Euclid's Elements: Proposition I

From a given line, construct an equilateral triangle with that line as a side.

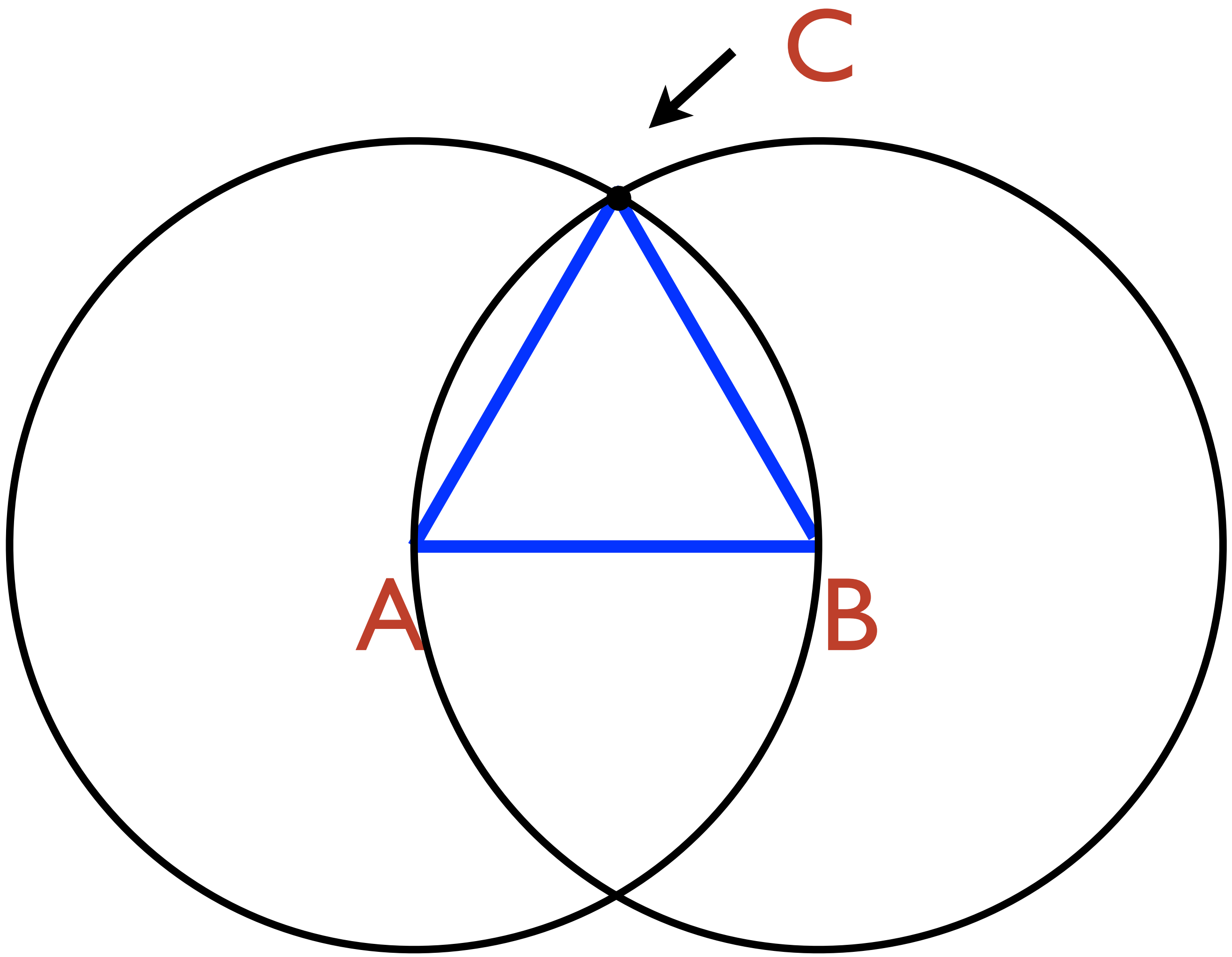
You can construct a straight line between any two points (postulate 1).

You can create a circle with any center and radius (postulate 3). [Def: in a circle all lines from center to outside are equal length]

Things which equal the same thing equal one another (common notion 1).

1) Draw a circle with A at the center, and radius AB

2) Draw a circle with B at the center, and radius AB



3) Let C be where the circles intersect and draw AC and BC. Since AB=AC and BA (=AB) =BC,

ABC is an equilateral triangle

PROPERTIES OF RELATIONS

Wednesday, 17 November

Common Properties

Reflexivity $\forall x\, R(x,x)$

Anti-Reflexivity $\forall x\, \neg R(x,x)$

Non-reflexive $\neg \forall x\, R(x,x) \Leftrightarrow \exists x\, \neg R(x,x)$

Symmetry $\forall x \forall y (R(x,y) \rightarrow R(y,x))$

Asymmetry $\forall x \forall y (R(x,y) \rightarrow \neg R(y,x))$

Non-symmetric $\neg \forall x \forall y (R(x,y) \rightarrow \neg R(y,x)) \Leftrightarrow$
$\exists x \exists y (R(x,y) \wedge \neg R(y,x))$

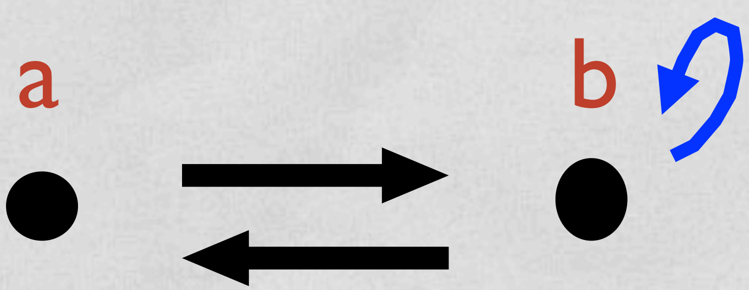
Anti-symmetry $\forall x \forall y [(R(x,y) \wedge R(y,x)) \rightarrow x=y]$

Reflexive

Anti-symmetric

So asymmetric +

Non-symmetric



~~Irreflexive~~

So non-reflexive (Still)

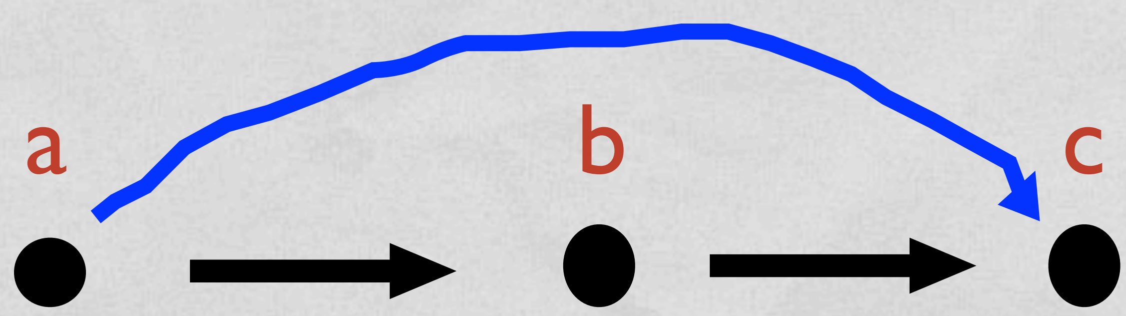
Symmetric

Common Properties

Transitivity $\forall x \forall y \forall z\, [(R(x,y) \wedge R(y,z)) \rightarrow R(x,z)]$

Anti-Transitivity $\forall x \forall y \forall z [(R(x,y) \wedge R(y,z)) \rightarrow \neg R(x,z)]$

Non-Transitivity $\exists x \exists y \exists z [(R(x,y) \wedge R(y,z)) \wedge \neg R(x,z)]$



If we assume transitivity

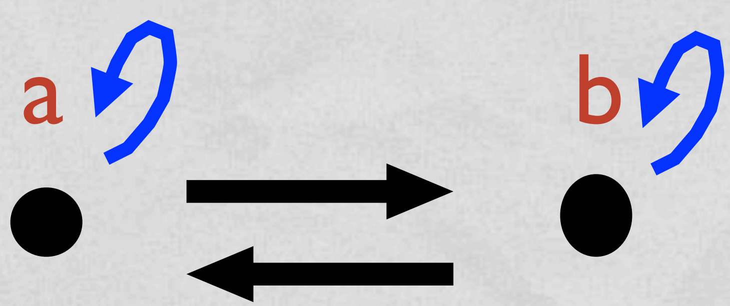
$[R(a,b) \wedge R(b,c)] \rightarrow R(a,c)$

Common Properties

Transitivity $\forall x \forall y \forall z\ [(R(x,y) \wedge R(y,x)) \rightarrow R(x,z)]$

Anti-Transitivity $\forall x \forall y \forall z [(R(x,y) \wedge R(y,x)) \rightarrow \neg R(x,z)]$

Non-Transitivity $\exists x \exists y \exists z [(R(x,y) \wedge R(y,x)) \wedge \neg R(x,z)]$

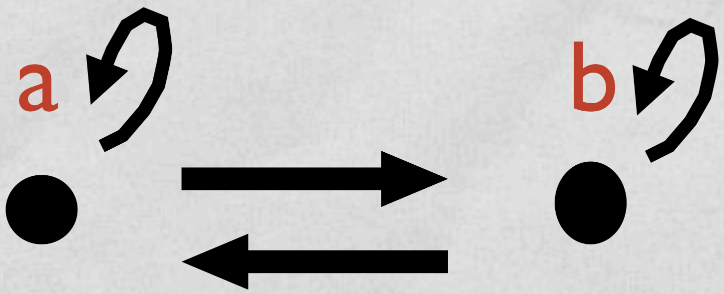


If we assume transitivity

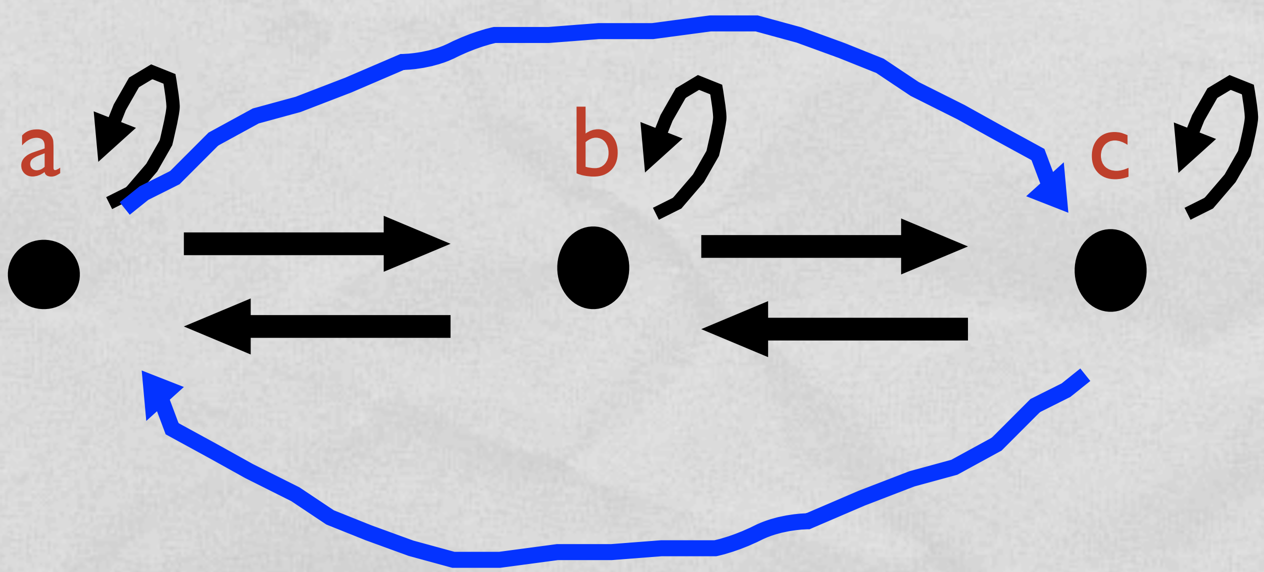
$[R(a,b) \wedge R(b,a)] \rightarrow R(a,a)$

Equivalence Relations

Equivalence relations are those that are reflexive, symmetric, and transitive - not necessarily universal

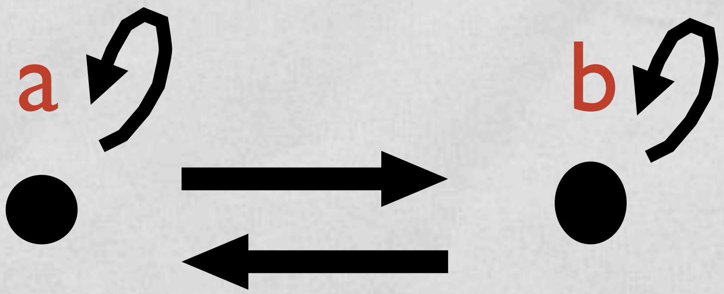


or

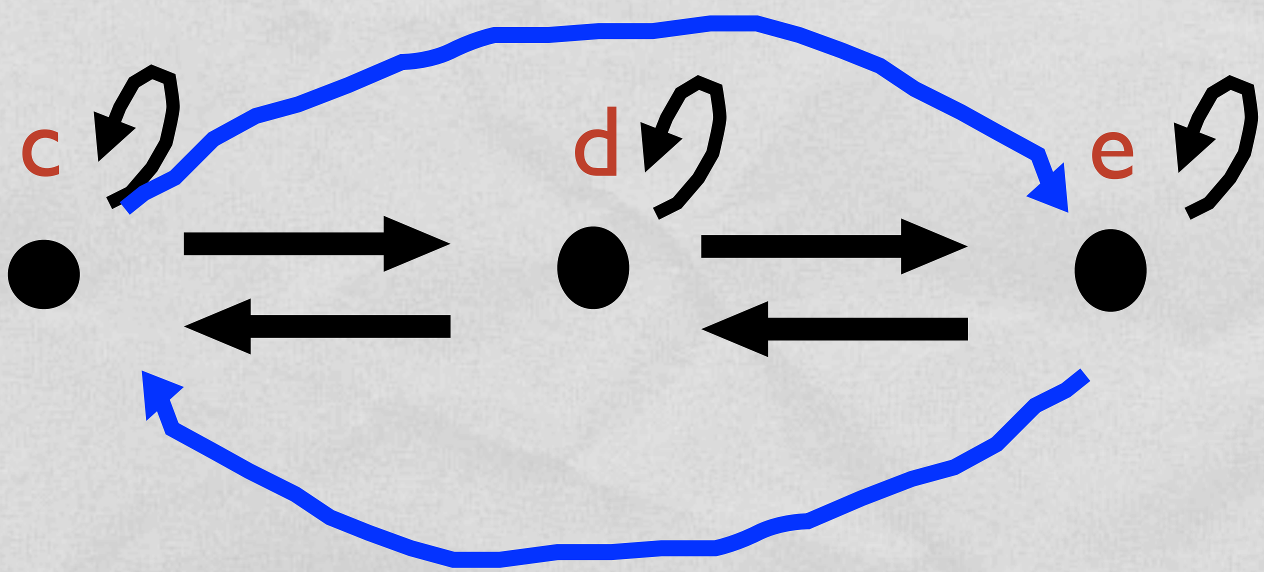


EQUIVALENCE RELATIONS

Equivalence relations are those that are reflexive, symmetric, and transitive - not necessarily universal



R <u>partitions</u> the space into groups



Common Properties

There are many other properties which are important enough to have names:

Seriality (or totality) $\forall x \exists y\ R(x,y)$

Connected (or total) $\forall x \forall y (R(x,y) \vee R(y,x))$

Trichotomous $\forall x \forall y \forall y (R(x,y) \vee R(y,x) \vee x=y)$

Euclidean $\forall x \forall y \forall y [(R(x,y) \wedge R(x,z)) \rightarrow R(y,z)]$

Dense $\forall x \forall y [R(x,y) \rightarrow \exists z (R(x,z) \wedge R(z,y))]$