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What Every Young Person Ought to Know About Naked-Eye Astronomy

In order to provide a starting point for an understanding of ancient astronomical texts, I shall begin by presenting, in all brevity, the basic elements of naked-eye astronomy. I shall, of course, deal principally, but not entirely, with phenomena of interest to ancient astronomers. Among these are many phenomena, such as the first or last visibility of a planet or the moon, that the modern astronomer shuns since they take place near the horizon and further depend on imperfectly understood criteria. Thus, these phenomena are not commonly discussed in the modern astronomical literature and, more seriously, we lack modern standards with which we may measure the quality of the ancient results.

My presentation is entirely descriptive. I do not attempt to explain why planets become retrograde, or that celestial bodies really do not rise and set but that the earth's rotation makes it appear that they do. Whoever does not feel comfortable talking about the solar system without taking a detour via the sun is welcome to do so.

The Celestial Sphere, Fixed Stars, Daily Rotation

We have no simple means of judging distances to celestial bodies; we can only determine the directions toward them. To describe what we thus observe, we introduce the *celestial sphere* as a spherical surface with its center at the observer's eye and of unit radius—which unit is irrelevant, but it is often thought more comfortable to choose a very large one. A celestial object is then mapped at, or identified with, the point on this spherical surface at which the line of sight to the object pierces the celestial sphere. The study of the behavior of this map of celestial objects on the celestial sphere is called *spherical astronomy.*

If all celestial bodies visible to the naked eye are thus mapped on the celestial sphere, it becomes apparent that the vast majority of them remain in fixed patterns with respect to each other: They form recognizable constellations that, in turn, remain unchangeably distributed. These celestial bodies are called the *fixed stars.* To the naked-eye observer there remain seven exceptional objects: the Sun, the Moon, and the five bright planets: Mercury, Venus, Mars, Jupiter, and Saturn; being exceptional, they are, of course, of particular interest.

Before describing the celestial sphere and its motion, I shall briefly introduce, informally and without proofs, some basic terminology and a few results from spherical geometry (i.e., the geometry on the surface of a sphere).

A plane cuts a spherical surface, if at all, in a circle (or in one point if the plane is tangent to the sphere). If the plane happens to pass through the center of the sphere, the section is called a *great circle*; otherwise the section is called a *small circle.* Great circles are fundamental to spherical geometry; indeed, they play much the same role that straight lines do in plane geometry. Thus, through two points on a spherical surface that are not diametrically opposite, passes one and only one great circle, and the shortest distance between two such points measured on the sphere is along the great circle joining them. By the "distance between two points on the sphere," we mean the shortest distance, namely, the length of the shorter of the two great circle arcs that join them; this distance is usually given not in linear measure but in degrees (thus, the longer of the two great circle arcs joining two points will be 360° minus the distance between the points).

Associated with a great circle are two points called its *poles*; they are the end points of that particular diameter of the sphere which is perpendicular to the plane of the great circle (i.e., they are to the great circle what the North and South poles are to the equator). Conversely, to two diametrically opposite points on the sphere there corresponds one, and only one, great circle whose poles they are. The distance from a pole to any point on the corresponding great circle is 90°.

Two great circles always intersect in diametrically opposite points, that is, they always bisect each other. The angle between two great circles is the same as the distance (in degrees) between their poles.

These few remarks may suffice for our present purposes, and we can now return to the celestial sphere and its behavior.

The fixed stars permit us to get a hold on the celestial sphere—they provide us with a coordinate system, if you will—and enable us to perceive its motion. Ignoring for the moment various very slow changes, we will observe that the celestial sphere, with the fixed stars fixed upon it, revolves about a fixed axis at a fixed rate of very nearly 366 1/4 (*sic*) revolutions per year, relative to familiar fixed objects in our surroundings. This axis, being a diameter of the celestial sphere, pierces it at two diametrically opposite points called the celestial *north* and *south poles* (the north pole is now very near the North Star, or Stella Polaris). The great circle corresponding to these poles (i.e., the great circle that slides in itself during the *daily rotation*, as this motion is usually named) is called the *celestial equator* or, simply, the *equator*.

A horizontal plane through the observer intersects the celestial sphere in a great circle called the *horizon*, and a vertical line, also through the observer, pierces the sphere at two points, *zenith* above and *nadir* below, which are the poles (the term "pole" is used here as in spherical geometry) belonging to the horizon. Celestial objects below the horizon are invisible since the line of sight to them would pass through the body of the earth: The plane of the horizon is ideally tangent to the spherical earth.

The vertical plane through the observer and the north pole, which because it is vertical also contains the zenith, intersects the celestial sphere in a great circle called the *meridian* and meets the horizon in two points: the *north* and *south points*.

The vertical plane through the observer perpendicular to the plane of the meridian intersects the celestial sphere in a great circle called the *first vertical*, which meets the horizon in the *east* and *west points*. The equator passes through the east and west points.

For a given place of observation, the directions to the north, south, east, and west points as just defined, remain fixed in relation to characteristic features of the visible neighboring terrain. Furthermore, the elevation of the north pole does not change.* These two facts together imply that the axis through the north and south poles, about which the daily rotation takes

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*Here there is no need for the cautionary remark about long-term place, remains fixed for an observer at a given locality in relation to his or her terrestrial surroundings. The elevation of the north pole above the horizon, in angular measure, is called the terrestrial latitude of the place of observation and is usually denoted by ϕ. For Babylon we have $\phi = 32\frac{1}{2}^\circ$, very nearly.

In relation to a given horizon, the fixed stars are divided into three classes: those that are always above the horizon; those that are sometimes above and sometimes below the horizon; and those that are always below the horizon.

The stars in the first category are called *circumpolar*. They are all within a cap with the north pole as its center (for observers in the northern hemisphere of the earth) and a radius equal to the observer's terrestrial latitude ϕ. Thus, the farther north you live, the larger the region of the circumpolar stars.

A corresponding cap of equal size but centered on the south pole contains the fixed stars that are never above the horizon.

All the fixed stars in the belt between these two circumpolar caps will cross the horizon in the east and in the west and by the daily rotation will be carried in a path partly above and partly below the horizon. This daily path—a circle on the celestial sphere—is traversed by the fixed star in slightly less than 24 hours. A star on the equator is as long in time above as below the horizon, for its diurnal path is bisected by the horizon. Again for an observer in the northern hemisphere, if a star is north of the equator, it spends more time above the horizon than below—the difference is larger the nearer the star is to the circumpolar cap—and symmetrically for a star south of the equator.

Since a star is visible only at night and when above the horizon, it follows that circumpolar stars are visible every night of the year. Those circumpolar about the south pole are never visible. A star in between is visible more nights the farther north it is, for the interval of nighttime shifts throughout the year in relation to a star's horizon crossings, as we shall see.

The sizes of the circumpolar caps and of the horizon-crossing belt vary with terrestrial latitude, as mentioned. To illustrate this, let us consider two extremal situations.

First, for an observer on the earth's North Pole, where the terrestrial latitude ϕ is 90°, zenith and north pole coincide. The circumpolar caps meet at the horizon, and the horizon-crossing belt vanishes. All visible stars are circumpolar, their diurnal paths are parallel to the horizon, and no stars ever cross the horizon. One has a chance of seeing only the stars on the northern celestial hemisphere, whereas the rest are never above the horizon.

Second, for an observer on the terrestrial equator, it is the circumpolar caps that vanish while the horizon-crossing belt fills the entire celestial sphere. The north and south poles are in the horizon, and all stars spend equal time above and below the horizon. One has, theoretically, an equal chance of seeing all stars, though in practice those stars near the poles never get very far above the horizon.

The diurnal circle of a star in the horizon-crossing belt intersects the meridian in the points of *upper* and *lower culmination* at which the star is at its greatest elevation above, and lowest depression below, the horizon. For a circumpolar star these two points mark its greatest and smallest distance from the horizon, respectively.

Sun, Ecliptic, Seasons

For two reasons it is natural to begin an introduction to spherical astronomy, as I have, with the fixed stars. First, their behavior is simpler than that of the other celestial bodies: They remain fixed in relation to each other, and all join in the uniform diurnal rotation about a fixed axis. Second, they provide a convenient background against which the more complicated behavior of sun, moon, and planets can be perceived and described. It must be emphasized that once a celestial body has been placed among the fixed stars it will, of course, partake of the same daily rotation that they are subject to, in addition to any motion that it will have relative to them.

The first example of this approach is the case of the sun. Let us imagine, contrary to our everyday experience, that it were possible to see the sun and fixed stars at the same time. (This is now so for an observer outside the earth's atmosphere.) We would then note that the sun, day by day, moves eastward very slowly among the fixed stars, in the amount of about 1° per day. It returns exactly to its original place after the lapse of one year—that is, indeed, the definition of the year or, more precisely, the *sidereal* year—having traced out in that interval a path among the stars which is a great circle. Year after year the sun travels precisely the same great circle, which is called the *ecliptic*.

In the time it takes the sun to complete one revolution in the ecliptic, namely, in one year, the fixed stars revolve, as said, very nearly 366 1/4 times relative to, say, the meridian. This is the same as saying that a year has about 365 1/4 days, for the sun has in that period revolved the same number of times as the stars less the one revolution it itself has performed relative to them, but in the opposite sense of the daily rotation.

The ecliptic plays a fundamental role as reference circle in ancient astronomy. This is not surprising, for not only is it the path of the sun, but the moon and the planets are always within a belt extending at most 10° on either side of it.

The ecliptic has an inclination toward the equator of about $23\frac{1}{2}^\circ$. The two diametrically opposite points of intersection between ecliptic and equator are called the *equinoxes*. When in its travel the sun happens to be in either one of these, its diurnal path is the equator itself which is bisected by the horizon, so day equals night in duration. When in its yearly motion the sun crosses the equator from the south to the north, it is *vernal* or *spring equinox*—this term is commonly applied both to the phenomenon and to the point on the celestial sphere—and the other crossing is called *autumnal* or *fall equinox*.

The point halfway between the equinoxes and at which the sun is farthest north of the equator is the *summer solstice*, and the diametrically opposite point is the *winter solstice*. When the sun is at these points, the duration of daylight is longest and shortest, respectively.

Let us, once more, consider the previous two extremal situations. First, for an observer on the North Pole of the earth, where zenith and celestial north pole coincide, as do horizon and celestial equator, the sun will appear in the horizon for the first time at "vernal equinox" and will not set until "autumnal equinox." From the vernal equinox the sun slowly gains elevation above the horizon until it reaches an altitude of $23\frac{1}{2}^\circ$—the amount of inclination of the ecliptic against the equator—at summer solstice. In the course of 24 hours the sun will be seen above every point of the horizon, and the shadow cast by a vertical stick will revolve 360°. From summer solstice the sun slowly works its way back down to the horizon, and from fall equinox to spring equinox it will be invisible.

On the earth's equator, where the celestial equator and the first vertical coincide, day and night are always equal, and the "equinoxes" are marked by the sun passing through zenith at noon. At the solstices the sun crosses the meridian farthest north or south of zenith.

Two further special cases are of interest. One is to find the zones on the earth where the sun becomes circumpolar just once a year. For this to happen the circumpolar cap whose radius is the terrestrial latitude ϕ must reach the sun when farthest (i.e., $23\frac{1}{2}^\circ$) from the equator. Thus, ϕ must be $90^\circ - 23\frac{1}{2}^\circ = 66\frac{1}{2}^\circ$. At this northern latitude the sun will just reach, but not cross, the horizon at midnight on summer solstice, and symmetrically for the southern hemisphere. The two circles on earth of these latitudes are called the *polar circles*.

The other special case is to determine where on the earth the sun just reaches the zenith once a year at solstice. It is readily seen that this happens at terrestrial latitude $23\frac{1}{2}^\circ$ north or south. The corresponding two circles are called the *tropics*, the northern *of Cancer*, the southern *of Capricorn*, for reasons that will become clear later.

Finally, I shall mention a variant way of characterizing terrestrial latitude, namely, by giving the ratio of longest to shortest daylight. This works, of course, only for places between, but not on, the polar circles. Since, for reasons of symmetry, the shortest day at a given locality equals the shortest night in length, this ratio tells us how the sun's diurnal circle at summer solstice is divided by the horizon; it is then a fairly simple matter to determine, if one wishes, the elevation of the pole (i.e., the place's terrestrial latitude) by means of spherical trigonometry.

On the equator of the earth this ratio is 1 : 1. The farther north a locality, namely, the higher the North Pole is elevated above the horizon, the larger this ratio becomes until it loses definition on the polar circle, where the longest day is 24 hours and the shortest 0 hours.

We find this practice in ancient Greece, and a ratio of longest to shortest daylight of 3 : 2 for Babylon is, as we shall see, a fundamental parameter in Babylonian astronomy. I must emphasize, however, that we have no evidence whatsoever that the Babylonians were aware that this ratio changes as one travels north or south.

Synodic Cycle of a Star Near the Ecliptic

As an introduction to phenomena of the kind dealt with in Babylonian astronomy, let us consider a fixed star on or near the ecliptic; we shall be concerned with when, where, and how long it is visible from a place of observation of reasonable latitude.

Since the star is close to the sun's yearly path, at a particular time of year the sun and star nearly coincide on the celestial sphere or, as we say, are in conjunction. The star will be invisible, for whenever it is above the horizon, so is the sun. Star and sun will rise simultaneously on that day.

When the star is about to rise the next morning, the sun will have moved about 1° farther along the ecliptic so that the star rises a little before the sun. The following morning the sun will have moved yet another degree away from the star, so the interval from starrise to sunrise has lengthened. Eventually a morning will come when the star rises so long before sunrise that the sky is sufficiently dark for the star to be visible as it crosses the horizon, if only for a short while until the dawn extinguishes it. This is the phenomenon of *first visibility* (we usually denote it by the letter Γ).

From the morning of first visibility, the star will rise earlier and earlier, and we can follow it farther and farther from the eastern horizon along its diurnal arc before dawn makes it vanish.

Half a year after conjunction the sun reaches a point on the ecliptic 180° from the star. Since ecliptic and horizon, being great circles, always intersect in diametrically opposite points, we now have the situation that when the star is in the horizon, so is the sun. Thus, the star rises at sunset and sets at sunrise; further, the star will be in upper culmination at midnight. We say that now the star is in *opposition* to the sun, and we use the letter Θ for this phenomenon. We are able to see the star all night long—less the intervals of dawn and dusk.

As the sun now progresses in its yearly motion, it approaches the star from the other side, from the west. After each sunset, when it gets dark enough to see it, the star will already be well past the eastern horizon, and it will at its appearance get closer and closer to the western horizon across which it sets.

There will now come an evening when the star appears only just before it sets, while on the next evening the sun will have gotten so close to it that the sky is not dark enough for the star to be seen before it sets. This is the phenomenon of disappearance of the fixed star, and we denote it by the letter Ω.

The star will now remain invisible while the sun catches up with it. Star and sun will, once again, be in conjunction, and the cycle is closed, having occupied just one year.

What I have described here is called the fixed star's *synodic cycle*, consisting of the *synodic phenomena* conjunction, first appearance, opposition, last appearance, and conjunction—once again, all phenomena that place the star in special relation to the sun.

The visibility phenomena, first and last appearance, or Γ and Ω, are determined by several factors, some of which are difficult to control. First they depend on the brightness of the star: The brighter the star is, the shorter the sun has to be removed from it for it to be visible near the horizon. Second, they depend on the inclination of the ecliptic against the horizon when the star is rising or setting: The smaller the inclination, the farther the sun must be from the star to ensure the darkness necessary for visibility. Furthermore, these phenomena depend on the acuity of the observer's eyesight and the quality of the atmosphere; in the former there is clearly a personal variation, and the latter is difficult, if not impossible, to ascertain for a place of observation in antiquity.

Since the synodic period of a fixed star is one year, any one of the synodic phenomena can be used as a seasonal indicator. This is, indeed, an ancient practice. It is well known that the first appearance of Sothis, our Sirius, was used by the ancient Egyptians as a herald of the rising of the Nile—two seasonal phenomena that happened to coincide—and one finds various rustic tasks tied to first or last appearances of certain fixed stars or constellations in primitive societies as we learn, for instance, from Hesiod's *Works and Days*.

Finally, a remark about the term "synodic"; *synodos* in Greek means "getting together" or "meeting" and in astronomy is used particularly for the coincidence in position of a celestial object with the sun, that is, for conjunction. It is now used for any phenomenon linking a star, a planet, or the moon to the sun in a certain fashion.

Synodic Cycle of an Outer Planet

The planets are conspicuous among celestial bodies for at least the following three reasons: They are all very bright; they do not twinkle like the fixed stars; and they move relative to the fixed stars though always staying close to the ecliptic. They are divided into two classes consisting respectively of those that can reach opposition to the sun and those that cannot.

The members of the first class—we call them *outer planets*—are Mars, Jupiter, and Saturn, as far as naked-eye astronomy is concerned. The second class consists of the two *inner planets*, Mercury and Venus. Characteristic of them is the limit to how far they can get from the Sun: Venus never gets farther from the Sun than about 40°, and Mercury's limits are even narrower—about 25°. We shall first be concerned with the outer planets.

The synodic behavior of Saturn has much in common with that of a fixed star near the ecliptic. Like it, Saturn makes its first and last appearance (also denoted Γ and Ω), and near the middle of its interval of visibility it is in opposition (Θ) to the sun. The difference is that Saturn has a motion of its own: The general trend of this motion is that from Γ to Γ (or from Ω to Ω), that is, during one synodic period, Saturn moves about 12° along the ecliptic in the same direction—eastward—as that of the Sun's proper motion. This means that the synodic period of Saturn is some 12 days longer than a year, for the Sun will have to travel 12° in excess of one complete revolution in the ecliptic to catch up with Saturn again, and the Sun travels almost 1° per day.

Within each synodic period Saturn's motion is, however, more complicated; after its first appearance Γ, Saturn first moves *directly*, as we call eastward motion along the ecliptic, then comes to a halt among the fixed stars—it is now at its *first stationary point* Φ—then reverses its motion, or becomes retrograde, and then comes to a halt once again at its *second stationary point* Ψ, where its motion finally becomes direct, once again. Opposition happens in the middle of the retrograde arc. The synodic phenomena of Saturn are then, in their proper order,

Γ: first visibility
Φ: first stationary point ⎫
Θ: opposition ⎬ retrogradation
Ψ: second stationary point ⎭
Ω: disappearance

An actual run of Saturn for some three synodic periods is illustrated in Figure 1, where the dimensions perpendicular to the ecliptic are exaggerated four times for the sake of clarity. The dotted stretches from Ω to Γ are its arcs of invisibility in the middle of which it is in conjunction with the sun.

The other outer planets, Jupiter and Mars, behave qualitatively like Saturn. The character and sequence of their synodic

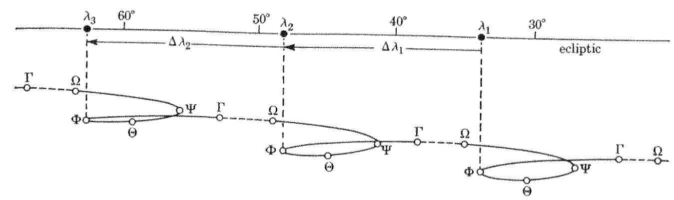

FIGURE 1.

phenomena are quite the same, but their *synodic arcs*—the progress from, say, Γ to Γ—are larger. Jupiter's synodic arc is some thirty-odd degrees, and that of Mars is even in excess of 360°. Thus, their synodic periods are longer than Saturn's , that of Mars even longer than two years. I shall later return to these matters and treat them from a more quantitative point of view.

Synodic Cycle of an Inner Planet

Since an inner planet is, as it were, tethered on a short leash to the sun, it will, in the long run, travel among the fixed stars as many times as the sun does or, on the average, once a year.

When an inner planet is, within its bounds, sufficiently west of the sun, it will be visible in the eastern sky for an interval before sunrise, and it is then called a *morning star*. When it is sufficiently east of the sun, it will be seen in the western sky for a while after sunset, and it is then called an *evening star*. Since Mercury is tied to the Sun within rather narrow limits, it is visible, if at all, only for relatively short intervals before sunrise or after sunset. The synodic phenomena of an inner planet are, in order,

Ξ: first appearance as an evening star;
greatest eastern elongation from the sun
Ψ: stationary point in the west ⎫
Ω: disappearance as an evening star; ⎪
inferior conjunction ⎬ retrogradation
Γ: first appearance as a morning star ⎪
Φ: stationary point in the east; ⎭
greatest western elongation from the sun
Σ: disappearance as a morning star;
superior conjunction

The phenomena that have not received Greek letters are not considered in Babylonian astronomy. At inferior and superior conjunction, an inner planet is, of course, invisible so it is not odd that they are not among the Babylonian synodic phenomena (the distinction between inferior and superior conjunction reflects our modern knowledge that the inner planets circle about the sun). One may find it strange, however, that an inner planet at greatest elongation from the sun—the situation when it is visible the longest—is not given special consideration.

The Moon's Synodic Course

The moon moves directly, namely, eastward, among the fixed stars at a rate of about 13° per day in an orbit that is inclined approximately 5° toward the ecliptic; the moon will thus return very nearly to the same position among the fixed stars after some $27\frac{1}{3}$ days, an interval that is called one *sidereal month*.

To get some sense of the moon's swiftness, one may consider that the moon's apparent diameter is slightly more than $\frac{1}{2}^\circ$, so the moon moves a distance of its own width in less than one hour. Thus, the moon's proper motion can be readily perceived in quite a short time if one refers it to neighboring fixed stars.

When we considered the synodic cycle of a fixed star or of an outer planet, the sun was the swifter body that ran away from, and caught up with, the other. With the moon the roles are reversed. If sun and moon are in conjunction near a fixed star at a certain moment, the moon will reach the star again after one sidereal month, or some $27\frac{1}{3}$ days, in the course of which the sun has moved ahead only some 27°. Thus, it will take the moon a little more than two extra days to reach the sun and, once more, be in conjunction. The interval in time from conjunction to conjunction is, then, somewhat over 29 days—on the average, 29.5309 days, to be more exact—and is called one *synodic month* or one *lunation*. In addition to the year and to the day, this is a most important time interval in ancient calendars, and fundamental in Babylonian astronomy.

When we now consider the sequence of synodic phenomena within each synodic cycle, we should recall that the moon's phase can be used as an additional indicator of its synodic state. Furthermore, it should be remembered that the moon is the only celestial body that may be seen with ease simultaneously with the sun, that is, in the daytime (one may actually at times follow the progress of Venus across the day sky, but it is difficult, and one has to know where to look).

At conjunction the moon is invisible, unless it happens to eclipse the sun; we shall return to the question of eclipses later. As the moon now moves away from the sun at the rate of some 12° per day—the difference between lunar and solar daily motion—it increases its chances for being visible in two ways: by getting farther from the sun, and by its lighted sickle growing in width. At the first or second sunset—in exceptional circumstances, perhaps the third—after conjunction, the thin crescent becomes visible in the west against the darkening evening sky before the moon sets following the sun. This is the *evening of first visibility* of the new moon and it marks the beginning of a new month in the Babylonian calendar. Determining in advance which evening the new moon will become visible is one of the chief goals of Babylonian lunar theory and, as we shall see, is a highly difficult task.

Some seven days after conjunction, the moon is 90° from the sun, and half its visible surface is lighted. We say that the moon is at *first quarter*. It will now rise near noon, when the sun culminates; at sunset the moon at first quarter will be near culmination, and it sets near midnight.

Half a synodic month, or 14 to 15 days, after conjunction, the moon reaches opposition to the sun. Its entire visible surface is lighted and we have *full moon*. The full moon rises at sunset and sets at sunrise. Eclipses of the moon occur at full moon. Both conjunctions and oppositions of the moon are called by the common term *syzygy* (strictly speaking, "syzygy" means precisely the same as conjunction, namely, the state of being yoked together).

Some 21 or 22 days after conjunction, the moon is at its *last quarter*, and a little before the next conjunction will be a morning when the waning sickle—the term "crescent" is still often used, though a misnomer—is seen to rise just before sunrise for the last time. This is the *morning of last visibility* of the moon, and it is followed by an interval during which the moon is too close to the sun, and its lighted part too slender, for it to be visible. When conjunction is reached in the middle of this interval of invisibility, the synodic cycle of the moon is closed.

To recapitulate, the moon's synodic phenomena are, in order,

conjunction
first visibility
first quarter
full moon or opposition
last quarter
last visibility
conjunction

As said, the term *synodic month* is applied to the time interval from one synodic phenomenon to the next synodic phenomenon of the same kind. But there is a certain difference to which I ought to draw attention. If the phenomenon is a syzygy—conjunction or opposition—a specific synodic month will be some fraction of a day in excess of 29 days; just how much will depend on various factors, as we shall see. But when the phenomenon is one that, like first visibility, can happen only at a certain time of day, here sunset, any synodic month must be a whole number of days—for first visibility either 29 or 30 days. In either case the *average* length of the synodic month will, of course, be the same, namely, 29.5309 days.

Lunar Orbit and Nodes

In the preceding discussion we ignored that the moon's orbit is inclined to the ecliptic, for the inclination is so small that it plays no significant role in a qualitative description of the moon's synodic phenomena. However, where the moon is in its inclined orbit can be decisive when one wishes to determine, for example, precisely on which evening the moon will first become visible, and for the prediction of eclipses it is absolutely essential to know how far away from the ecliptic the moon is.

The lunar orbit is, then, a great circle on the celestial sphere which is inclined to the ecliptic at an angle of ca. 5°. The two diametrically opposite points of intersection between the ecliptic and the moon's orbit are called the *lunar nodes*; the one where the moon crosses the ecliptic from the south to the north is called the *ascending node*, the other the *descending node*.

The first difficulty is that the nodes do not remain fixed among the fixed stars, but have an appreciable retrograde motion of not quite 2° per synodic month (this motion does not affect the orbit's inclination). Consequently, if the moon, a fixed star, and the ascending node happen to coincide on the celestial sphere at a certain moment, the moon will return to the ascending node almost 3 hours before it reaches its nearest approach to the fixed star, for the node has moved so as to meet it earlier. The period of the moon's return to a given node is called the *draconitic* (or dracontic, or nodical) *month* and it is a trifle more than $27\frac{1}{5}$ days in length.

Some 7 days after the moon has been at the ascending node, it will be as far north of the ecliptic as possible, namely 5°; after a further 7 days or, more precisely, after in all half a draconitic month, the moon will be at the descending node; after still another 7 days the moon will be as far south of the ecliptic as it can get; and after the lapse of a full draconitic month it will, once again, be at the ascending node.

Eclipses

A lunar eclipse happens when the earth intervenes between the sun and the moon and deprives the moon of the sun's light or, in other words, when the moon enters the earth's shadow. For this to take place two conditions must surely be satisfied: First, sun and moon must be seen in opposite directions by an observer on the earth, that is, the moon must be in opposition, or be full; second, since the line from the observer to the sun is always in the ecliptic, and since sun, earth (observer), and moon must be in a straight line, the moon must be at, or near, a node.

A lunar eclipse is, of course, a *real* eclipse in the sense that anyone who can but see the moon at the time—even an observer on the moon itself—will notice that the moon is deprived of the sun's light. For observers on the earth, this means simply that if only the eclipse occurs during their night, they should be able to see it.

A solar eclipse happens when the body of the moon intrudes between the observer and the sun. The character of a solar eclipse is thus quite different from that of a lunar eclipse. A solar eclipse is a more subjective phenomenon; for an observer somewhere in space it is signalled not, of course, by any change in the aspect of the sun, but by the appearance on the lighted surface of the earth of a black spot, the shadow cast by the moon. The conditions that ensure that a solar eclipse will occur at a given locality are, then, first the necessary ones that sun and moon are in conjunction, that the moon is sufficiently close to a node for its shadow to hit the earth, and that this happens while it is day at the place of observation. In addition, one must ascertain that the locality lies in the path of the moon's shadow. To do this, one must know, at least, the shape of the earth and the relative sizes and distances of sun, moon, and earth. If I may introduce a historical remark, Ptolemy's *Almagest* (ca. A.D. 150) is the earliest surviving work in which this problem is reasonably dealt with and on the basis of which one may predict a solar eclipse for a particular place with confidence. While the Babylonian texts yield solid predictions of lunar eclipses, they serve only to present necessary, but not sufficient, conditions for solar eclipses or, if you wish, to issue warnings, but not predictions, of solar eclipses.

The simple, necessary condition for solar as well as lunar eclipses is then that the moon at syzygy be near a node. A rough estimate of how often this happens is readily reached. Let us assume that a syzygy of a certain kind, say, a conjunction of sun and moon, takes place at a node, say, the ascending one. The moon will reach that node again after the lapse of a draconitic month, which, on the average, is 27.21 days. However, the moon will not catch up with the sun again until after a full synodic month, which, in the mean, is 29.53 days. At the next conjunction the moon will then be 2.32 days' travel past the ascending node. At the second conjunction, the moon will be twice that distance from the ascending node, and so on. At the sixth conjunction after the original one, the moon will be $6 \times 2.32 = 13.92$ days' travel from the ascending node; however, since half a draconitic month, or 13.61 days, brings the moon from the ascending to the descending node, the sixth conjunction will take place just past the descending node. Thus, six synodic months bring a conjunction—or an opposition—of sun and moon from one node to a little past the other, and eclipse possibilities will then happen at intervals of mostly six, but occasionally of only five, synodic months. An interval of only five months must intervene at times since the six-month interval is clearly a little too long and since, further, experience shows that two eclipses for a given locality never take place only one month apart. (Modern canons do show *solar* eclipses at an interval of only one month, but in general one of these will be visible only near the earth's North Pole and the other only near its South Pole.)

It is then proper to issue eclipse warnings at intervals of mostly six, but occasionally of only five, months; what one can guarantee is not, of course, that eclipses will happen at these syzygies, but rather that eclipses will not occur at any other. This holds for both solar and lunar eclipses.

Coordinate Systems and the Zodiac

So far I have tried to keep the discussion of spherical astronomy as qualitative as possible, but as the need arises to become more precise and quantitative, we must introduce coordinates.

The basic device for fixing a position on a spherical surface by a pair of numbers, or coordinates, is well known from geography, where a location on the earth's surface is identified by latitude and longitude. It is a great circle (see Figure 2) with a fixed point F on it and its two poles, P_1 and P_2. The coordinates of a point S on the sphere's surface are now found in the following fashion. Through the poles P_1 and P_2 and the point S is drawn a great circle that intersects the basic great circle at the point S' (of the two possibilities, S' is the one for which the arc $S'S$ is less than 90°). The two coordinates (x, y) of S are then simply

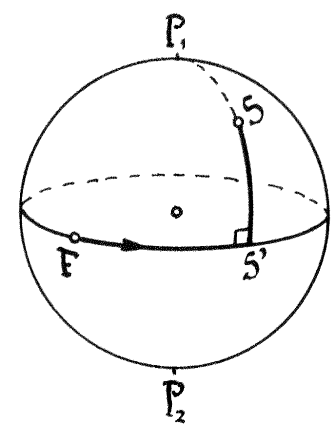

FIGURE 2.

$$x = FS',$$
$$y = S'S,$$

where both arcs are measured in degrees. To avoid ambiguity in the measure of the arc FS', the basic great circle is usually provided with a positive direction and FS' can then assume values up to 360°. One may, as an alternative, choose to keep the values of FS' below 180° and then indicate which way the arc is measured [this happens for terrestrial longitudes given in degrees (<180°) east or west of Greenwich]. For the other coordinate, the arc $S'S$, already less than 90°, one must indicate whether S lies toward P_1 or P_2 from S'. This may be done by a sign; say, positive toward P_1 and negative toward P_2, or by words such as north and south (as for terrestrial latitudes).

I shall mention here three coordinate systems on the celestial sphere. In the first the basic great circle is the ecliptic, the fixed point F is the vernal equinox (i.e., one of the points of intersection between the equator and the ecliptic), and the positive direction on the ecliptic is that of direct motion (i.e., toward the east). The coordinate measured on the ecliptic is called *celestial longitude*, or simply *longitude*, when no confusion with terrestrial longitude is likely. It is usually designated with the letter λ. The other coordinate, measured perpendicularly to the ecliptic, is called *celestial latitude* or simply *latitude*, with a similar caution as for longitude. It is positive for points north of the ecliptic, and negative for the other hemisphere. It is usually denoted by the letter β.

This ecliptic coordinate system is without question the most important in ancient astronomy; it is still used for problems involving sun, moon, and planets, for they are all on or near the ecliptic or, in other words, they have celestial latitudes that are always small ($\beta = +10°$ and $-10°$ are the extremal values for Venus, the planet showing the largest latitudes).

The longitude of the sun, whose latitude is always 0, increases by 360° in the course of a year. Direct motion of a planet means that its longitude increases; retrograde motion corresponds to decreasing longitude. The synodic arc of a planet—$\Delta\lambda$, as we have already called it—is the increment in the planet's longitude from one synodic phenomenon to the next of the same kind.

While we count longitudes from 0° to 360°, the classical manner of giving longitudes is in terms of a zodiacal sign and degrees within that sign. The ecliptic in this system is divided

into 12 parts of exactly 30° each, beginning with the vernal equinox, and they are named thus:

♈ Aries:	$0° \leqq \lambda < 30°$	♎ Libra:	$180° \leqq \lambda < 210°$
♉ Taurus:	$30° \leqq \lambda < 60°$	♏ Scorpio:	$210° \leqq \lambda < 240°$
♊ Gemini:	$60° \leqq \lambda < 90°$	♐ Sagittarius:	$240° \leqq \lambda < 270°$
♋ Cancer:	$90° \leqq \lambda < 120°$	♑ Capricorn:	$270° \leqq \lambda < 300°$
♌ Leo:	$120° \leqq \lambda < 150°$	♒ Aquarius:	$300° \leqq \lambda < 330°$
♍ Virgo:	$150° \leqq \lambda < 180°$	♓ Pisces:	$330° \leqq \lambda < 360°$

and I have affixed to them their now-standard sigilla. Thus, to give an example, Leo 27° is simply another way of denoting the longitude 147°.

The Babylonian zodiac does not agree exactly with this description; at the period, from which most of our texts come, the signs begin some 5° earlier than in the preceding list. We shall return to the reason for this later.

The longitude of the sun at vernal equinox is then Aries 0° or $\lambda = 0°$; at summer solstice Cancer 0°, or $\lambda = 90°$; at fall equinox Libra 0°, or $\lambda = 180°$; and at winter solstice Capricorn 0°, or $\lambda = 270°$.

I shall briefly mention two more coordinate systems. The first has as its basic great circle the celestial equator whose poles are the north and south poles, and the fixed point F is again the vernal equinox. The first coordinate is called *right ascension* and is measured from the vernal equinox toward the east so that the sun's right ascension always increases. Right ascension is usually denoted by the letter α. The other coordinate, measured perpendicularly to the equator, is called *declination* and is counted positive north and negative south of the equator. Declination is usually denoted by the letter δ.

The sun's right ascension at the four cardinal positions just mentioned is, respectively, 0°, 90°, 180°, and 270°; that is, its right ascension here is the same as its longitude, which, of course, is not so elsewhere. The declination of the sun at equinox is $\delta = 0°$, at summer solstice $\delta = 23\frac{1}{2}°$, and at winter solstice $\delta = -23\frac{1}{2}°$, where the value $23\frac{1}{2}°$ is the inclination of ecliptic to equator.

In the equator and the ecliptic systems, the coordinates of fixed stars will not change appreciably in the course of even several years; that they change at all is largely due to the precession of the equinoxes, which we will describe later. The longitudes, latitudes, right ascensions, and declinations of the other celestial bodies change at a moderate rate, most swiftly for the moon, whose longitude grows by about 13° per day, while solar longitude increases about 1° per day. The daily rotation has no effect in these coordinate systems at all.

That is not so in the last system I shall introduce. Here the horizon is the basic great circle whose poles are zenith and nadir, and the fixed point F is the south point. The first coordinate is called *azimuth* and is usually counted from the south point westward from 0° to 360°. It is denoted by Az. The other coordinate, measured perpendicularly to the horizon, is called the *altitude* and is denoted by h. It is counted positive for objects above the horizon, and negative for those below.

The horizon system is a local system; it belongs to the observer and his or her terrestrial surroundings. The altitude—what I earlier called elevation—of the north pole is ϕ, the terrestrial latitude of the observer, and the azimuth of an object not too close to a pole changes about 360° in one day due to the daily rotation.

Precession, Anomalistic Periods, and Various Other Refinements

In this section I deal with various refinements—long-term changes, and variation in velocities—which I have hitherto ignored for the sake of simplicity of presentation.

The first of these is the *precession of the equinoxes*, or simply the *precession*. Its effect is most easily described in the ecliptic system: The precession causes the longitude of all fixed stars to increase at a uniform, very slow rate of 1° in approximately $72\frac{2}{3}$ years, while their latitudes remain constant.

Of course, this phenomenon can be, and often is, described in a different manner. Since longitudes are measured from the vernal equinox, one can just as well say that the vernal equinox moves in the retrograde direction—it precedes—among the fixed stars along the ecliptic—a great circle that remains fixed among the fixed stars since their latitudes do not change. Thus, the equator, which serves to define the vernal equinox, moves slowly among the fixed stars but in such a fashion that its inclination to the ecliptic remains the same; the north pole travels slowly on a small circle of radius $23\frac{1}{2}^\circ$ and the northern pole of the ecliptic as its center.

Right ascension and declination of a fixed star will therefore both change, so it is more complicated to describe the effects of the precession in the equatorial than in the ecliptic system.

One consequence of the precession of the equinoxes is that we should distinguish between two kinds of year. One is the period of the sun's return to a fixed star, and it is called the *sidereal year*; the other is the period of the sun's return in longitude (i.e., to the vernal equinox) and, since this governs the seasons, it is called the *tropical year*. The latter is slightly shorter than the former for the sun has to travel slightly farther to catch up with a fixed star than with the vernal equinox, which moves slightly to meet it during each of its revolutions. Values of the two are

$$1 \text{ sidereal year} = 365.256 \text{ days},$$

$$1 \text{ tropical year} = 365.242 \text{ days}.$$

Similarly, we distinguish between a *sidereal* and a *tropical* month, the periods of the moon's return to a fixed star and to the same longitude, respectively. Their values are

$$1 \text{ sidereal month} = 27.32166 \text{ days},$$

$$1 \text{ tropical month} = 27.32158 \text{ days}.$$

I must most emphatically warn that my inclusion of precession in this presentation does not imply that it plays a role in Babylonian astronomical schemes. There is no evidence whatever that the phenomenon of precession was known to Babylonian astronomers; all indications are that their longitudes are sidereal longitudes—that their zodiac was fixed among the fixed stars, that their year was the sidereal year, and that they were unaware of any difference between this and the seasonal or tropical year. It was Hipparchus (ca. 150 B.C.) who first drew the distinction between the tropical and the sidereal year or who, in other words, discovered the precession. I introduced the precession in this presentation to make clear the natural discrepancy between Babylonian and modern longitudes, a discrepancy that increases with time at the rate of 1° in 72 to 73 years.

Anomalistic Periods

In the previous discussion of the motions of sun and moon among the fixed stars, I have been content to give crude estimates of average daily progress. In a more precise account one must recognize that the velocity—which in spherical astronomy means, as it must, apparent angular velocity—of either body is not constant, but changes periodically by sensible amounts.

The sun's daily progress varies between some 1.02° (or 1°1′) and 0.95° (or 57′) in a regular manner; the period of this variation, that is, the time interval from high velocity to high velocity, or from low velocity to low velocity, is called one anomalistic year. Its length is

$$1 \text{ anomalistic year} = 365.2596^d$$

(where ^{d} = days) which is very close to the value of the sidereal year. Within the historically relevant period we may ignore this difference and shall, with the Babylonian astronomers, consider solar velocity as a function of Babylonian, namely sidereal, longitude.

For the moon the situation is quite different. Its daily progress varies between some 15° and $11\frac{1}{2}^\circ$ (it makes no sense to present sharper limits here), and the period from high velocity to high velocity, called the *anomalistic month*, is

$$1 \text{ anomalistic month} = 27.5545^d.$$

This is substantially more than either the sidereal or the tropical month—by some 6 hours—and the anomalistic month was recognized as a separate and important parameter in Babylonian lunar theory, as we shall see. Abandoning for the moment the conventions of spherical astronomy, I may state that the moon assumes its least velocity when at the point of its orbit farthest from the earth, or at its apogee, and its greatest velocity when nearest, or at its perigee; that the anomalistic month is longer than the tropical month means, then, that the apogee and the perigee of the moon both advance steadily in the ecliptic.

Synodic Arcs of the Planets

A quantitative investigation of the synodic arc $\Delta\lambda$ of a given planet (i.e., the increment in the planet's longitude when it moves from one synodic phenomenon to the next of the same kind) will show that it is not constant but varies according to where in the ecliptic the planet happens to be. To say this in other words, $\Delta\lambda$ is a function of λ, the longitude at which the arc $\Delta\lambda$ begins. That this is correct when longitudes are counted sidereally as in Babylonian astronomy can be established on the basis of modern planetary theory, but I shall not do so here, though the necessary arguments are fairly simple.

Long-Term Changes

There are several long-term, or exceedingly slow, changes in the various parameters I have mentioned, in the lengths of the several kinds of month, in the length of the year, in the amount of the precession, in the relative positions of fixed stars, in the places where synodic arcs achieve their maxima, and so on. These minute variations, some of which are called *secular* since they are only felt after the lapse of centuries, are of prime interest to astronomers. I shall nonetheless pass them by, for in the time interval of Babylonian astronomy, and within its margin of precision, they play no significant role. The causes of some of these very slow changes are still not quite under control; indeed, some of their magnitudes are not yet unequivocally established. Ancient observations, once properly understood and properly treated, must play a central role in these matters, but our understanding of ancient theoretical astronomy clearly does not depend on such issues at all.