PUZZLE

Suppose that in the country of Knights and Knaves you meet three individuals, A, B, and C. You discover that at least one of them is a Knight and at least one of them is a Knave.

A says "B or C is a Knight" and
B says "A or C is a Knight."

Which of them are Knights and which are Knaves?

The Boolean Connectives

Friday, 3 September

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- Note: SameSize(¬a, b) is NOT a sentence.

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  ¬Cube(a) ∧ a=b

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  ¬(Cube(a) ∧ a=b)

  ¬(Cube(a) ∧ a=b) ∨ (Larger(b, c) ∨ ¬Medium(b))

Complex Sentences

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- Boolean connectives are one way of turning atomic sentences into complex sentences.
- If Φ is a sentence and Ψ is a sentence then $(\Phi \vee \Psi)$ is a sentence, etc.

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    - $(A \vee B)$ and C are the conjuncts
- Parentheses are used to determine the order of the connectives and disambiguate sentences.

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$\neg(A \vee (B \wedge \neg C)) \wedge (\neg(D \wedge E) \vee F)$

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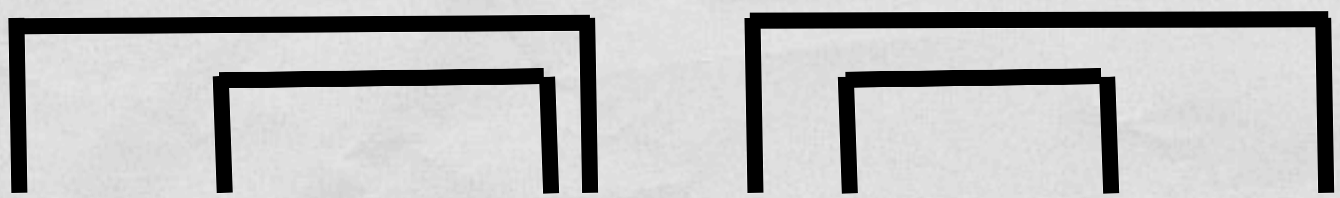

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$\neg(A \lor (B \land \neg C)) \land (\neg(D \land E) \lor F)$

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$(A \lor (B \land \neg C))$

$\neg(D \land E)$

F

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A $\quad$ $(B \land \neg C)$ $\quad$ $(D \land E)$

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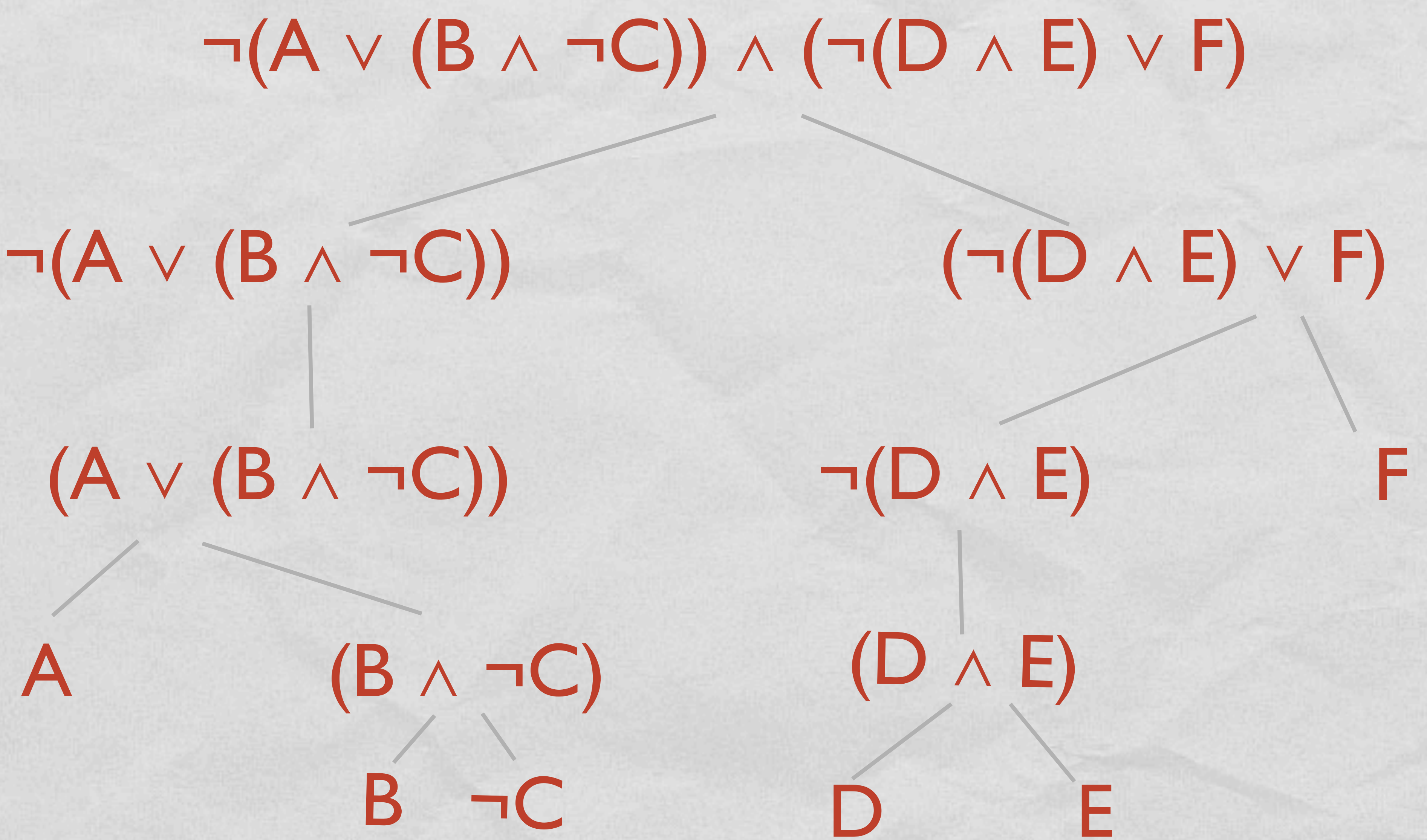

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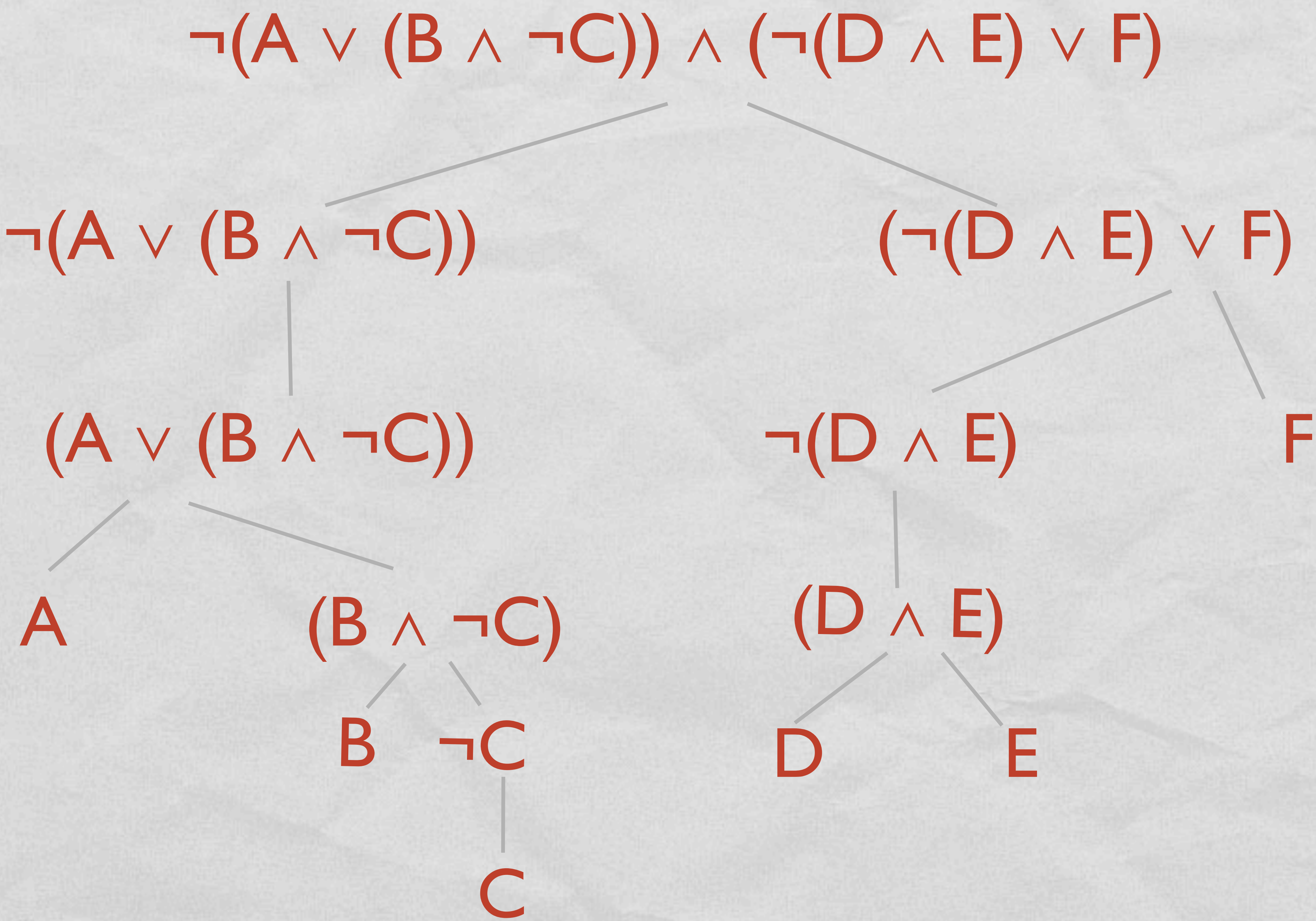

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A	¬A
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FALSE	TRUE

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- The sentence A ∧ B is true iff A and B are both true.

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- We translate $A \wedge B$ into English as 'it is the case that A and it is the case that B'.
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TRUE	FALSE	FALSE
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- 'And' in English can link two names or properties. For example, Sam and Sarah had breakfast; Sam had breakfast and went to the park.
  In FOL, conjunction only links two sentences.
- In English, 'and' is often used to imply causation or a temporal sequence.
  'And' does not have this implication in FOL.

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Disjunction

- In English, 'A or B' is often used to mean that either A is true, or B is true, but not both (exclusive or).

  In FOL, exclusive or (exactly one of) could be expressed as $(A \lor B) \land \neg(A \land B)$

  The simple disjunction $(A \lor B)$ is always the inclusive or (at least one of and maybe both).

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