QUESTION TO PONDER...

Is the binary relation "Is the same species as" (biological species, like human, chicken, *T. Rex*) an equivalence relation?

If yes, this relation fully partitions the space of all organisms past and present into species. Does this mean that some organisms must have offspring that are different species from them?

SETS AND MODELS

Monday, 22 November

Semantics of FOL

An argument is FOL valid if any <u>interpretation</u> that makes all the premises true also makes the conclusion true.

But what counts as an interpretation? If "L(x,y) means 'x loves y' " can be part of an interpretation, then how can we determine whether L(a,b) is true on a given interpretation? (not to mention determining whether things were true on all interpretations...)

Solution: Be more explicit, formal, and abstract

Formal Semantics

- An interpretation (or structure in LPL chap 18) has:
  - A set of objects as the domain
  - A set of objects for each one place predicate (the set of things that satisfy the predicate)
  - A set of ordered pairs for each two place predicate [n-tuples for the n-place predicates]
  - A function which maps names into objects in the domain
  - Assigns a function (domain to domain) for each function in the language

Varieties of Interpretations

Give an interpretation that shows that the following argument is invalid:

$\exists x(P(x) \wedge Q(x))$

$\exists x(Q(x) \wedge R(x))$

$\vdash \exists x(P(x) \wedge R(x))$

Domain: Natural numbers

P(x): Even numbers

Q(x): Prime numbers

R(x): Odd numbers

Varieties of Interpretations

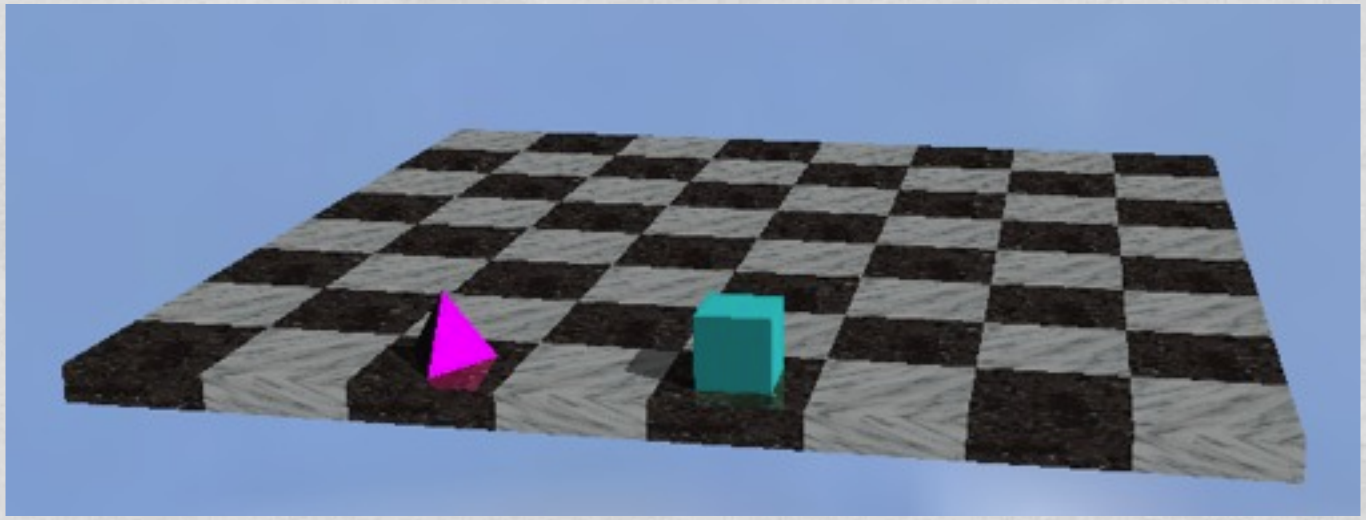
Give an interpretation that shows that the following argument is invalid:

$\exists x(P(x) \wedge Q(x))$

$\exists x(Q(x) \wedge R(x))$

$\vdash \exists x(P(x) \wedge R(x))$

Let P(x)=x is a tet
Q(x) = x is small
R(x) = x is a cube



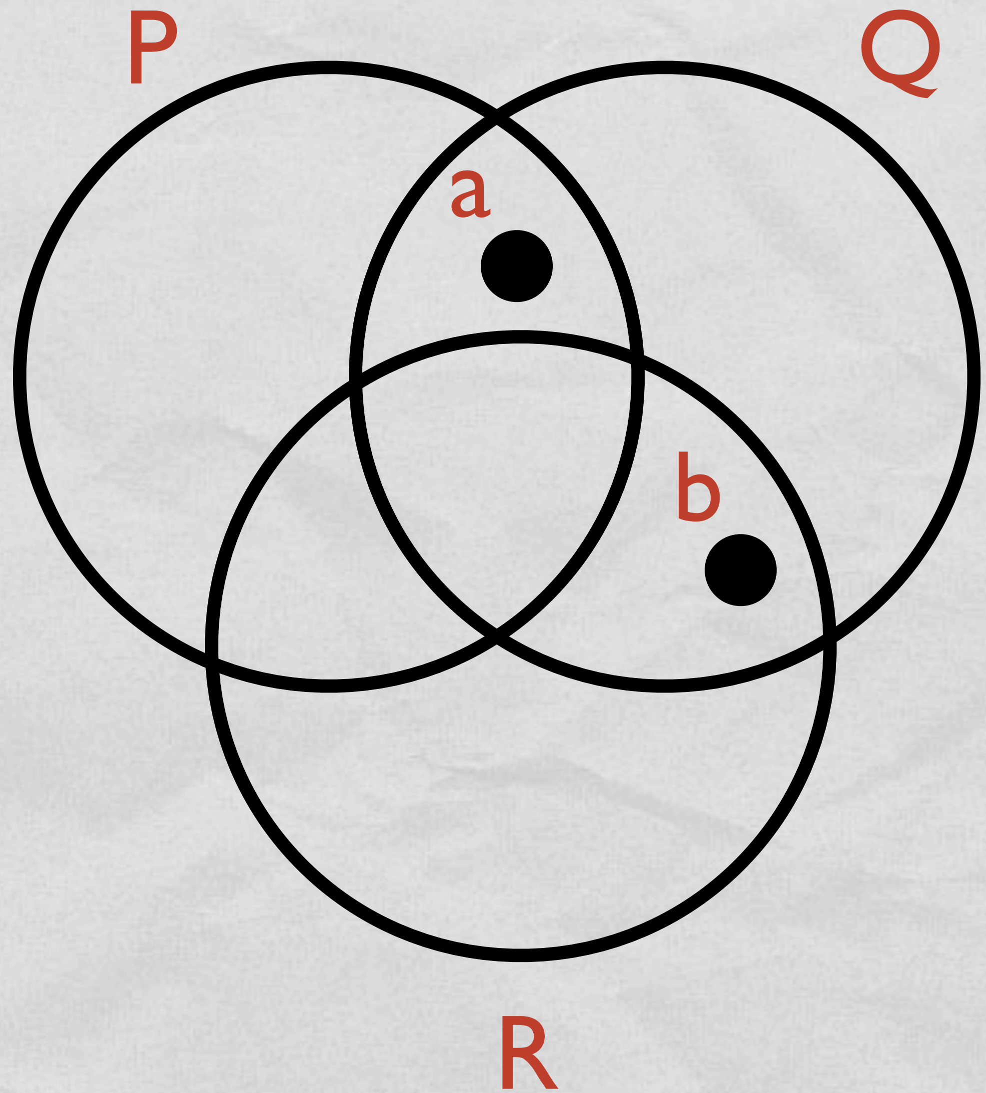
Varieties of Interpretations

Give an interpretation that shows that the following argument is invalid:

$\exists x(P(x) \wedge Q(x))$

$\exists x(Q(x) \wedge R(x))$

$\vdash \exists x(P(x) \wedge R(x))$



Varieties of Interpretations

Give an interpretation that shows that the following argument is invalid:

$\exists x(P(x) \wedge Q(x))$

$\exists x(Q(x) \wedge R(x))$

$\vdash \exists x(P(x) \wedge R(x))$

Domain: {a,b}

P(x): {a}

Q(x): {a,b}

R(x): {b}

Called the extension of R

Varieties of Interpretations

Give an interpretation that shows that the following argument is invalid:

$\exists x(P(x) \wedge Q(x))$

$\exists x(Q(x) \wedge \neg R(x))$

$\vdash \exists x(P(x) \wedge R(x))$

Domain: {a,b}

P(x): {a}

Q(x): {a,b}

R(x): { } also written $\varnothing$

R equals the empty set

2-Place Relations

Give an interpretation that shows that the following argument is invalid:

$\exists x \exists y(P(x) \wedge Q(y) \wedge R(x,y))$

$\exists x\, R(x,x)$

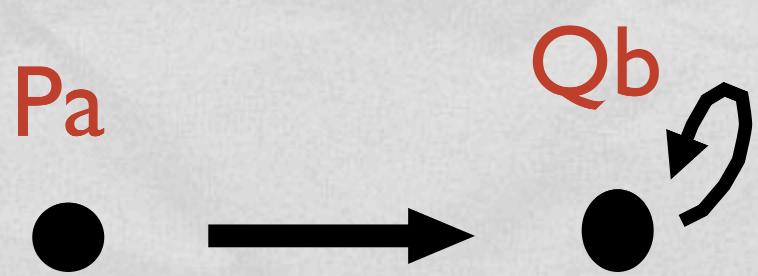
$\vdash\ \exists x \forall y\, R(x,y)$

Domain: {a,b}

P(x): {a}

Q(x): {b}

R(x,y): {<a,b>, <b,b>}



There are two <u>ordered pairs</u> in R

Names as Domain Elements

Names are straightforward and easy to interpret domain elements. But are bad choices when identity is involved. -- e.g. show that these are consistent:

$P(a) \land Q(b) \land R(a,b))$

$\exists x(Q(x) \land \neg R(a,x))$

Domain: {a,b,c}

P(x): {a}

Q(x): {b,c}

R(x,y): {<a,b>} ~~<a,c>~~

a: a

b: b

constants (names) → a: a, b: b ← objects

Names as Domain Elements

Names are straightforward and easy to interpret domain elements. But are bad choices when identity is involved. -- e.g. show that these are consistent:

$P(a) \land Q(b) \land R(a,b))$

$\exists x(Q(x) \land \neg R(a,x))$

$\exists x \exists y \forall z(z=x \lor z=y)$

These are consistent - but only if a=b

Domain: {a,b,c}

P(x): {a}

Q(x): {b,c}

R(x,y): {<a,b>}

~~<a,c>~~

a: a

b: b

NAMES AS DOMAIN ELEMENTS

Names are straightforward and easy to interpret domain elements. But are bad choices when identity is involved. -- e.g. show that these are consistent:

$P(a) \wedge Q(b) \wedge R(a,b))$

$\exists x(Q(x) \wedge \neg R(a,x))$

$\exists x \exists y \forall z(z=x \vee z=y)$

These are consistent - but only if a=b

Domain: {1,2,3}

P(x): {1}

Q(x): {2,3}

R(x,y): {<1,2>} ~~<1,3>~~

a: 1

b: 2

Old model (no good)

Names as Domain Elements

Names are straightforward and easy to interpret domain elements. But are bad choices when identity is involved. -- e.g. show that these are consistent:

$P(a) \wedge Q(b) \wedge R(a,b))$

$\exists x(Q(x) \wedge \neg R(a,x))$

$\exists x \exists y \forall z(z=x \vee z=y)$

These are consistent - but only if a=b

Domain: {1,3}

P(x): {1}

Q(x): {1,3}

R(x,y): {<1,1>} ~~<1,3>~~

a: 1

b: 1

New model

ADVANTAGES OF SETS

Give a model of the following set of sentences:

$\exists x \exists y(R(x,y) \wedge S(y,x))$

$\exists x \exists y(B(a,x,y) \wedge D(a,b,x,y))$

$\forall x \forall y(R(x,y) \rightarrow R(y,x))$

Domain: {1,2}

R(x,y): {<1,2>}

S(x,y): {<2,1>}

Takes care of P1

Advantages of Sets

Give a model of the following set of sentences:

$\exists x \exists y(R(x,y) \land S(y,x))$

$\exists x \exists y(B(a,x,y) \land D(a,b,x,y))$

$\forall x \forall y(R(x,y) \rightarrow R(y,x))$

Takes care of P1, P2

Domain: {1,2,3,4}

R(x,y): {<1,2>}

S(x,y): {<2,1>}

B(x,y,z): {<1,2,3>}

D(x,y,z): {<1,2,3,4>}

a: 1

b: 2

Advantages of Sets

Give a model of the following set of sentences:

$\exists x \exists y (R(x,y) \wedge S(y,x))$

$\exists x \exists y (B(a,x,y) \wedge D(a,b,x,y))$

$\forall x \forall y (R(x,y) \rightarrow R(y,x))$

Domain: $\{1,2,3,4\}$

R(x,y): $\{\langle 1,2\rangle, \langle 2,1\rangle\}$

S(x,y): $\{\langle 2,1\rangle\}$

B(x,y,z): $\{\langle 1,2,3\rangle\}$

D(x,y,z): $\{\langle 1,2,3,4\rangle\}$

a: 1
b: 2

Added for P3

Set Theory Notation and Logic

Sets are just generic collections of objects

{1,2,3} is the set which has three members - 1, 2, and 3

We say 2 is a member of {1,2,3} by writing "$2 \in \{1,2,3\}$"

Of course there is this mathematical theory - set theory - written in FOL where there are axioms giving the formal definition of set. In set theory there is one 2-place predicate E(x,y) and $a \in b$ is just shorthand for E(a,b)

Set Theory Notation and Logic

We want sets to just be defined by their members

{1,2,3} = {2,3,1} = {1,2,2,3,3,1,2,3} because:

- $1 \in$ the set
- $2 \in$ the set
- $3 \in$ the set

Nothing else is in the set

To formalize this we have the first axiom: Extentionality

$$\forall x \forall y[\forall z((z \in x \leftrightarrow z \in y) \rightarrow x=y)]$$