Puzzle

There are three defendants – A, B, and C – and the following facts are known:

1. If A is innocent, then both B and C are guilty.
2. If A is guilty, then B is also guilty.
3. If C is guilty, then B is innocent.

Note that you do not know how many of these defendants are guilty. It may be 0, 1, 2, or all 3.

Who is innocent and who is guilty?

CONDITIONALS

Wednesday, 8 September

The Conditional

- A connective is truth-functional if the truth or falsity of compound sentences is completely determined by the truth values of the constituents.
- Today we will introduce two new truth-functional connectives to our formal language.
- The first is called the material conditional, designated with the symbol $\rightarrow$.
- If A and B are sentences, then A $\rightarrow$ B is a sentence.

The Conditional

- We can translate $A \rightarrow B$ into English as 'if A, then B'.
- A is called the antecedent, B is called the consequent.
- If Alice is tall then Bill is: $A \rightarrow B$
- Bill is tall if Alice is: $A \rightarrow B$
- If Bill and Alice are both tall, then neither Charlie nor David are: $(B \wedge A) \rightarrow \neg(C \vee D)$

Other Conditionals

- Alice will go only if Tom does:
  - $A \rightarrow T$
  - $\neg T \rightarrow \neg A$
- Alice will go unless Tom does
  - $\neg T \rightarrow A$
  - $\neg A \rightarrow T$
  - Think "Alice will go (<u>IF NOT</u>) Tom

The Conditional

- The sentence A → B is true iff whenever A is true, B is also true.
- Truth table for the material conditional:

A	B	A → B
TRUE	TRUE	TRUE
TRUE	FALSE	FALSE
FALSE	TRUE	TRUE
FALSE	FALSE	TRUE

The Conditional

- In English, 'if… then' statements can imply causation.
  In FOL, they do not.
- In English, the truth of 'if… then' statements depend on both the antecedent and the consequent.
  In FOL, if the antecedent is false, then the conditional is always true.

  In FOL, if the consequent is true, then the conditional is always true.
- A→B just means either A is false or B is true

Necessary and Sufficient Conditions

- In English, the expression "only if" implies that we are introducing a necessary condition.
- "You can be a lawyer only if you have a J.D." This means that having a J.D. is necessary in order to be a lawyer.
- You might still not be a lawyer, for example if you haven't passed the bar exam. Having a J.D. is no *guarantee* that you are a lawyer.
- Only one situation is ruled out: it is false that you could *not* have a J.D. and yet be a lawyer anyway.
- 'A only if B' is roughly 'B is necessary for A': $A \rightarrow B$

Necessary and Sufficient Conditions

- A sufficient condition, on the other hand, does guarantee that something else will happen.
- "You are rich if you are a millionaire."
- Yet it might still be the case that you are rich even if you aren't a millionaire.
- Only one situation is ruled out: it can't be the case that you are a millionaire and yet you aren't rich.
- 'A is sufficient for B' is roughly 'If A then B': $A \rightarrow B$

Necessary and Sufficient Conditions

• If P, then Q:	$P \rightarrow Q$
• A only if B:	$A \rightarrow B$
• P if Q:	$Q \rightarrow P$
• Unless B, A:	$\neg B \rightarrow A$
• P if not Q:	$\neg Q \rightarrow P$
• A is necessary for B:	$B \rightarrow A$
• P is sufficient for Q:	$P \rightarrow Q$

The Biconditional

- Another new connective: the biconditional (↔).
- If A and B are sentences, then A ↔ B is a sentence.
- A sentence of the form P ↔ Q is true iff P and Q have the same truth value.
- Generally, the English expression used to express the biconditional is "if and only if".

The Biconditional

- Truth table for the biconditional:

A	B	$A \leftrightarrow B$
TRUE	TRUE	TRUE
TRUE	FALSE	FALSE
FALSE	TRUE	FALSE
FALSE	FALSE	TRUE

- $A \leftrightarrow B$ is logically equivalent to $(A \rightarrow B) \wedge (B \rightarrow A)$

Equivalence

- We saw some equivalences already:
  - Neither A nor B
    - $\neg(A \vee B)$ is equivalent to $\neg A \wedge \neg B$
  - Not both A and B
    - $\neg(A \wedge B)$ is equivalent to $\neg A \vee \neg B$
    - These two equivalences are called "DeMorgan's Laws"

EQUIVALENCE

- We denote FOL equivalences using the symbol ⇔, e.g., ¬¬P ⇔ P
- When two sentences are logically equivalent, they have the same truth conditions, i.e., are true in the same circumstances.
- DeMorgan'sLaws:
¬(P∧Q) ⇔ (¬P∨¬Q) ¬(P∨Q) ⇔ (¬P∧¬Q)

EQUIVALENCE

- There are LOTS of equivalences. Some more obvious than others.
- Either Alice or Bill went to the party
  - $P(a) \vee P(b)$
  - $P(b) \vee P(a)$
  - $\neg\neg P(a) \vee \neg\neg\neg\neg P(b)$
  - $\neg[\neg P(a) \wedge \neg P(b)]$
  - $[\neg P(a) \rightarrow P(b)] \wedge [\neg P(b) \rightarrow P(a)]$
  - $[\neg P(a) \rightarrow P(b)]$

Truth Tables

- Truth tables show how the truth value of a complex sentence depends on the truth values of its components.
- They also help us keep track of relationships that exist between the truth values of different sentences.
- So, for example, we can use truth tables to show logical equivalence.
- Two sentences are logically equivalent if they have the same truth values in all possible circumstances.

Truth Tables

- Example: joint truth table for $\neg(P \wedge Q)$ and$(\neg P \vee \neg Q)$

Truth Tables

- Example: joint truth table for $\neg(P \wedge Q)$ and $(\neg P \vee \neg Q)$ First, give truth conditions of the atomic sentences:

P	Q	$\neg (P \wedge Q)$	$(\neg P \vee \neg Q)$
T	T	T T	T T
T	F	T F	T F
F	T	F T	F T
F	F	F F	F F

Truth Tables

- Example: joint truth table for ¬(P∧Q) and (¬P∨¬Q) Then assign truth conditions of the combinations:

P	Q	¬ (P ∧ Q)	(¬ P ∨ ¬ Q)
T	T	**F** T **T** T	**F** T **F** **F** T
T	F	**T** T **F** F	**F** T **T** **T** F
F	T	**T** F **F** T	**T** F **T** **F** T
F	F	**T** F **F** F	**T** F **T** **T** F

TRUTH TABLES

- Example: joint truth table for ¬(P∧Q) and (¬P∨¬Q) This shows that the two sentences are equivalent.

P	Q	¬ (P ∧ Q)	(¬ P ∨ ¬ Q)
T	T	**F** T **T** T	**F** T **F** **F** T
T	F	**T** T **F** F	**F** T **T** **T** F
F	T	**T** F **F** T	**T** F **T** **F** T
F	F	**T** F **F** F	**T** F **T** **T** F

Truth Tables

We will construct truth tables in Boole.