

# PUZZLE

Mr. X always tells the truth on Monday, Tuesday, and Wednesday and never tells the truth on the other days.

Ms. Y always tells the truth on Thursday, Friday, and Saturday and never tells the truth on the other days.

On a certain day, you meet Mr. X and Ms. Y and they both say, "Yesterday I didn't tell the truth."

What day of the week is it?

# THE LOGIC OF ATOMIC SENTENCES: PROOFS OF (IN)VALIDITY

Wednesday, 1 September

# VALIDITY IN FOL

- A sentence  $S$  is a logical consequence of sentences  $P_1 \dots P_n$  iff the argument with  $P_1 \dots P_n$  as the premises and  $S$  as the conclusion is valid.
  - A formal deduction in  $\mathcal{F}$  proves validity.
- A sentence  $S$  is a nonconsequence of sentences  $P_1 \dots P_n$  iff the argument with  $P_1 \dots P_n$  as the premises and  $S$  as the conclusion is invalid.
  - A counterexample (such as a world in Tarski's World) proves invalidity.

# FITCH-STYLE DEDUCTIVE SYSTEM

1. P

2. Q

3.  $S_1$

4.  $S_2$

...

n.  $S_n$

Justification 1

Justification 2

Justification n

# FITCH-STYLE DEDUCTIVE SYSTEM

Rules of the system F:

- = Intro
- = Elim
- Reit (Reiteration): “we have already shown that P”

# THE COMPUTER PROGRAM FITCH

Premises →

Steps →

Conclusion →

The screenshot shows a software interface for a proof assistant. At the top, there is a toolbar with various navigation and editing icons. Below the toolbar is a menu with options like Tet, Small, LeftOf, SameCol, Adjoi, Cube, Medium, RightOf, SameRow, Betwe, Dodec, Large, FrontOf, Smaller, SameSi, SameSize, BackOf, and Larger. On the right side of the menu are buttons for 'Check Step', 'Verify Proof', and 'Goal Constraints'. The main area contains a list of logical steps:

- SameSize(b,b)
- a = b
- a = a
- b = a
- SameSize(a,b)

To the right of the list, there are justifications for the last three steps:

- ✓ ▾ = Intro
- ✓ ▾ = Elim
- ✓ ▾ = Elim

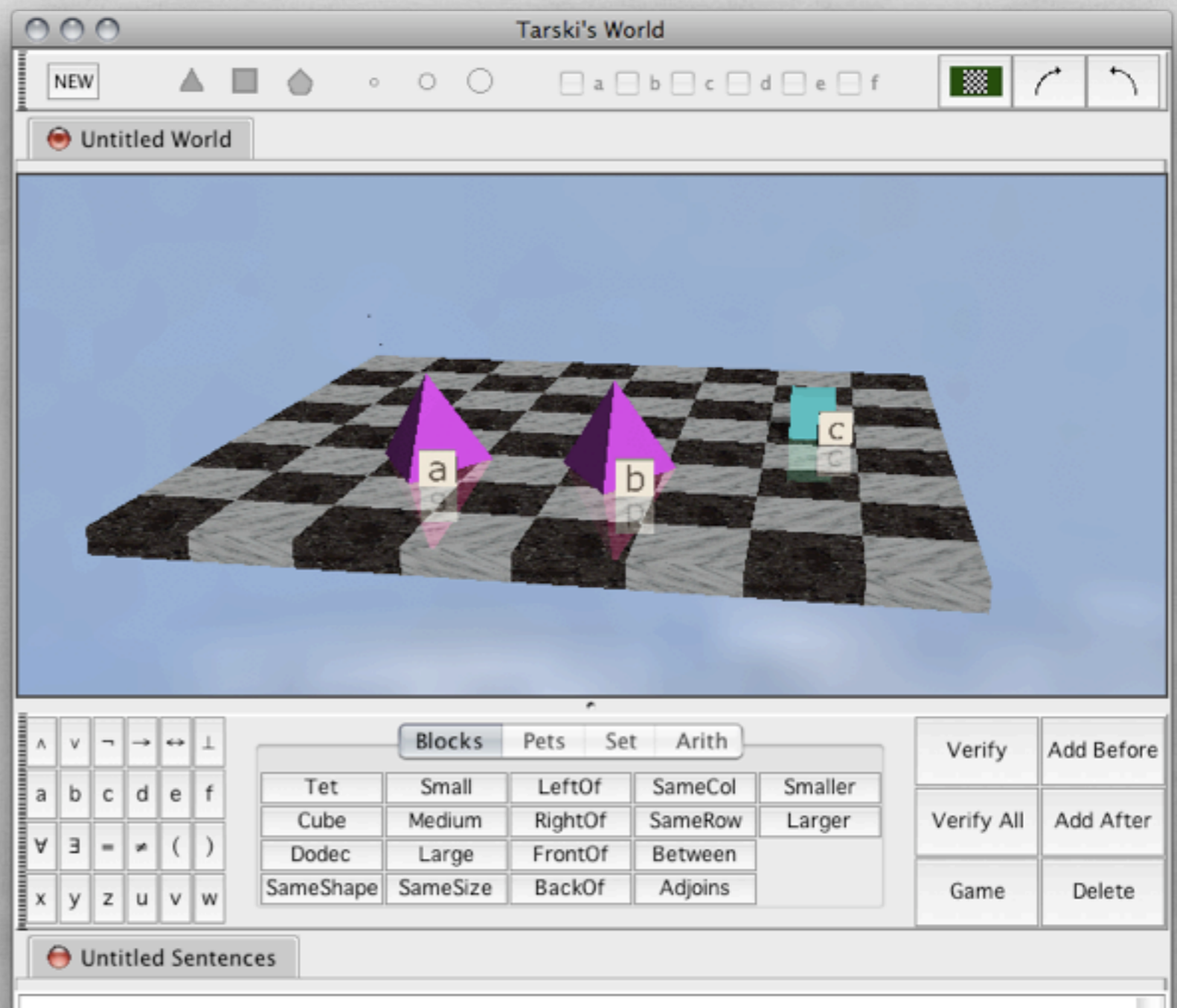
An arrow points from the word 'Justifications' to the justification for the final step. At the bottom of the interface, there is a 'Goals' section with a light blue background, containing the text 'SameSize(a,b)' and a checkmark.

Justifications

# PROVING NONCONSEQUENCE

## Example:

1. SameSize(a, b)
2. Small(c)
3. Small(a)



# EXAMPLES

Example:

1. LeftOf(a, b)
2. LeftOf(a, c)
3.  $b=d$
4. LeftOf(d,c)

**Invalid**

Example:

1. SameSize(a, b)
2. SameSize(a, c)
3. Medium(b)
4. Medium(c)

**Valid**



# EXAMPLES

Example:

1. SameCol(a, b)
2. SameCol(c, d)
3.  $b=c$
4. SameCol(a, d)

**Valid**

Example:

1. Larger(a, b)
2. Smaller(b, c)
3. Medium(c)
4. Large(a)

**Invalid**