware of the break.

Copernican Astronomy – The Sun

Copernicus' argument permits an orbital motion of the earth in a vastly expanded universe, but the point is academic unless the orbital motion can be shown to be compatible with the observed motions of the sun and other planets. It is to those motions that Copernicus turns in Chapters 10 and 11 of his First Book. We may best begin with an expanded paraphrase of Chapter 11, in which

Copernicus describes the orbital motion of the earth and considers its effect upon the apparent position of the sun. For the moment assume, as shown in Figure 28, that the centers of the universe, the sun, and the earth's orbit all coincide. In the diagram the plane of the ecliptic is viewed from a position near the north celestial pole; the sphere of the stars is stationary; the earth travels regularly eastward in its orbit once in a year; and it simultaneously spins eastward on its axis once in every 23 hours 56 minutes. Provided that the earth's orbit is much

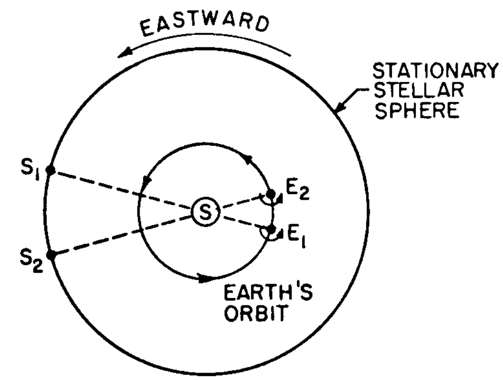

Figure 28. As the earth moves in its Copernican orbit from E_1 to E_2, the apparent position of the central sun, S, seen against the sphere of the stars shifts from S_1 to S_2.

smaller than the sphere of the stars, the axial rotation of the earth will account precisely for the diurnal circles of the sun, moon, and planets, as well as for those of the stars, because from any position in the earth's orbit all of these bodies must be seen against the sphere of the stars and must seem to move with it as the earth rotates.

In the diagram the earth is shown in two positions which it occupies thirty days apart. In each position the sun is viewed against the sphere of the stars, and both apparent positions of the sun must lie on the ecliptic, which is now defined as the line in which the plane of the earth's motion (a plane that includes the sun) intersects the sphere. But as the earth has moved eastward from position E_1 to position E_2 in the diagram, the sun has apparently moved eastward along the ecliptic from position S_1 to position S_2. Copernicus' theory therefore predicts just the same eastward annual motion of the sun along the ecliptic as the Ptolemaic theory. It also predicts, as we shall discover immediately, the same seasonal variation of the height of the sun in the sky.

Figure 29 shows the earth's orbit viewed from a point in the celestial sphere slightly north of the autumnal equinox. The earth is drawn at the four positions occupied successively at the vernal equinox, the summer solstice, the autumnal equinox, and the winter solstice. In all four of these positions, as throughout its motion, the earth's axis remains parallel to an imaginary line passing through the sun and tilted 23½° from a perpendicular to the plane of the ecliptic. Two

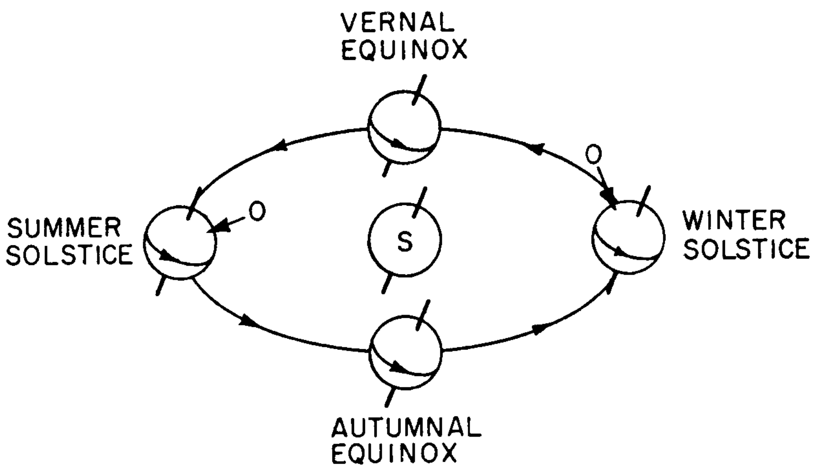

Figure 29. The earth's annual motion around its Copernican orbit. At all times the earth's axis stays parallel to itself or to the stationary line drawn through the sun. As a result an observer *O* at noon in middle-northern latitudes finds the sun much more nearly overhead at the summer than at the winter solstice.

little arrows in the diagram show the position of a terrestrial observer in middle-northern latitudes at local noon on June 22 and December 22, the two solstices. Lines from the sun to the earth (not shown in the diagram) indicate the direction of the rays of the noon sun, which is clearly more nearly over the observer's head during the summer solstice than during the winter solstice. A similar construction determines the sun's elevation at the equinoxes and at intermediate seasons.

The seasonal variation of the sun's elevation can therefore be completely diagnosed from Figure 29. In practice, however, it is simpler to revert to the Ptolemaic explanation. Since in every season

the sun appears to occupy the same position among the stars in the Copernican as in the Ptolemaic system, it must rise and set with the same stars in both systems. The correlation of the seasons with the apparent position of the sun along the ecliptic cannot be affected by the transition. With respect to the apparent motions of the sun and stars the two systems are equivalent, and the Ptolemaic is simpler.

The last diagram also reveals two other interesting features of Copernicus' system. Since it is the rotation of the earth that produces the diurnal circles of the stars, the earth's axis must point to the center of those circles in the celestial sphere. But, as the diagram indicates, the earth's axis never does point to quite the same positions on the celestial sphere from one year's end to the next. According to the Copernican theory the extension of the earth's axis traces, during the course of a year, two small circles on the sphere of the stars, one around the north celestial pole and one around the south. To an observer on the earth the center of the diurnal circles of the stars should itself seem to move in a small circle about the celestial pole once each year. Or, to put the same point in a way more closely related to observation, each of the stars should seem slightly to change its position on the sphere of the stars (or with respect to the observed pole of the sphere) during the course of a year.

This apparent motion, which cannot be seen with the naked eye and which was not even seen with telescopes until 1838, is known as

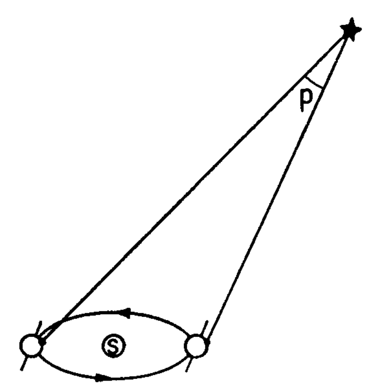

Figure 30. The annual parallax of a star. Because the line between a terrestrial observer and a fixed star does not stay quite parallel to itself as the earth moves in its orbit, the star's apparent position on the stellar sphere should shift by an angle *p* during an interval of six months.

the parallactic motion. Because two lines drawn to a star from diametrically opposite points on the earth's orbit are not quite parallel (Figure 30), the apparent angular position of the star viewed from the earth should be different at different seasons. But if the distance to the star is very much greater than the distance across the earth's orbit, then the angle of parallax, *p* in Figure 30, will be very, very small, and the change in the apparent position of the star will not be appreciable. The parallactic motion is not apparent only because the stars are so very far away relative to the dimensions of the earth's orbit. The situation is precisely equivalent to the one we discussed above when considering why the earth's motion did not seem to change the intersection of the horizon plane and the sphere of the stars. In fact, we are dealing with the same problem. But the present version of the problem is a more important one, because near the horizon it is very difficult to make the precise measurements of stellar position required to discover whether the horizon bisects the stellar sphere. Unlike the rising and setting of the equinoxes, discussed above, the search for parallactic motions need not be restricted to the horizon. Parallax therefore provides a much more sensitive observational check upon the minimum size of the sphere of the stars relative to the size of the earth's orbit than is provided by the position of the horizon, and the Copernican estimates of the sphere's size given above ought really to have been derived from a discussion of parallax.

The second point illuminated by considering Figure 29 is not about the skies at all but about Copernicus. We described the orbital motion illustrated in the diagram as a single motion by which the earth's center is carried in a circle about the sun while its axis remains always parallel to a fixed line through the sun. Copernicus describes the same physical motion as consisting of two simultaneous mathematical motions. That is why he gives the earth a total of three circular motions. And the reasons for his description give another significant illustration of the extent to which his thought was bound to the traditional patterns of Aristotelian thought. For him the earth is a planet which is carried about the central sun by a sphere just like the one that used to carry the sun about the central earth. If the earth were firmly fixed in a sphere, its axis would not always stay parallel to the same line through the sun; it would instead be carried about by the sphere's rotation and would occupy the positions shown

in Figure 31*a*. After the earth had revolved 180° about the sun, the earth's axis would still be tilted 23½° away from the perpendicular but in a direction opposite to that in which it had begun. To undo this change in the direction of the axis, caused by the rotation of the

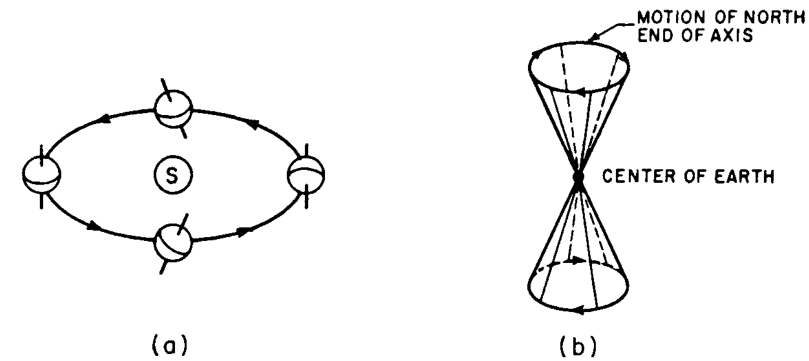

Figure 31. Copernicus' "second" and "third" motions. The second motion, that of a planet fixed in a rotating sun-centered sphere, is shown in (*a*). This motion does not keep the earth's axis parallel to itself, so that the conical third motion shown in (*b*) is required to bring the axis back into line.

sphere that carries the earth, Copernicus requires a third circular motion, this one applied to the axis of the earth only and shown in Figure 31*b*. It is a conical motion, which carries the north end of the axis once westward each year, and thus just compensates for the effect on the earth's axis of the orbital motion.

Copernican Astronomy — The Planets

So far the conceptual scheme developed by Copernicus is just as effective as Ptolemy's, but it is surely no more so, and it seems a good deal more cumbersome. It is only when the planets are added to Copernicus' universe that any real basis for his innovation becomes apparent. Consider, for example, the explanation of retrograde motion to which Copernicus alluded without discussion at the end of Chapter 5 in his introductory First Book. In the Ptolemaic system the retrograde motion of each planet is accounted for by placing the planet on a major epicycle whose center is, in turn, carried about the earth by the planet's deferent. The combined motion of these two circles produces the characteristic looped patterns discussed in Chapter 3.

In Copernicus' system no major epicycles are required. The retrograde or westward motion of a planet among the stars is only an apparent motion, produced, like the apparent motion of the sun around the ecliptic, by the orbital motion of the earth. According to Copernicus the motion that Ptolemy had explained with major epicycles was really the motion of the earth, attributed to the planets by a terrestrial observer who thought himself stationary.

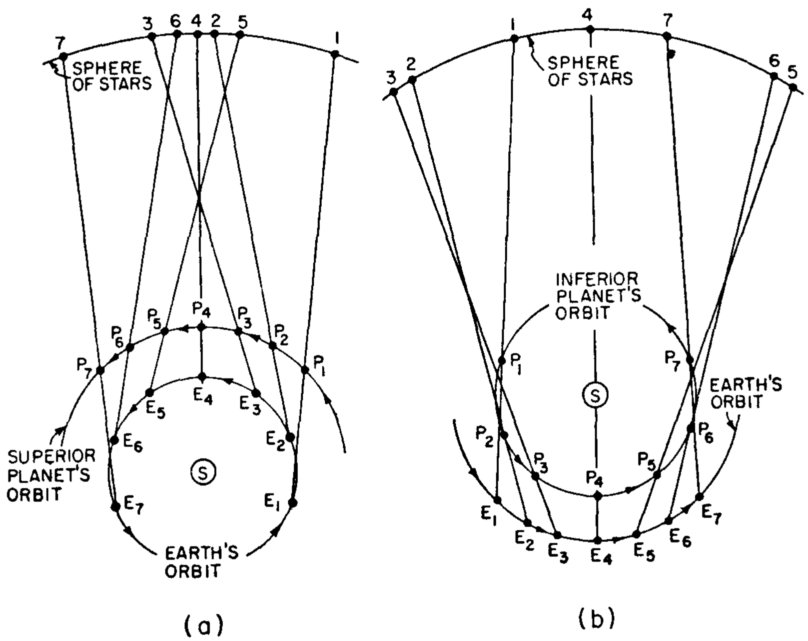

Figure 32. The Copernican explanation of retrograde motion for (*a*) superior planets and (*b*) inferior planets. In each diagram the earth moves steadily on its orbit from E_1 to E_7 and the planet moves from P_1 to P_7. Simultaneously the planet's apparent position against the stellar sphere shifts eastward from 1 to 7, but as the two planets pass there is a brief westward retrogression from 3 to 5.

The basis of Copernicus' contention is illustrated and clarified by Figures 32*a* and 32*b*. Successive apparent positions of a moving superior planet viewed from a moving earth against the fixed background provided by the stellar sphere are shown in the first diagram; the second shows successive apparent positions of an inferior planet. Only the orbital motions are indicated; the earth's diurnal rotation, which produces the rapid apparent westward motion of the sun, planets, and stars together, is omitted. In both diagrams successive positions

of the earth in its sun-centered circular orbit are indicated by the points $E_1, E_2, \ldots, E_7$; the corresponding consecutive positions of the planets are marked $P_1, P_2, \ldots, P_7$; and the corresponding apparent positions of the planet, discovered by extending a line from the earth through the planet until it intersects the stellar sphere, are labeled 1, 2, . . . , 7. In each case the more central planet moves more rapidly in its orbit. Inspection of the diagram indicates that the apparent motion of the planet among the stars is normal (eastward) from 1 to 2 and from 2 to 3; then the planet appears to retrogress (move westward) from 3 to 4 and from 4 to 5; and finally it reverses its motion again and moves normally from 5 to 6 and from 6 to 7. As the earth completes the balance of its orbit, the planet continues in normal motion, moving eastward most rapidly when it lies diametrically across the sun from the earth.

Therefore, in Copernicus' system, planets viewed from the earth should appear to move eastward most of the time; they retrogress only when the earth, in its more rapid orbital motion, overtakes them (superior planets) or when they overtake the earth (inferior planets). Retrograde motion can occur only when the earth is nearest to the planet whose motion is observed, and this is in accord with observations. Superior planets, at least, are most brilliant when they move westward. The first major irregularity of planetary motion has been explained qualitatively without the use of epicycles.

Figure 33 indicates how Copernicus' proposal accounts for a second major irregularity of the planetary motions – the discrepancy between the times required for successive trips of a planet around the ecliptic. In the diagram it is assumed that the earth completes $1\frac{1}{4}$ eastward trips about its orbit while the planet, in this case a superior planet, travels eastward through its orbit once. Suppose that at the start of the series of observations the earth is at E_1 and the planet at P. The planet is then in the middle of a retrogression and appears silhouetted against the stationary stellar sphere at 1. When the planet has completed one revolution in its orbit and returned to P, the earth has made $1\frac{1}{4}$ trips around its orbit and reached E_2. The planet therefore is seen at 2, west of position 1 at which it started. It has not yet completed a full journey around the ecliptic, and its first full trip will therefore consume more time than the planet required to revolve once in its orbit.

As the planet makes its second trip about its orbit, the earth again makes more than one orbital revolution and reaches E_3 when the planet has returned to *P* again. This time the planet is seen silhouetted at 3, to the east of position 2. It has completed more than one journey around the ecliptic while moving only once through its orbit, and its second journey around the ecliptic was therefore a very rapid one. After a third revolution the planet is again at *P*, but it appears at position 4, east of 3, and its journey around the ecliptic was therefore

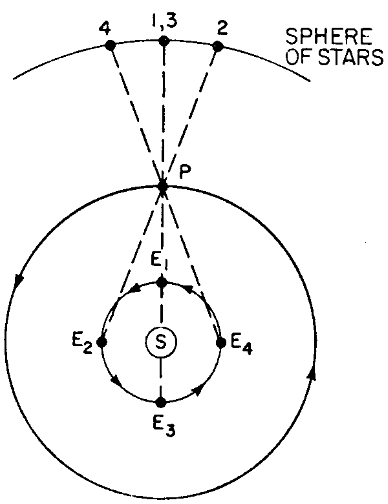

Figure 33. The Copernican explanation of variations in the time required for a superior planet to complete successive journeys around the ecliptic. While the planet moves once eastward around its orbit from *P* to *P*, the earth makes 1¼ eastward revolutions from E_1 to E_1 and on to E_2. During this interval the apparent position of the planet among the stars moves eastward from 1 to 2, slightly less than a full trip. During the planet's next revolution the earth moves from E_2 to E_2 and on to E_3, so that its apparent position among the stars shifts from 2 to 1 and on to 1 again, slightly more than one full trip around the ecliptic.

again a fast one. After a fourth revolution in its orbit the planet again appears at 1, west of 4, and its final trip was therefore slow. The planet has completed four trips about its orbit and four trips around the ecliptic at the same instant. The average time required by a superior planet to circle the ecliptic is therefore identical with the planet's orbital period. But the time required for an individual trip may be considerably greater or considerably less than the average. A similar argument will account for the similar irregularities of an inferior planet's motion.

Retrograde motion and the variation of the time required to circle

the ecliptic are the two gross planetary irregularities which in antiquity had led astronomers to employ epicycles and deferents in treating the problem of the planets. Copernicus' system explains these same gross irregularities, and it does so without resorting to epicycles, or at least to major epicycles. To gain even an approximate and qualitative account of the planetary motions Hipparchus and Ptolemy had required twelve circles – one each for the sun and moon, and two each for the five remaining "wanderers." Copernicus achieved the same qualitative account of the apparent planetary motions with only seven circles. He needed only one sun-centered circle for each of the six known planets – Mercury, Venus, Earth, Mars, Jupiter, and Saturn – and one additional earth-centered circle for the moon. To an astronomer concerned only with a qualitative account of the planetary motions, Copernicus' system must seem the more economical.

But this apparent economy of the Copernican system, though it is a propaganda victory that the proponents of the new astronomy rarely failed to emphasize, is largely an illusion. We have not yet begun to deal with the full complexity of Copernicus' planetary astronomy. The seven-circle system presented in the First Book of the *De Revolutionibus*, and in many modern elementary accounts of the Copernican system, is a wonderfully economical system, but it does not work. It will not predict the position of planets with an accuracy comparable to that supplied by Ptolemy's system. Its accuracy is comparable to that of a simplified twelve-circle version of Ptolemy's system – Copernicus can give a more economical *qualitative* account of the planetary motions than Ptolemy. But to gain a reasonably good *quantitative* account of the alteration of planetary position Ptolemy had been compelled to complicate the fundamental twelve-circle system with minor epicycles, eccentrics, and equants, and to get comparable results from his basic seven-circle system Copernicus, too, was forced to use minor epicycles and eccentrics. His full system was little if any less cumbersome than Ptolemy's had been. Both employed over thirty circles; there was little to choose between them in economy. Nor could the two systems be distinguished by their accuracy. When Copernicus had finished adding circles, his cumbersome sun-centered system gave results as accurate as Ptolemy's, but it did not give more accurate results. Copernicus did not solve the problem of the planets.

The full Copernican system is described in the latter books of the *De Revolutionibus*. Fortunately we need only illustrate the sorts of complexities there developed. Copernicus' system was not, for example, really a sun-centered system at all. To account for the increased rate at which the sun travels through the signs of the zodiac during the winter, Copernicus made the earth's circular orbit eccentric, displacing its center from the sun's. To account for other irregularities, indicated by ancient and contemporary observations of the sun's motion, he kept this displaced center in motion. The center of the earth's eccentric was placed upon a second circle whose motion continually varied the extent and direction of the earth's eccentricity. The final system employed to compute the earth's motion is represented approximately in Figure 34*a*. In the diagram, S is the sun, fixed in space; the point *O*, which itself moves slowly about the sun, is the center of a slowly rotating circle that carries the moving center O_E of the earth's eccentric; *E* is the earth itself.

Similar complexities were necessitated by the observed motions of the other heavenly bodies. For the moon Copernicus used a total of three circles, the first centered on the moving earth, the second centered on the moving circumference of the first, and the third on the

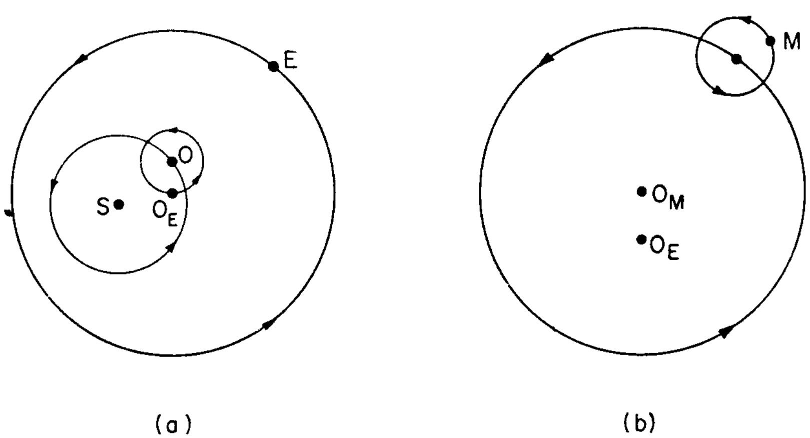

Figure 34. Copernicus' account of the motion of (*a*) the earth and (*b*) Mars. In (*a*) the sun is at S, and the earth, *E*, revolves on a circle whose center, O_E, revolves slowly about a point *O*, which in turn revolves on a sun-centered circle. In (*b*) Mars is placed on an epicycle revolving on a deferent whose center, O_M, maintains a fixed geometric relation to the moving center O_E of the earth's orbit.

circumference of the second. For Mars and most of the other planets he employed a system much like that illustrated in Figure 34*b*. The center of Mars's orbit, O_M, is displaced from the center of the earth's orbit, O_E, and is moved with it; the planet itself is placed at *M*, not on the eccentric but on an epicycle, which rotates eastward in the same direction and with the same period as the eccentric. Nor do the complexities end here. Still other devices, fully equivalent to Ptolemy's, were required to account for the north and south deviations of each planet from the ecliptic.

Even this brief sketch of the complex system of interlocking circles employed by Copernicus to compute planetary position indicates the third great incongruity of the *De Revolutionibus* and the immense irony of Copernicus' lifework. The preface to the *De Revolutionibus* opens with a forceful indictment of Ptolemaic astronomy for its inaccuracy, complexity, and inconsistency, yet before Copernicus' text closes, it has convicted itself of exactly the same shortcomings. Copernicus' system is neither simpler nor more accurate than Ptolemy's. And the methods that Copernicus employed in constructing it seem just as little likely as the methods of Ptolemy to produce a single consistent solution of the problem of the planets. The *De Revolutionibus* itself is not consistent with the single surviving early version of the system, described by Copernicus in the early manuscript *Commentariolus*. Even Copernicus could not derive from his hypothesis a single and unique combination of interlocking circles, and his successors did not do so. Those features of the ancient tradition which had led Copernicus to attempt a radical innovation were not eliminated by that innovation. Copernicus had rejected the Ptolemaic tradition because of his discovery that "the Mathematicians are inconsistent in these [astronomical] investigations" and because "if their hypotheses were not misleading, all inferences based thereon might surely be verified." A new Copernicus could have turned the identical arguments against him.

The Harmony of the Copernican System

Judged on purely practical grounds, Copernicus' new planetary system was a failure; it was neither more accurate nor significantly simpler than its Ptolemaic predecessors. But historically the new system was a great success; the *De Revolutionibus* did convince a few of Copernicus' successors that sun-centered astronomy held the key to the problem of the planets, and these men finally provided the simple and accurate solution that Copernicus had sought. We shall examine their work in the next chapter, but first we must try to discover why they became Copernicans – in the absence of increased economy or precision, what reasons were there for transposing the earth and the sun? The answer to this question is not easily disentangled from the technical details that fill the *De Revolutionibus*, because, as Copernicus himself recognized, the real appeal of sun-centered astronomy was aesthetic rather than pragmatic. To astronomers the initial choice between Copernicus' system and Ptolemy's could only be a matter of taste, and matters of taste are the most difficult of all to define or debate. Yet, as the Copernican Revolution itself indicates, matters of taste are not negligible. The ear equipped to discern geometric harmony could detect a new neatness and coherence in the sun-centered astronomy of Copernicus, and if that neatness and coherence had not been recognized, there might have been no Revolution.

We have already examined one of the aesthetic advantages of Copernicus' system. It explains the principal *qualitative* features of the planetary motions without using epicycles. Retrograde motion, in particular, is transformed to a natural and immediate consequence of the geometry of sun-centered orbits. But only astronomers who valued qualitative neatness far more than quantitative accuracy (and there were a few – Galileo among them) could consider this a convincing argument in the face of the complex system of epicycles and eccentrics elaborated in the *De Revolutionibus*. Fortunately there were other, less ephemeral, arguments for the new system. For example, it gives a simpler and far more natural account than Ptolemy's of the motions of the inferior planets. Mercury and Venus never get very far from the sun, and Ptolemaic astronomy accounts for this observation by tying the deferents of Mercury, Venus, and the sun together so that the center of the epicycle of each inferior planet always lies on a straight line between the earth and the sun (Figure 35*a*). This alignment of the centers of the epicycles is an "extra" device, an *ad hoc* addition to the geometry of earth-centered astronomy, and there is no need for such an assumption in Copernicus' system. When, as in

Figure 35*b*, the orbit of a planet lies entirely within the earth's orbit, there is no way in which the planet can appear far from the sun. Maximum elongation will occur when, as in the diagram, the line from the earth to the planet is tangent to the planet's orbit and the angle *SPE* is a right angle. Therefore the angle of elongation, *SEP*, is the largest angle by which the inferior planet can deviate from the sun. The basic geometry of the system fully accounts for the way in which Mercury and Venus are bound to the sun.

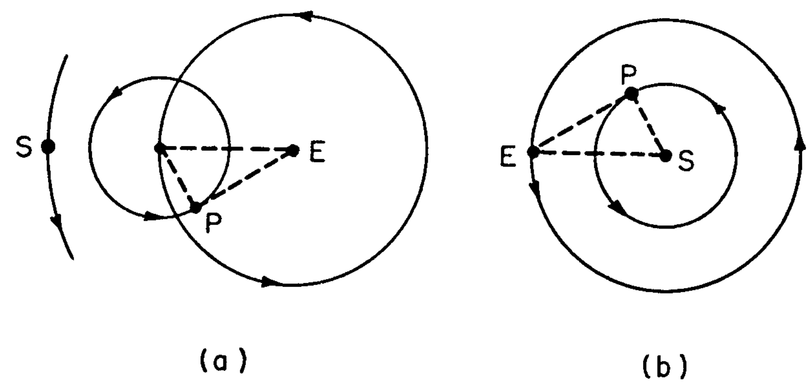

Figure 35. Limited elongation of inferior planets explained in (*a*) the Ptolemaic and (*b*) the Copernican systems. In the Ptolemaic system the angle between the sun, S, and the planet, *P*, must be restricted by keeping the center of the epicycle on the line between the earth and the sun. In the Copernican system, with the planet's orbit entirely contained by the earth's, no such restriction is necessary.

Copernican geometry illuminates another even more important aspect of the behavior of the inferior planets, namely, the order of their orbits. In the Ptolemaic system the planets were arranged in earth-centered orbits so that the average distance between a planet and the earth increased with the time required for the planet to traverse the ecliptic. The device worked well for the superior planets and for the moon, but Mercury, Venus, and the sun all require 1 year for an average journey around the ecliptic, and the order of their orbits had therefore always been a source of debate. In the Copernican system there is no place for similar debate; no two planets have the same orbital period. The moon is no longer involved in the problem, for it travels about the earth rather than about the central sun. The superior planets, Mars, Jupiter, and Saturn, preserve their old order about the new center, because their orbital periods are the same as the average lengths of time they need to circle the ecliptic. The earth's orbit lies inside of Mars's, since the earth's orbital period, 1 year, is less than Mars's 687 days. It only remains to place Mercury and Venus in the system, and their order is, for the first time, uniquely determined.

This can be seen as follows. Venus is known to retrogress every 584 days, and since retrograde motion can be observed only when Venus passes the earth, 584 days must be the time Venus requires to lap the earth once in their common circuit of the sun. Now in 584 days the earth has traversed its orbit $\frac{584}{365}$ $(=1\frac{219}{365})$ times. Since Venus has lapped the earth once during this interval, it must have circled its orbit $2\frac{219}{365}(=\frac{949}{365})$ times in just 584 days. But a planet that circles its orbit $\frac{949}{365}$ times in 584 days must require $584\times\frac{365}{949}(=225)$ days to circle its orbit once. Therefore, since Venus's period, 225 days, is less than earth's, Venus's orbit must be inside the earth's, and there is no ambiguity. A similar calculation places Mercury's orbit inside Venus's and closest to the sun. Since Mercury retrogresses, and therefore laps the earth, every 116 days, it must complete its orbit just $1\frac{116}{365}(=\frac{481}{365})$ times in 116 days. Therefore it will complete its orbit just once in $116\times\frac{365}{481}$ $(=88)$ days. Its orbital period of 88 days is the shortest of all, and it is therefore the planet closest to the sun.

So far we have ordered the sun-centered planetary orbits with the same device used by Ptolemaic astronomers to order earth-centered orbits: planets farther from the center of the universe take longer to circle the center. The assumption that the size of the orbit increases with orbital period can be applied more fully in the Copernican than in the Ptolemaic system, but in both systems it is initially arbitrary. It seems natural that planets should behave this way, like Vitruvius' ants on a wheel, but there is no necessity that they do so. Perhaps the assumption is entirely gratuitous, and the planets, excepting the sun and moon, whose distances can be directly determined, have another order.

The response to this suggested reordering constitutes another very important difference between the Copernican and the Ptolemaic systems, and one which, as we discovered in his preface, Copernicus

himself particularly emphasizes. In the Ptolemaic system the deferent and epicycle of any one planet can be shrunk or expanded at will without affecting either the sizes of the other planetary orbits or the position at which the planet, viewed from a central earth, appears against the stars. The order of the orbits *may be* determined by assuming a relation between size of orbit and orbital period. In addition, the relative dimensions of the orbits *may be* worked out with the aid of the further assumption, discussed in Chapter 3, that the minimum distance of one planet from the earth is just equal to the maximum distance between the earth and the next interior planet. But though both of these seem natural assumptions, neither is necessary. The Ptolemaic system could predict the same apparent positions for the planets without making use of either. In the Ptolemaic system the appearances are not dependent upon the order or the sizes of the planetary orbits.

There is no similar freedom in the Copernican system. If all the planets revolve in approximately circular orbits about the sun, then both the order and the relative sizes of the orbits can be determined directly from observation without additional assumptions. Any change in order or even in relative size of the orbits will upset the whole system. For example, Figure 36*a* shows an inferior planet, *P*, viewed from the earth at the time when it reaches its maximum elongation from the sun. The orbit is assumed circular, and the angle *SPE* must therefore be a right angle when the angle of elongation, *SEP*, reaches its maximum value. The planet, the sun, and the earth form a right triangle one of whose acute angles, *SEP*, can be directly measured. But knowledge of one acute angle of a right triangle determines the ratio of the lengths of the sides of that triangle. Therefore the ratio of the radius of the inferior planet's orbit, *SP*, to the radius of the earth's orbit, *SE*, can be computed from the measured value of the angle *SEP*. The relative sizes of the earth's orbit and the orbits of both inferior planets can be discovered from observation.

An equivalent determination can be made for a superior planet, though the techniques are more complex. One possible technique is illustrated in Figure 36*b*. Suppose that at some determined instant of time the sun, the earth, and the planet all lie on the straight line *SEP*; this is the orientation in which the planet lies diametrically across the ecliptic from the sun and is in the middle of a retrograde motion. Since the earth traverses its orbit more rapidly than any su-

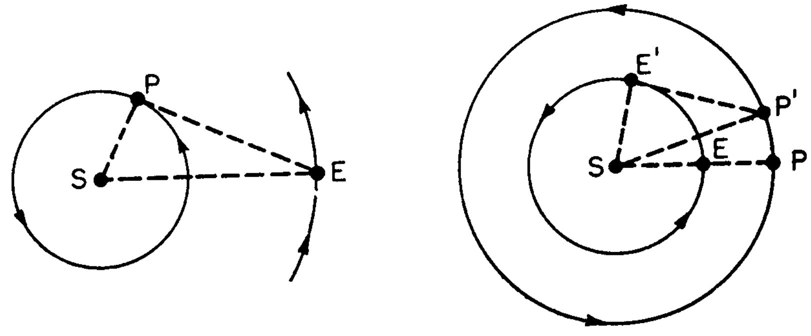

Figure 36. Determining the relative dimensions of orbits in the Copernican system: (*a*) for an inferior planet; (*b*) for a superior planet.

perior planet, there must be some later instant of time when the earth at *E′* and the planet at *P′* will form a right angle *SE′P′* with the sun, and since *SE′P′* is the angle between the sun and the superior planet viewed from the earth, it can be directly determined and the time required to achieve it can be measured. The angle *ESE′* can now be determined, for it must bear the same ratio to 360° as the time required by the earth to move from *E* to *E′* bears to the 365 days that the earth requires to complete its orbit. The angle *PSP′* can be determined in just the same way, since the time required by the planet to complete its orbit is already known, and the time occupied by the planet in going from *P* to *P′* is the same as that needed by the earth to go from *E* to *E′*. With *PSP′* and *ESE′* known, the angle *P′SE′* can be found by subtraction. Then we again have a right triangle, *SE′P′*, with one acute angle, *P′SE′*, known, and the ratio of the radius of the planet's orbit, *SP′*, to that of the earth's orbit, *SE′*, can therefore be determined just as for an inferior planet.

By techniques like this the distances to all the planets can be determined in terms of the distance between the earth and the sun, or in terms of any unit, like the stade, in which the radius of the earth's orbit has been measured. Now, for the first time, as Copernicus says in his prefatory letter, "the orders and magnitudes of all stars and spheres . . . become so bound together that nothing in any part thereof could be moved from its place without producing confusion of all the other parts and of the universe as a whole." Because the

relative dimensions of the planetary orbits are a direct consequence of the first geometric premises of sun-centered astronomy, the new astronomy has for Copernicus a naturalness and coherence that were lacking in the older earth-centered version. The structure of the heavens can be derived from Copernicus' system with fewer extraneous or *ad hoc* assumptions like plenitude. That is the new and aesthetic harmony which Copernicus emphasizes and illustrates so fully in the tenth chapter of his introductory First Book, to which we now turn, having first learned enough about the new system (as Copernicus' lay readers had not) to understand what he is talking about.

10. *Of the Order of the Heavenly Bodies.*

No one doubts that the Sphere of the Fixed Stars is the most distant of visible things. As for the order of the planets, the early Philosophers wished to determine it from the magnitude of their revolutions. They adduce the fact that of objects moving with equal speed, those farther distant seem to move more slowly (as is proved in Euclid's *Optics*). They think that the Moon describes her path in the shortest time because, being nearest to the Earth, she revolves in the smallest circle. Farthest they place Saturn, who in the longest time describes the greatest circuit. Nearer than he is Jupiter, and then Mars.

Opinions differ as to Venus and Mercury which, unlike the others, do not altogether leave the Sun. Some place them beyond the Sun, as Plato in *Timaeus*; others nearer than the Sun, as Ptolemy and many of the moderns. Alpetragius [a twelfth-century Moslem astronomer] makes Venus nearer and Mercury farther than the Sun. If we agree with Plato in thinking that the planets are themselves dark bodies that do but reflect light from the Sun, it must follow, that if nearer than the Sun, on account of their proximity to him they would appear as half or partial circles; for they would generally reflect such light as they receive upwards, that is toward the Sun, as with the waxing or waning Moon. [See the discussion of the phases of Venus in the next chapter. Neither this effect nor the following is distinctly visible without the telescope.] Some think that since no eclipse even proportional to their size is ever caused by these planets, they can never be between us and the Sun. . . . [Copernicus proceeds to note many difficulties in the arguments usually used to determine the relative order of the sun and the inferior planets. Then he continues:]

Unconvincing too is Ptolemy's proof that the Sun moves between those bodies that do and those that do not recede from him completely [that is, between the superior planets which can assume any angle of elongation and the inferior planets whose maximum elongation is limited]. Consideration of the case of the Moon, which does so recede, exposes its falseness. Again, what cause can be alleged, by those who place Venus nearer than the Sun, and Mercury next, or in some other order? Why should not these planets also follow separate paths, distinct from that of the Sun, as do the other planets [whose deferents are not tied to the sun's]? And this might be said even if their relative swiftness and slowness did not belie their alleged order. Either then the Earth cannot be the center to which the order of the planets and their Spheres is related, or certainly their relative order is not observed, nor does it appear why a higher position should be assigned to Saturn than to Jupiter, or any other planet.

Therefore I think we must seriously consider the ingenious view held by Martianus Capella [a Roman encyclopedist of the fifth century who recorded a theory of the inferior planets probably first suggested by Heraclides] . . . and certain other Latins, that Venus and Mercury do not go round the Earth like the other planets but run their courses with the Sun as center, and so do not depart from him farther than the convexity of their Spheres allows. . . . What else can they mean than that the center of these Spheres is near the Sun? So certainly the circle of Mercury must be within that of Venus, which, it is agreed, is more than twice as great.

We may now extend this hypothesis to bring Saturn, Jupiter and Mars also into relation with this center, making their Spheres great enough to contain those of Venus and Mercury and the Earth. . . . These outer planets are always nearer to the Earth about the time of their evening rising, that is, when they are in opposition to the Sun, and the Earth between them and the Sun. They are more distant from the Earth at the time of their evening setting, when they are in conjunction with the Sun and the Sun between them and the Earth. These indications prove that their center pertains rather to the Sun than to the Earth, and that this is the same center as that to which the revolutions of Venus and Mercury are related.

[Copernicus' remarks do not actually "prove" a thing. The Ptolemaic system explains these phenomena as completely as the Copernican, but the Copernican explanation is again more natural, for, like the Copernican explanation of the limited elongation of the inferior planets, it depends only on the geometry of a sun-centered astronomical system, not on the particular orbital periods assigned to the planets. Copernicus' remarks will be clarified by reference to Figure 32*a*. A superior planet retrogresses when the earth overtakes it, and under these circumstances it must be simultaneously closest to the earth and across the ecliptic from the sun. In the Ptolemaic system a retrogressing superior planet must be closer to the earth than at any other time, and it is in fact also across the sky from the sun. But it is only across the sky from the sun because the rates of rotation of its deferent and epicycle have particular values that happen to put the planet back in opposition to the sun whenever the epicycle brings the planet back close to the central earth. If, in the Ptolemaic system, the period of epicycle or deferent were quantitatively slightly different, then the qualitative regularity that puts a retrogressing superior planet across the sky from the sun would not occur. In the Copernican system it must occur regardless of the particular rates at which the planets revolve in their orbits.]

But since all these [Spheres] have one center it is necessary that the space between the convex side of Venus's Sphere and the concave side of Mars's must also be viewed as a Sphere concentric with the others, capable of receiving the Earth with her satellite the Moon and whatever is contained within the Sphere of the Moon – for we must not separate the Moon from the Earth, the former being beyond all doubt nearest to the latter, especially as in that space we find suitable and ample room for the Moon.

We therefore assert that the center of the Earth, carrying the Moon's path, passes in a great circuit among the other planets in an annual revolution round the Sun; that near the Sun is the center of the Universe; and that whereas the Sun is at rest, any apparent motion of the Sun can be better explained by motion of the Earth. Yet so great is the Universe that though the distance of the Earth from the Sun is not insignificant compared with the size of any other planetary path, in accordance with the ratios of their sizes, it is insignificant compared with the distances of the Sphere of the Fixed Stars.

I think it easier to believe this than to confuse the issue by assuming a vast number of Spheres, which those who keep Earth at the center must do. We thus rather follow Nature, who producing nothing vain or superfluous often prefers to endow one cause with many effects. Though these views are difficult, contrary to expectation, and certainly unusual, yet in the sequel we shall, God willing, make them abundantly clear at least to mathematicians.

Given the above view – and there is none more reasonable – that the periodic times are proportional to the sizes of the Spheres, then the order of the Spheres, beginning from the most distant is as follows. Most distant of all is the Sphere of the Fixed Stars, containing all things, and being therefore itself immovable. It represents that to which the motion and position of all the other bodies must be referred . . . . Next is the planet Saturn, revolving in 30 years. Next comes Jupiter, moving in a 12-year circuit; then Mars, who goes round in 2 years. The fourth place is held by the annual revolution [of the Sphere] in which the Earth is contained, together with the Sphere of the Moon as on an epicycle. Venus, whose period is 9 months, is in the fifth place, and sixth is Mercury, who goes round in the space of 80 days.

In the middle of all sits Sun enthroned. In this most beautiful temple could we place this luminary in any better position from which he can illuminate the whole at once? He is rightly called the Lamp, the Mind, the

Ruler of the Universe; Hermes Trismegistus names him the Visible God, Sophocles' Electra calls him the All-seeing. So the Sun sits as upon a royal throne ruling his children the planets which circle round him. The Earth has the Moon at her service. As Aristotle says, in his *On [the Generation of] Animals*, the Moon has the closest relationship with the Earth. Meanwhile the Earth conceives by the Sun, and becomes pregnant with an annual rebirth.

So we find underlying this ordination an admirable symmetry in the Universe, and a clear bond of harmony in the motion and magnitude of the Spheres such as can be discovered in no other wise. For here we may observe why the progression and retrogression appear greater for Jupiter than Saturn, and less than for Mars, but again greater for Venus than for Mercury [a glance at Figure 32 will show that the closer the orbit of a planet is to the orbit of the earth, the larger the apparent retrograde motion of that planet must be – an additional harmony of Copernicus' system]; and why such oscillation appears more frequently in Saturn than in Jupiter, but less frequently in Mars and Venus than in Mercury [the earth will lap a slowly moving superior planet more frequently than it laps a rapid one, and conversely for an inferior planet]; moreover why Saturn, Jupiter and Mars are nearer to the Earth at opposition to the Sun than when they are lost in or emerge from the Sun's rays. Particularly Mars, when he shines all night [and is therefore in opposition], appears to rival Jupiter in magnitude, being only distinguishable by his ruddy color; otherwise he is scarce equal to a star of the second magnitude, and can be recognized only when his movements are carefully followed. All these phenomena proceed from the same cause, namely Earth's motion.

That there are no such phenomena for the fixed stars proves their immeasurable distance, because of which the outer sphere's [apparent] annual motion or its [parallactic] image is invisible to the eyes. For every visible object has a certain distance beyond which it can no more be seen, as is proved in optics. The twinkling of the stars, also, shows that there is still a vast distance between the farthest of the planets, Saturn, and the Sphere of the Fixed Stars [for if the stars were very near Saturn, they should shine as he does], and it is chiefly by this indication that they are distinguished from the planets. Further, there must necessarily be a great difference between moving and non-moving bodies. So great is this divine work of the Great and Noble Creator!

Throughout this crucially important tenth chapter Copernicus' emphasis is upon the "admirable symmetry" and the "clear bond of harmony in the motion and magnitude of the Spheres" that a sun-centered geometry imparts to the appearances of the heavens. If the sun is the center, then an inferior planet cannot possibly appear far from the sun; if the sun is the center, then a superior planet must be in opposition to the sun when it is closest to the earth; and so on and on. It is through arguments like these that Copernicus seeks to persuade his contemporaries of the validity of his new approach. Each argument cites an aspect of the appearances that can be explained by *either* the Ptolemaic *or* the Copernican system, and each then proceeds to point out how much more harmonious, coherent, and natural the Copernican explanation is. There are a great many such arguments. The sum of the evidence drawn from harmony is nothing if not impressive.

But it may well be nothing. "Harmony" seems a strange basis on which to argue for the earth's motion, particularly since the harmony is so obscured by the complex multitude of circles that make up the full Copernican system. Copernicus' arguments are not pragmatic. They appeal, if at all, not to the utilitarian sense of the practicing astronomer but to his aesthetic sense and to that alone. They had no appeal to laymen, who, even when they understood the arguments, were unwilling to substitute minor celestial harmonies for major terrestrial discord. They did not necessarily appeal to astronomers, for the harmonies to which Copernicus' arguments pointed did not enable the astronomer to perform his job better. New harmonies did not increase accuracy or simplicity. Therefore they could and did appeal primarily to that limited and perhaps irrational subgroup of mathematical astronomers whose Neoplatonic ear for mathematical harmonies could not be obstructed by page after page of complex mathematics leading finally to numerical predictions scarcely better than those they had known before. Fortunately, as we shall discover in the next chapter, there were a few such astronomers. Their work is also an essential ingredient of the Copernican Revolution.

Revolution by Degrees

Because he was the first fully to develop an astronomical system based upon the motion of the earth, Copernicus is frequently called the first modern astronomer. But, as the text of the *De Revolutionibus* indicates, an equally persuasive case might be made for calling him the last great Ptolemaic astronomer. Ptolemaic astronomy meant far more than astronomy predicated on a stationary earth, and it is only with respect to the position and motion of the earth that Copernicus broke with the Ptolemaic tradition. The cosmological frame in which his astronomy was embedded, his physics, terrestrial and celestial, and even the mathematical devices that he employed to make his system give adequate predictions are all in the tradition established by ancient and medieval scientists.

Though historians have occasionally grown livid arguing whether Copernicus is really the last of the ancient or the first of the modern astronomers, the debate is in principle absurd. Copernicus is neither an ancient nor a modern but rather a Renaissance astronomer in whose work the two traditions merge. To ask whether his work is really ancient or modern is rather like asking whether the bend in an otherwise straight road belongs to the section of road that precedes the bend or to the portion that comes after it. From the bend both sections of the road are visible, and its continuity is apparent. But viewed from a point before the bend, the road seems to run straight to the bend and then to disappear; the bend seems the last point in a straight road. And viewed from a point in the next section, after the bend, the road appears to begin at the bend from which it runs straight on. The bend belongs equally to both sections, or it belongs to neither. It marks a turning point in the direction of the road's progress, just as the *De Revolutionibus* marks a shift in the direction in which astronomical thought developed.

To this point in this chapter we have emphasized primarily the ties between the *De Revolutionibus* and the earlier astronomical and cosmological tradition. We have minimized, as Copernicus himself does, the extent of the Copernican innovation, because we have been concerned to discover how a potentially destructive innovation could be produced by the tradition that it was ultimately to destroy. But, as we shall soon discover, this is not the only legitimate way to view the *De Revolutionibus*, and it is not the view taken by most later Copernicans. For Copernicus' sixteenth- and seventeenth-century followers, the primary importance of the *De Revolutionibus* derived from its single novel concept, the planetary earth, and from the novel astronomical consequences, the new harmonies, which Copernicus had derived from that concept. To them Copernicanism meant the threefold motion of the earth and, initially, that alone. The traditional conceptions with which Copernicus had clothed his innovation were not to his followers essential elements of his work, simply because, as traditional elements, they were not Copernicus' contribution to science. It was not because of its traditional elements that people quarreled about the *De Revolutionibus*.

That is why the *De Revolutionibus* could be the starting point for a new astronomical and cosmological tradition as well as the culmination of an old one. Those whom Copernicus converted to the concept of a moving earth began their research from the point at which Copernicus had stopped. Their starting point was the earth's motion, which was all they necessarily took from Copernicus, and the problems to which they devoted themselves were not the problems of the old astronomy, which had occupied Copernicus, but the problems of the new sun-centered astronomy, which they discovered in the *De Revolutionibus*. Copernicus presented them with a set of problems that neither he nor his predecessors had had to face. In the pursuit of those problems the Copernican Revolution was completed, and a new astronomical tradition, deriving from the *De Revolutionibus*, was founded. Modern astronomy looks back to the *De Revolutionibus* as Copernicus had looked back to Hipparchus and Ptolemy.

Major upheavals in the fundamental concepts of science occur by degrees. The work of a single individual may play a preëminent role in such a conceptual revolution, but if it does, it achieves preëminence either because, like the *De Revolutionibus*, it initiates revolution by a small innovation which presents science with new problems, or because, like Newton's *Principia*, it terminates revolution by integrating concepts derived from many sources. The extent of the innovation that any individual can produce is necessarily limited, for each individual must employ in his research the tools that he acquires from a traditional education, and he cannot in his own lifetime replace them all. It seems therefore that many of the elements in the *De Revolutionibus* which, in the earlier parts of this chapter, we pointed to as incongruities are not really incongruities at all. The *De Revolutionibus* seems incongruous only to those who expect to find the entire Copernican Revolution in the work which gives that revolution its name, and such an expectation derives from a misunderstanding of the way in which new patterns of scientific thought are produced. The limitations of the *De Revolutionibus* might better be regarded as essential and typical characteristics of any revolution-making work.

Most of the apparent incongruities in the *De Revolutionibus* reflect the personality of its author, and Copernicus' personality seems entirely appropriate to his seminal role in the development of astronomy. Copernicus was a dedicated specialist. He belonged to the revived Hellenistic tradition of mathematical astronomy which emphasized the mathematical problem of the planets at the expense of cosmology. For his Hellenistic predecessors the physical incongruity of an epicycle had not been an important drawback of the Ptolemaic system, and Copernicus displayed a similar indifference to cosmological detail when he failed to note the incongruities of a moving earth in an otherwise traditional universe. For him, mathematical and celestial detail came first; he wore blinders that kept his gaze focused upon the mathematical harmonies of the heavens. To anyone who did not share his specialty Copernicus' view of the universe was narrow and his sense of values distorted.

But an excessive concern with the heavens and a distorted sense of values may be essential characteristics of the man who inaugurated the revolution in astronomy and cosmology. The blinders that restricted Copernicus' gaze to the heavens may have been functional. They made him so perturbed by discrepancies of a few degrees in astronomical prediction that in an attempt to resolve them he could embrace a cosmological heresy, the earth's motion. They gave him an eye so absorbed with geometrical harmony that he could adhere to his heresy for its harmony alone, even when it had failed to solve the problem that had led him to it. And they helped him evade the nonastronomical consequences of his innovation, consequences that led men of less restricted vision to reject his innovation as absurd.

Above all, Copernicus' dedication to the celestial motions is responsible for the painstaking detail with which he explored the mathematical consequences of the earth's motion and fitted those consequences to an existing knowledge of the heavens. That detailed technical study is Copernicus' real contribution. Both before and after Copernicus there were cosmologists more radical than he, men who with broad brush strokes sketched an infinite and multipopulated universe. But none of them produced work resembling the later books of the *De Revolutionibus*, and it is these books which, by showing for the first time that the astronomer's job could be done, and done more harmoniously, from a moving earth, provided a stable base from which to launch a new astronomical tradition. Had Copernicus' cosmological First Book appeared alone, the Copernican Revolution would and should be known by someone else's name.

6

THE ASSIMILATION OF COPERNICAN ASTRONOMY

The Reception of Copernicus' Work

Copernicus died in 1543, the year in which the *De Revolutionibus* was published, and tradition tells us that he received the first printed copy of his life's work on his deathbed. The book had to fight its battles without further help from its author. But for those battles Copernicus had constructed an almost ideal weapon. He had made the book unreadable to all but the erudite astronomers of his day. Outside of the astronomical world the *De Revolutionibus* created initially very little stir. By the time large-scale lay and clerical opposition developed, most of the best European astronomers, to whom the book was directed, had found one or another of Copernicus' mathematical techniques indispensable. It was then impossible to suppress the work completely, particularly because it was in a printed book and not, like Oresme's work or Buridan's, in a manuscript. Whether intentionally or not, the final victory of the *De Revolutionibus* was achieved by infiltration.

For two decades before the publication of his principal work Copernicus had been widely recognized as one of Europe's leading astronomers. Reports about his research, including his new hypothesis, had circulated since about 1515. The publication of the *De Revolutionibus* was eagerly awaited. When it appeared, Copernicus' contemporaries may have been skeptical of its main hypothesis and disappointed in the complexity of its astronomical theory, but they were nevertheless forced to recognize Copernicus' book as the first European astronomical text that could rival the *Almagest* in depth and completeness. Many advanced astronomical texts written during the fifty years after Copernicus' death referred to him as a "second