PUZZLE

You meet A and B in the land of knights and knaves.

A says “I am a knight if and only if B is also a knight.”

B says “A and I are of different kinds.”

Who is what?

Truth Tables

Friday, 10 September

Complex Sentences

- The truth-value of a complex sentence is a function of the truth-value of its parts.
- Assume that A and B are both true. What is the value of $(A \vee B) \rightarrow (A \wedge \neg B)$?
- Here, the antecedent, $(A \vee B)$ is true since A and B are both true. The consequent is false since it is a conjunction where the first conjunct (A) is true but the second conjunct ($\neg B$) is false since B is true.
- Conditionals with true antecedents and false consequents are false, so the whole sentence is false.

Truth Tables

- Example: partial truth table for (A ∨ B)→(A ∧ ¬B)

A	B	(A ∨ B) → (A ∧ ¬ B)
T	T	T **F** F F

Truth Tables

- Truth tables show how the truth value of a complex sentence depends on the truth values of its components.
- They also help us keep track of relationships that exist between the truth values of different sentences.
- So, for example, we can use truth tables to show logical equivalence.
- Two sentences are logically equivalent if they have the same truth values in all possible circumstances.

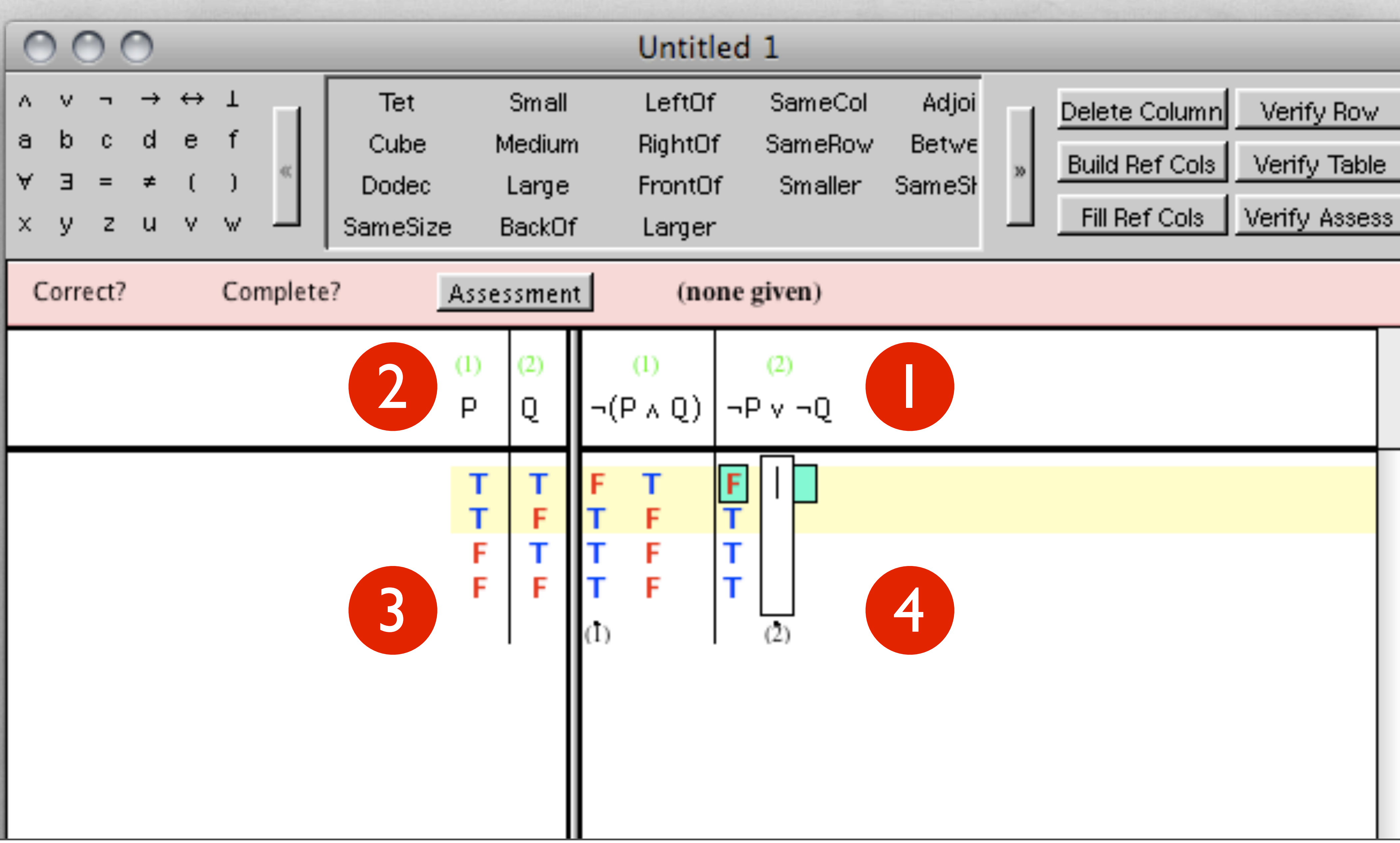
Truth Tables

- Example: joint truth table for ¬(P∧Q) and (¬P∨¬Q) This shows that the two sentences are equivalent.

P	Q	¬ (P ∧ Q)	(¬ P ∨ ¬ Q)
T	T	F T T T	F T F F T
T	F	T T F F	F T T T F
F	T	T F F T	T F T F T
F	F	T F F F	T F T T F

Truth Tables

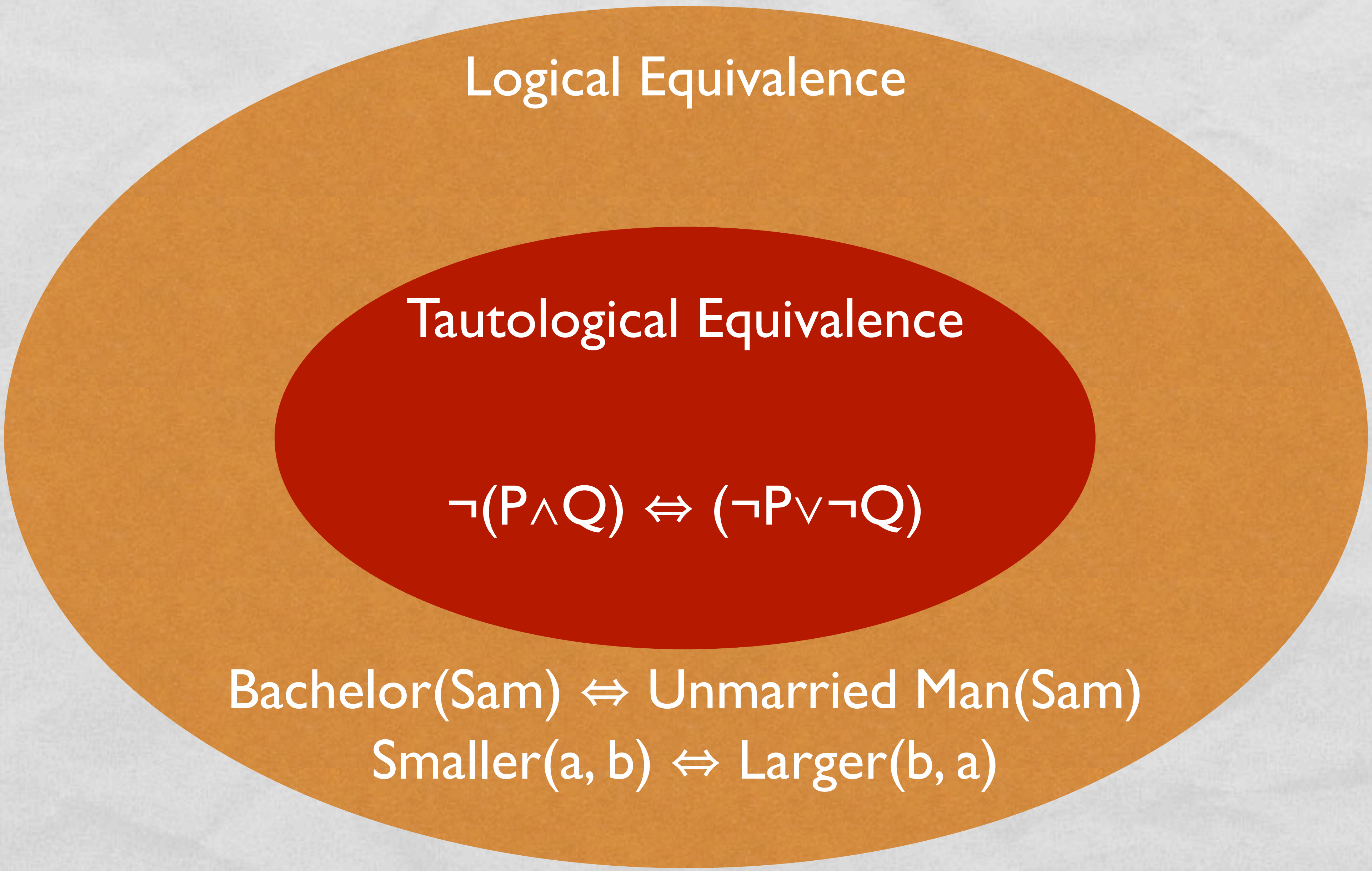
We will construct truth tables in Boole.



Tautological versus Logical

- Two sentences are logically equivalent if they have the same truth conditions, i.e., are true in the same circumstances.
- Two sentences are tautologically equivalent if they have matching truth tables, i.e., the same truth values for all combinations of atomic sentences' truth values.
- Tautological equivalence results simply from the meanings of the truth-functional connectives.
(Ex: DeMorgan's Laws, double negation, contraposition)

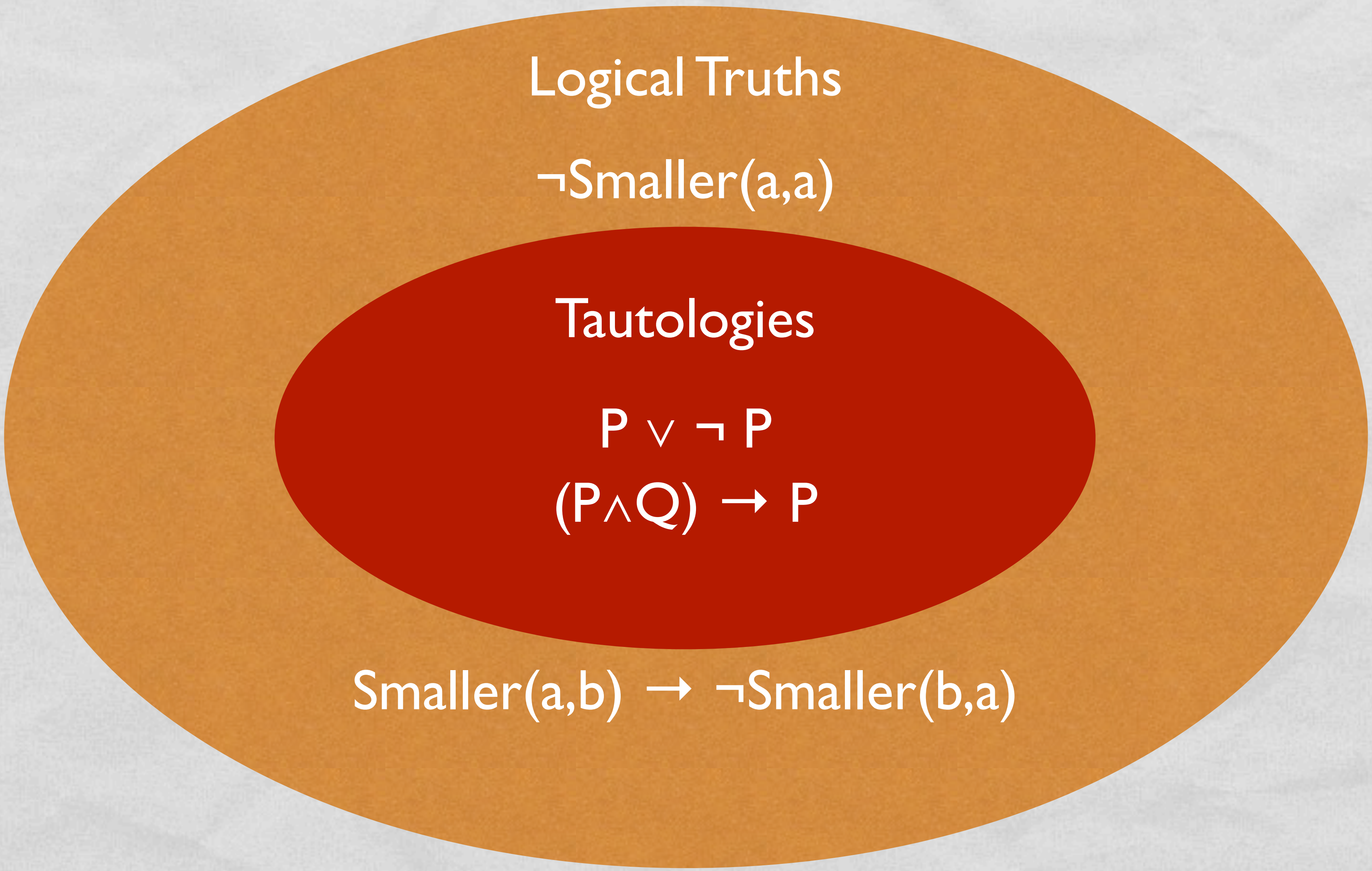
Tautological versus Logical



Tautological versus Logical

- A sentence is a logical truth iff it is logically necessary, i.e., is a logical consequence of any set of sentences. (It is impossible for a logical truth to be false.)
- A sentence is a tautology iff every row of its truth table assigns TRUE to that sentence.
- Tautologies are true in virtue of the meanings of the truth-functional connectives alone. (Ex: $P \lor \neg P$)

Tautological versus Logical



Logical and Tautological Consequence

- A sentence S is a <u>logical consequence</u> of a set of sentences $P_1 \ldots P_n$ iff whenever $P_1 \ldots P_n$ are true, S is also true.
- If an argument is valid, then the conclusion is a logical consequence of the premises.
- A sentence Q is a <u>tautological consequence</u> of a set of sentences $P_1 \ldots P_n$ iff every row of the truth table where $P_1 \ldots P_n$ are all true, Q is also true.
- Note: The Ps and Qs might be complex sentences.

Logical and Tautological Consequence

- Example:

  1) A

  2) $A \rightarrow B$

  3) $\neg B \vee C$

  4) Conclusion: C

Logical and Tautological Consequence

A	B	C	A	A→B	¬B ∨ C	C
T	T	T	T	T	F **T**	T
T	T	F	T	T	F **F**	F
T	F	T	T	F	T **T**	T
T	F	F	T	F	T **T**	F
F	T	T	F	T	F **T**	T
F	T	F	F	T	F **F**	F
F	F	T	F	T	T **T**	T
F	F	F	F	T	T **T**	F

No row is T, T, T, F

Logical and Tautological Consequence

A	B	C	A	A→B	B ∨ C	C
T	T	T	T	T	**T**	T
T	T	F	T	T	**T**	F
T	F	T	T	F	**T**	T
T	F	F	T	F	**F**	F
F	T	T	F	T	**T**	T
F	T	F	F	T	**T**	F
F	F	T	F	T	**T**	T
F	F	F	F	T	**T**	F

Second row is T, T, T, F
So NOT valid

Conditionals and Logical Consequence

- A sentence Q is a logical consequence of a set of sentences $P_1, P_2 \ldots P_n$ iff it is impossible for the premises to be true and the consequent to be false.
- This is exactly the same as the falsity of $(P_1 \wedge P_2 \wedge \ldots \wedge P_n) \rightarrow Q$
- $(P_1 \wedge P_2 \wedge \ldots \wedge P_n) \rightarrow Q$ is a logical truth iff Q is a logical consequence of $P_1, P_2 \ldots P_n$.

Conditionals and Logical Consequence

- $P \leftrightarrow Q$ is a logical truth iff P and Q are logically equivalent (have the same truth values).
- In other words, $P \leftrightarrow Q$ is a logical truth iff $P \Leftrightarrow Q$.
  - NOTE: $P \leftrightarrow Q$ might just happen to be true without P and Q being equivalent
- Recall: $A \leftrightarrow B \Leftrightarrow (A \rightarrow B) \wedge (B \rightarrow A)$.
- Similarly, A is logically equivalent to B iff A is a logical consequence of B and B is a logical consequence of A.