Puzzle

In a certain place, all the inhabitants are either Knights or Knaves. Knights always tell the truth and Knaves never tell the truth.

You meet two inhabitants, A and B. B says "Both of us are Knaves." What, if anything, can you infer from this?

The Logic of Atomic Sentences

Monday, 30 August

Chapter 1 skills

- A one-place predicate with one object has a truth-value
- Predicate: 'is prime'

  3 is prime - True

  4 is prime - False

Chapter 1 skills

- A one-place predicate with one object has a truth-value
- Predicate: 'is prime'

- A two-place predicate with a pair of objects has a truth-value
- Predicate: 'is less than'
- 2 is less than 3 - True
- 4 is less than 3 - False

Chapter 1 skills

- A one-place function takes a term as input and gives back an object
- Function: 'squared'
- squared(5) refers to 25
- A two-place function takes two terms and gives back one object
- Function: 'addition'
- addition(2,3) refers to 5

CHAPTER 1 SKILLS

- A sentence in FOL is true or false in a particular world.
- Tarski's World can give examples of worlds.
- NOTE: Tarski's World uses a specific language - the blocks language. We want to study the properties of all first order languages.

Arguments

- An argument is a series of sentences in which one (the conclusion) is meant to follow from the others (the premises).

Arguments

- <u>Example (adapted from cartoon):</u>
  (a) All penguins are black and white.
  (b) All old movies are black and white.
  (c) Thus, all penguins are old movies.

- <u>Example (adapted from Lewis Carroll):</u>
  (a) All babies are illogical persons.
  (b) Illogical persons are despised.
  (c) Nobody is despised who can manage a crocodile.
  (d) It follows that no baby can manage a crocodile.

Validity and Soundness

- An argument is <u>valid</u> if and only if (iff) the conclusion is guaranteed to be true, assuming the premises are true.
- When an argument is valid, the conclusion is a <u>logical consequence</u> of the premises.

Validity and Soundness

- Is this argument valid?
  (a) All penguins are black and white.
  (b) All old movies are black and white.
  (c) Thus, all penguins are old movies.

- What about this one?
  (a) All babies are illogical persons.
  (b) Illogical persons are despised.
  (c) Nobody is despised who can manage a crocodile.
  (d) It follows that no baby can manage a crocodile.

Validity and Soundness

- An argument is valid iff the conclusion is guaranteed to be true, assuming the premises are true.
- When an argument is valid, the conclusion is a logical consequence of the premises.
- An argument is sound iff (a) the agument is valid, AND (b) the premises are all true.

Validity and Soundness

- Is this argument sound?
  (a) All penguins are black and white.
  (b) All old movies are black and white.
  (c) Thus, all penguins are old movies.

NO - it is not even valid, so it can't be sound

Validity and Soundness

- How about this one?
  (a) All past and current presidents of the U.S. are male.
  (b) Bill Clinton is a past president of the U.S.
  (c) So, Bill Clinton is male.

- And this one?
  (a) All past and current presidents of the U.S. are male.
  (b) Bill Clinton is a past president of the U.S.
  (c) So, Bill Clinton is from Arkansas.

Validity and Soundness

- <u>How about this one?</u>
  (a) All past and current presidents of the U.S. are male.
  (b) Bill Clinton is a past president of the U.S.
  (c) So, Bill Clinton is male.

- <u>And this one?</u>
  (a) All past and current presidents of the U.S. are male.
  (b) Hillary Clinton is a past president of the U.S.
  (c) So, Hillary Clinton is male.

Methods of Proof

- An argument is invalid when the conclusion does not follow from the premises.
- A <u>counterexample</u> is a situation in which the premises are true but the conclusion is false.
- (a) All penguins are black and white.
  (b) All old movies are black and white.
  (c) Thus, all penguins are old movies.

  Counterexample: the actual world

METHODS OF PROOF

(a) All students are drinking coffee or diet coke.
(b) Gina is a student.
(c) Gina is drinking coffee.
(d) So, Gina is not drinking diet coke.

Counterexample?

Methods of Proof

- When an argument is valid, the conclusion is a logical consequence of the premises.
- A <u>proof</u> is a step-by-step demonstration that the conclusion must follow from the premises.
- <u>Informal</u> proofs use ordinary language and reasoning; <u>formal</u> proofs use a fixed system of presentation and set of rules.

Methods of Proof

- (a) All babies are illogical persons.
  (b) Illogical persons are despised.
  (c) Nobody is despised who can manage a crocodile.
  (d) It follows that no baby can manage a crocodile.

Informal proof: Suppose that the premises are true. Then it follows from the fact that all babies are illogical that all babies are despised, since all illogical persons are despised. But nobody is despised who can manage a crocodile; so if any baby could manage a crocodile, they would not be despised. Since all babies are despised, no baby can manage a crocodile.

Methods of Proof

- A <u>formal system of deduction</u> uses a fixed set of rules to specify what counts as acceptable steps in a proof.
- Each step of a proof must be justified by these rules, and the rules must be carried out precisely.
- Formal proofs do not allow shortcuts.
- We will be using $\mathcal{F}$ as our formal system of deduction. Fitch is a computer program which (partially) implements this.

IDENTITY RULES IN $\mathcal{F}$

- = Elim (Indiscernibility of Identicals):
  If b=c, then whatever holds of b holds of c.
- = Intro (Reflexivity of Identity):
  Sentences of the form b=b are always true.

Identity

Example: Proof of the Symmetry of Identity -

From b=c prove c=b

(so actually, a specific instance of symmetry)

- Informal proof: Suppose that b=c. We know by the reflexivity of identity that b=b. Now substitute c for the first b in b=b using the indiscernibility of identicals. We get c=b.

IDENTITY

Example: Proof of the Symmetry of Identity

Informal proof: Suppose that b=c. We know by the reflexivity of identity that b=b. Now substitute c for the first b in b=b using the indiscernibility of identicals. We get c=b.

1. b=c Premise
2. b=b, by = Intro
3. c=b, by = Elim on 1, 2

Identity

Proof of the Transitivity of Identity

1. a=b
2. b=c

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a=c