The Hardest Logic Puzzle Ever

Three gods A, B, and C are called, in some order, True, False, and Random. True always speaks truly, False always speaks falsely, but whether Random speaks truly or falsely is a completely random matter. Your task is to determine the identities of A, B, and C by asking three yes/no questions; each question must be put to exactly one god. The gods understand English, but will answer all questions in their own language, in which the words for *yes* and *no* are 'da' and 'ja', in some order. You do not know which word means which.

SOLUTION - BREAK IT INTO STEPS

Next step (working backwards)

If you know you are talking to either a knight or a knave (but you don't know which) how can you figure out what they are?

-- Remember, they will answer 'Bal' or 'Da'.

Solution - break it into steps

Answer:

Does "'Bal' means yes" have the same truth value as 2+2 = 4?

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The knight will answer 'Bal' and the knave will answer 'Da'.

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Does "'Bal' means yes" have the same truth value as 2+2 = 4?

The knight will answer 'Bal' and the knave will answer 'Da'.

SOLUTION - BREAK IT INTO STEPS

Answer:

Does "'Bal' means yes" have the same truth value as 2+2 = 4?

The knight will answer 'Bal' and the knave will answer 'Da'.

Notice now that we can solve the puzzle in two questions if we can just make sure we aren't talking to the Normal.

SOLUTION - BREAK IT INTO STEPS

Answer:

Does "'Bal' means yes" have the same truth value as 2+2 = 4?

The knight will answer 'Bal' and the knave will answer 'Da'.

Notice now that we can solve the puzzle in two questions if we can just make sure we aren't talking to the Normal.

Using and Building Diagrams -- More examples

Wednesday, 10 November

Diagrams

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- Since a diagram is an interpretation, if any diagram can make all the premises of an argument true but the conclusion false, that argument is invalid.

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- Since a diagram is an interpretation, if any diagram can make all the premises of an argument true but the conclusion false, that argument is invalid.

- Diagrams can also be used as 'guides' to what can be proved from a set of premises. If you are forced to add something to a diagram, then you could prove that it follows (and sometimes the diagram helps you figure out how).

1. $\forall x \exists y\, R(x,y)$
2. $\forall x\ \neg R(x,x)$
3. $\forall x \forall y \forall z (x=y \lor y=z \lor x=z)$

$\exists x \exists y (x \neq y \land R(x,y) \land R(y,x))$

1. $\forall x \exists y\, R(x,y)$

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3. $\forall x \forall y \forall z (x=y \lor y=z \lor x=z)$

$\exists x \exists y (x \neq y \land R(x,y) \land R(y,x))$

1. ∀x∃y R(x,y)

2. ∀x ¬R(x,x)

3. ∀x∀y∀z(x=y ∨ y=z ∨ x=z)

4. ∃y R(a,y) ∀ Elim 1

∃x∃y(x≠y ∧ R(x,y) ∧ R(y,x))

1. $\forall x \exists y\, R(x,y)$

2. $\forall x\, \neg R(x,x)$

3. $\forall x \forall y \forall z(x=y \lor y=z \lor x=z)$

a

4. $\exists y\, R(a,y)$ — $\forall$ Elim 1

5. [b] $R(a,b)$

$\exists x \exists y(x \neq y \land R(x,y) \land R(y,x))$

$\exists x \exists y(x \neq y \land R(x,y) \land R(y,x))$ — $\exists$ Elim

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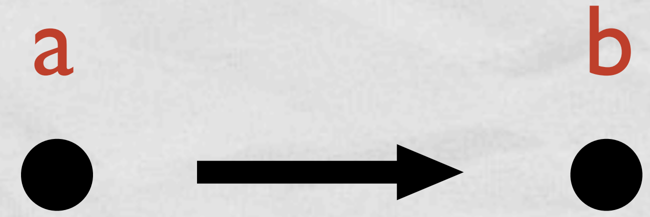


1. $\forall x \exists y\, R(x,y)$

2. $\forall x\, \neg R(x,x)$

3. $\forall x \forall y \forall z (x=y \lor y=z \lor x=z)$

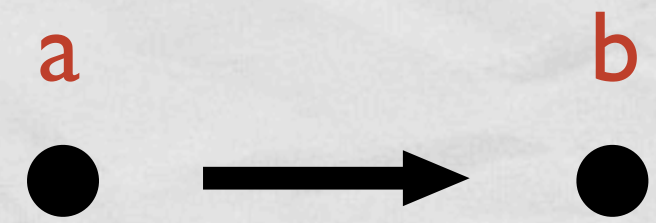
4. $\exists y\, R(a,y)$ — ∀ Elim 1

5. [b] $R(a,b)$

6. $\neg R(a,a)$ — ∀ Elim 2

$\exists x \exists y (x \neq y \land R(x,y) \land R(y,x))$

$\exists x \exists y (x \neq y \land R(x,y) \land R(y,x))$ — ∃ Elim



Wednesday, November 10, 2010

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3. $\forall x \forall y \forall z (x=y \lor y=z \lor x=z)$

4. $\exists y\, R(a,y)$ — ∀ Elim 1

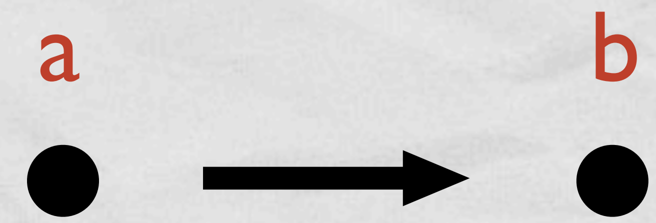
5. [b] $R(a,b)$

6. $\neg R(a,a)$ — ∀ Elim 2

7. $a \neq b$ — NI 5,6 — FO con

$\exists x \exists y (x \neq y \land R(x,y) \land R(y,x))$

$\exists x \exists y (x \neq y \land R(x,y) \land R(y,x))$ — ∃ Elim



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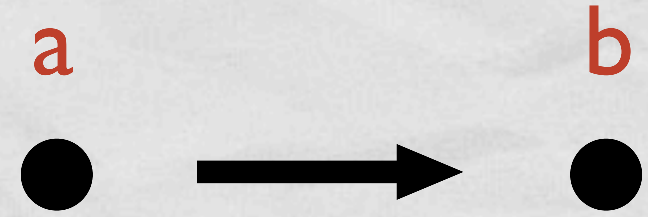

4. $\exists y\, R(a,y)$ — $\forall$ Elim 1

  5. $\boxed{b}$ $R(a,b)$

  6. $\neg R(a,a)$ — $\forall$ Elim 2

  7. $a \neq b$ — NI 5,6 — FO con

  8. $\exists y\, R(b,y)$ — $\forall$ Elim 1

  $\exists x \exists y (x \neq y \wedge R(x,y) \wedge R(y,x))$

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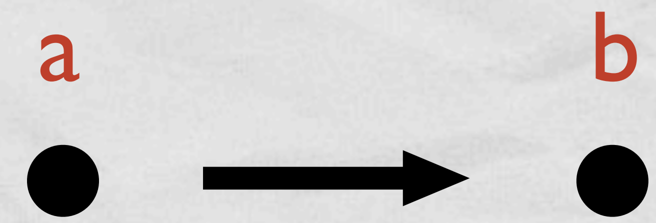

4. $\exists y\, R(a,y)$ — ∀ Elim 1

5. [b] $R(a,b)$

6. $\neg R(a,a)$ — ∀ Elim 2

7. $a \neq b$ — NI 5,6 — FO con

8. $\exists y\, R(b,y)$ — ∀ Elim 1

9. [c] $R(b,c)$

$\exists x \exists y(x \neq y \land R(x,y) \land R(y,x))$

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3. $\forall x \forall y \forall z(x=y \lor y=z \lor x=z)$

---

4. $\exists y\, R(a,y)$ — $\forall$ Elim 1
    5. [b] $R(a,b)$

    ---

    6. $\neg R(a,a)$ — $\forall$ Elim 2
    7. $a \neq b$ — NI 5,6 — FO con
    8. $\exists y\, R(b,y)$ — $\forall$ Elim 1
        9. [c] $R(b,c)$

        $\exists x \exists y(x \neq y \land R(x,y) \land R(y,x))$
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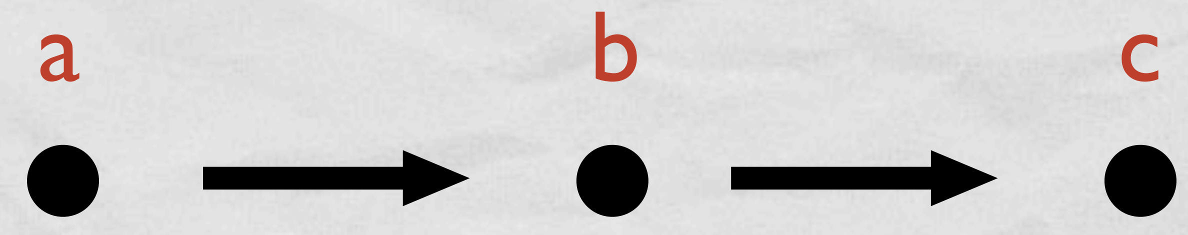

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3. $\forall x \forall y \forall z (x=y \lor y=z \lor x=z)$

---

4. $\exists y\, R(a,y)$ ∀ Elim 1

5. [b] $R(a,b)$

6. $\neg R(a,a)$ ∀ Elim 2

7. $a \neq b$ NI 5,6 FO con

8. $\exists y\, R(b,y)$ ∀ Elim 1

9. [c] $R(b,c)$

10. $\neg R(b,b)$ ∀ Elim 2

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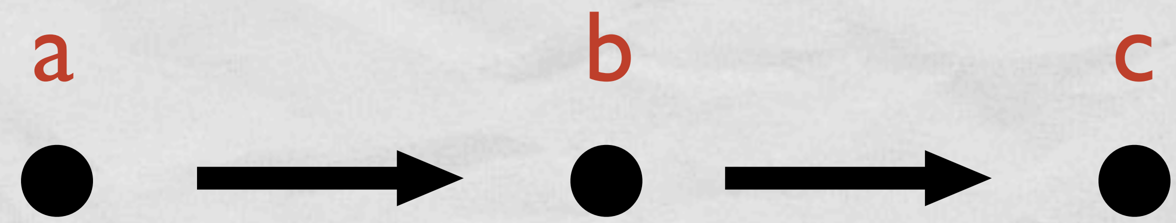
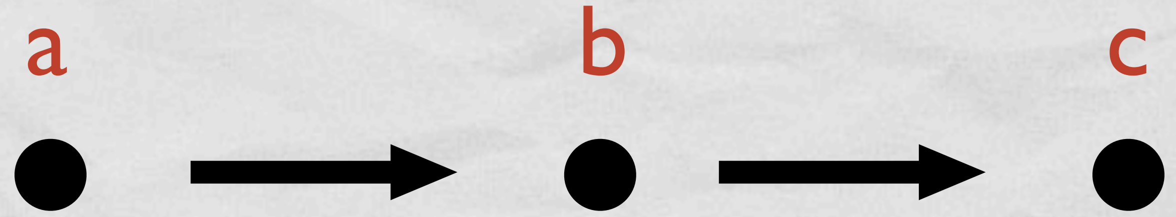


1. ∀x∃y R(x,y)

2. ∀x ¬R(x,x)

3. ∀x∀y∀z(x=y ∨ y=z ∨ x=z)



4. ∃y R(a,y) ∀ Elim 1

5. [b] R(a,b)

6. ¬R(a,a) ∀ Elim 2

7. a≠b NI 5,6 FO con

8. ∃y R(b,y) ∀ Elim 1

9. [c] R(b,c)

10. ¬R(b,b) ∀ Elim 2

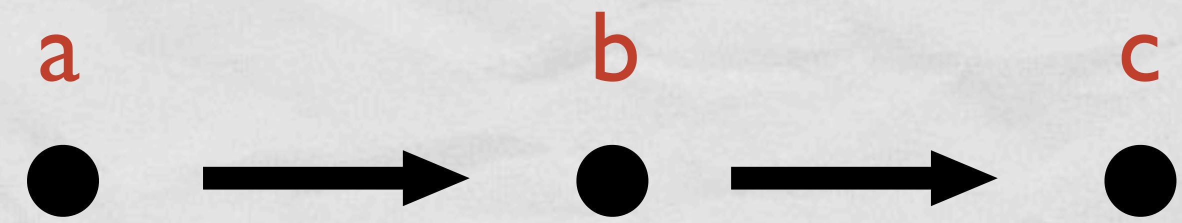
11. b≠c NI 9,10 FO con

∃x∃y(x≠y ∧ R(x,y) ∧ R(y,x))

∃x∃y(x≠y ∧ R(x,y) ∧ R(y,x)) ∃ Elim

∃x∃y(x≠y ∧ R(x,y) ∧ R(y,x)) ∃ Elim

1. $\forall x \exists y\, R(x,y)$
2. $\forall x\ \neg R(x,x)$
3. $\forall x \forall y \forall z (x=y \lor y=z \lor x=z)$



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4. $\exists y\, R(a,y)$ — $\forall$ Elim 1
   5. $\boxed{b}$ $R(a,b)$
   ---
   6. $\neg R(a,a)$ — $\forall$ Elim 2
   7. $a \neq b$ — NI 5,6 — FO con
   8. $\exists y\, R(b,y)$ — $\forall$ Elim 1
      9. $\boxed{c}$ $R(b,c)$
      ---
      10. $\neg R(b,b)$ — $\forall$ Elim 2
      11. $b \neq c$ — NI 9,10 — FO con
      12. $a=b \lor b=c \lor a=c$ — $\forall$ Elim 3 x3

      $\exists x \exists y (x \neq y \land R(x,y) \land R(y,x))$
   $\exists x \exists y (x \neq y \land R(x,y) \land R(y,x))$ — $\exists$ Elim
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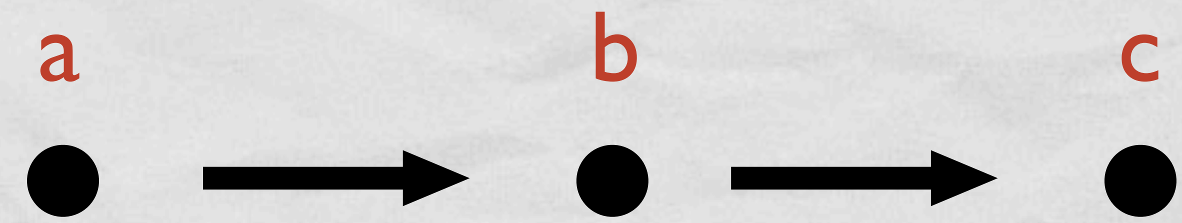

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6. $\neg R(a,a)$ — $\forall$ Elim 2

7. $a \neq b$ — NI 5,6 — FO con

8. $\exists y\, R(b,y)$ — $\forall$ Elim 1

9. [c] $R(b,c)$

10. $\neg R(b,b)$ — $\forall$ Elim 2

11. $b \neq c$ — NI 9,10 — FO con

12. $a=b \vee b=c \vee a=c$ — $\forall$ Elim 3 x3

13. $a=c$ — Taut Con 7,11,12

$\exists x \exists y (x \neq y \wedge R(x,y) \wedge R(y,x))$

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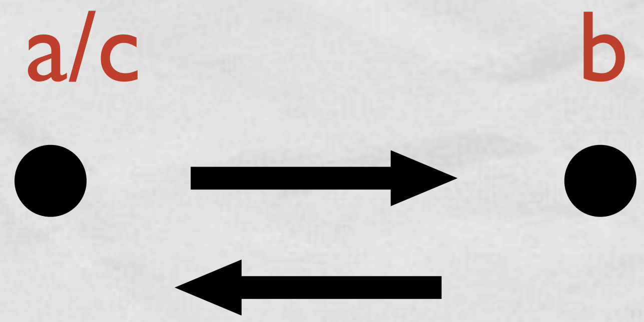
3. $\forall x \forall y \forall z(x=y \vee y=z \vee x=z)$



4. $\exists y\, R(a,y)$ — $\forall$ Elim 1

5. [b] $R(a,b)$

6. $\neg R(a,a)$ — $\forall$ Elim 2

7. $a \neq b$ — NI 5,6 — FO con

8. $\exists y\, R(b,y)$ — $\forall$ Elim 1

9. [c] $R(b,c)$

10. $\neg R(b,b)$ — $\forall$ Elim 2

11. $b \neq c$ — NI 9,10 — FO con

12. $a=b \vee b=c \vee a=c$ — $\forall$ Elim 3 x3

13. $a=c$ — Taut Con 7,11,12

$\exists x \exists y(x \neq y \wedge R(x,y) \wedge R(y,x))$

$\exists x \exists y(x \neq y \wedge R(x,y) \wedge R(y,x))$ — $\exists$ Elim

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1. $\forall x \exists y\, R(x,y)$

2. $\forall x\, \neg R(x,x)$

3. $\forall x \forall y \forall z (x=y \lor y=z \lor x=z)$

4. $\exists y\, R(a,y)$ — $\forall$ Elim 1

5. [b] $R(a,b)$

6. $\neg R(a,a)$ — $\forall$ Elim 2

7. $a \neq b$ — NI 5,6 — FO con

8. $\exists y\, R(b,y)$ — $\forall$ Elim 1

9. [c] $R(b,c)$

10. $\neg R(b,b)$ — $\forall$ Elim 2

11. $b \neq c$ — NI 9,10 — FO con

12. $a=b \lor b=c \lor a=c$ — $\forall$ Elim 3 x3

13. $a=c$ — Taut Con 7,11,12

14. $R(b,a)$ — = Elim 9,13

$\exists x \exists y (x \neq y \land R(x,y) \land R(y,x))$

$\exists x \exists y (x \neq y \land R(x,y) \land R(y,x))$ — $\exists$ Elim

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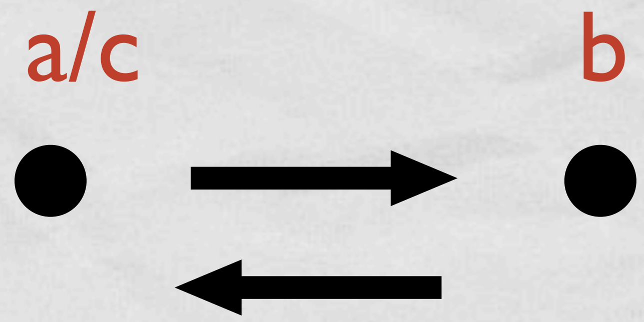
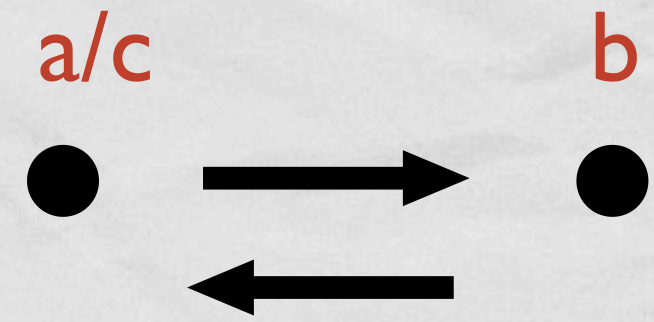


1. $\forall x \exists y\, R(x,y)$

2. $\forall x\ \neg R(x,x)$

3. $\forall x \forall y \forall z (x=y \lor y=z \lor x=z)$



4. $\exists y\, R(a,y)$ — $\forall$ Elim 1

5. [b] $R(a,b)$

6. $\neg R(a,a)$ — $\forall$ Elim 2

7. $a \neq b$ — NI 5,6 — FO con

8. $\exists y\, R(b,y)$ — $\forall$ Elim 1

9. [c] $R(b,c)$

10. $\neg R(b,b)$ — $\forall$ Elim 2

11. $b \neq c$ — NI 9,10 — FO con

12. $a=b \lor b=c \lor a=c$ — $\forall$ Elim 3 x3

13. $a=c$ — Taut Con 7,11,12

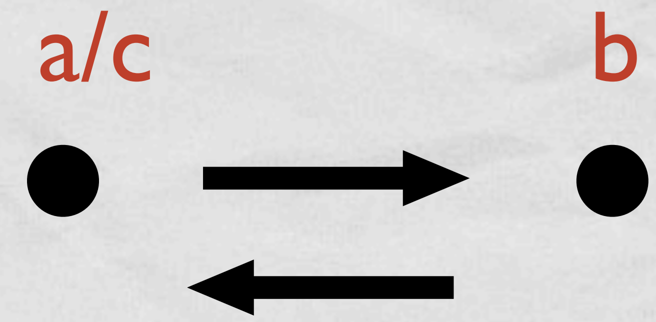
14. $R(b,a)$ — = Elim 9,13

15. $a \neq b \land R(a,b) \land R(b,a)$ — $\land$ Intro 5,7,14

$\exists x \exists y (x \neq y \land R(x,y) \land R(y,x))$

$\exists x \exists y (x \neq y \land R(x,y) \land R(y,x))$ — $\exists$ Elim

$\exists x \exists y (x \neq y \land R(x,y) \land R(y,x))$ — $\exists$ Elim



1. $\forall x \exists y\, R(x,y)$
2. $\forall x\, \neg R(x,x)$
3. $\forall x \forall y \forall z (x=y \lor y=z \lor x=z)$

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4. $\exists y\, R(a,y)$ — $\forall$ Elim 1
5. [b] $R(a,b)$

---

6. $\neg R(a,a)$ — $\forall$ Elim 2
7. $a \neq b$ — NI 5,6 — FO con
8. $\exists y\, R(b,y)$ — $\forall$ Elim 1
9. [c] $R(b,c)$

---

10. $\neg R(b,b)$ — $\forall$ Elim 2
11. $b \neq c$ — NI 9,10 — FO con
12. $a=b \lor b=c \lor a=c$ — $\forall$ Elim 3 x3
13. $a=c$ — Taut Con 7,11,12
14. $R(b,a)$ — = Elim 9,13
15. $a \neq b \land R(a,b) \land R(b,a)$ — $\land$ Intro 5,7,14
16. $\exists x \exists y (x \neq y \land R(x,y) \land R(y,x))$ — $\exists$ Intro 15 x2
17. $\exists x \exists y (x \neq y \land R(x,y) \land R(y,x))$ — $\exists$ Elim 8,9-16
18. $\exists x \exists y (x \neq y \land R(x,y) \land R(y,x))$ — $\exists$ Elim 4,5-17

1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y))$
2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge \neg A(y,x))$
3. $\exists x\, T(x)$

$\exists x \exists y(x \neq y \wedge M(x) \wedge M(y))$

1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y))$

2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge \neg A(y,x))$

3. $\exists x\, T(x)$

4. $\boxed{a}\ T(a)$

$\exists x \exists y(x \neq y \wedge M(x) \wedge M(y))$ $\exists$ Elim

1. $\forall x(T(x) \rightarrow \exists y(M(y) \land A(x,y)))$

2. $\forall x(M(x) \rightarrow \exists y(T(y) \land \neg A(y,x)))$

3. $\exists x\, T(x)$

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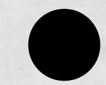
4. [a] $T(a)$

$\exists x \exists y(x \neq y \land M(x) \land M(y))$ ∃ Elim

Ta



1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y))$

2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge \neg A(y,x))$

3. $\exists x\, T(x)$

Ta

●

4. [a] $T(a)$

5. $T(a) \rightarrow \exists y(M(y) \wedge A(a,y)$ ∀ Elim 1

$\exists x \exists y(x \neq y \wedge M(x) \wedge M(y))$ ∃ Elim

1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y))$

2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge \neg A(y,x))$

3. $\exists x\, T(x)$

Ta

4. $\boxed{a}\ T(a)$

5. $T(a) \rightarrow \exists y(M(y) \wedge A(a,y)$ — $\forall$ Elim 1

6. $\exists y(M(y) \wedge A(a,y)$ — $\rightarrow$ Elim 4,5

$\exists x \exists y(x \neq y \wedge M(x) \wedge M(y))$ — $\exists$ Elim

1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y))$

2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge \neg A(y,x))$

3. $\exists x\, T(x)$

Ta

●

4. [a] $T(a)$

5. $T(a) \rightarrow \exists y(M(y) \wedge A(a,y)$ — ∀ Elim 1

6. $\exists y(M(y) \wedge A(a,y)$ — → Elim 4,5

7. [b] $M(b) \wedge A(a,b)$

$\exists x \exists y(x \neq y \wedge M(x) \wedge M(y))$ — ∃ Elim + ∃ Elim

1. $\forall x(T(x) \to \exists y(M(y) \wedge A(x,y))$

2. $\forall x(M(x) \to \exists y(T(y) \wedge \neg A(y,x))$

3. $\exists x\, T(x)$

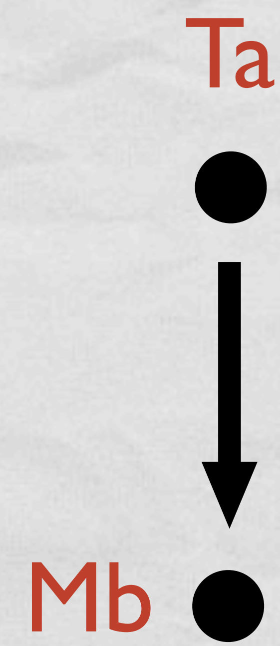
4. [a] $T(a)$

5. $T(a) \to \exists y(M(y) \wedge A(a,y)$ — $\forall$ Elim 1

6. $\exists y(M(y) \wedge A(a,y)$ — $\to$ Elim 4,5

7. [b] $M(b) \wedge A(a,b)$

$\exists x \exists y(x \neq y \wedge M(x) \wedge M(y))$ — $\exists$ Elim + $\exists$ Elim



1. ∀x(T(x) → ∃y(M(y) ∧ A(x,y))

2. ∀x(M(x) → ∃y(T(y) ∧ ¬A(y,x))

3. ∃x T(x)

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4. [a] T(a)

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5. T(a) → ∃y(M(y) ∧ A(a,y)) ∀ Elim 1

6. ∃y(M(y) ∧ A(a,y) → Elim 4,5

7. [b] M(b) ∧ A(a,b)

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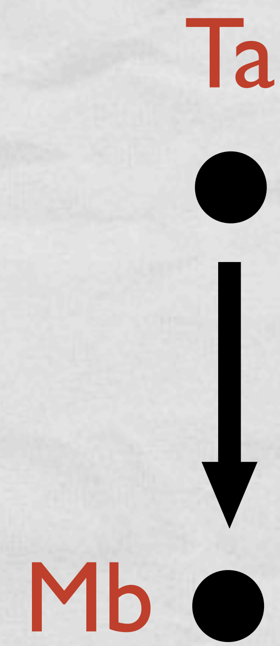
8. M(b) → ∃y(T(y) ∧ ¬A(y,b) ∀ Elim 2

∃x∃y(x≠y ∧ M(x) ∧ M(y)) ∃ Elim + ∃ Elim



1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y))$

2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge \neg A(y,x))$

3. $\exists x\, T(x)$

---

4. $\boxed{a}$ $T(a)$

---

5. $T(a) \rightarrow \exists y(M(y) \wedge A(a,y)$ — $\forall$ Elim 1

6. $\exists y(M(y) \wedge A(a,y)$ — $\rightarrow$ Elim 4,5

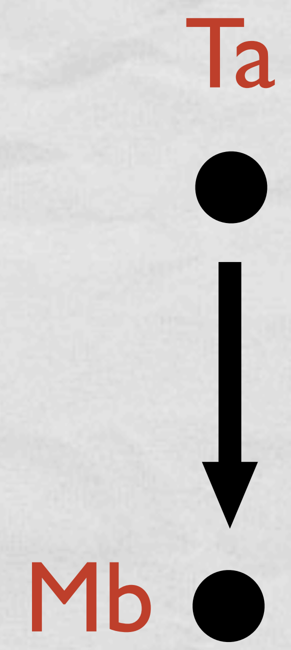
7. $\boxed{b}$ $M(b) \wedge A(a,b)$

---

8. $M(b) \rightarrow \exists y(T(y) \wedge \neg A(y,b)$ — $\forall$ Elim 2

9. $\exists y(T(y) \wedge \neg A(y,b)$ — Taut Con 7,8

$\exists x \exists y(x \neq y \wedge M(x) \wedge M(y))$ — $\exists$ Elim + $\exists$ Elim



1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y)))$

2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge \neg A(y,x)))$

3. $\exists x\, T(x)$

4. [a] $T(a)$

5. $T(a) \rightarrow \exists y(M(y) \wedge A(a,y))$ — $\forall$ Elim 1

6. $\exists y(M(y) \wedge A(a,y))$ — $\rightarrow$ Elim 4,5

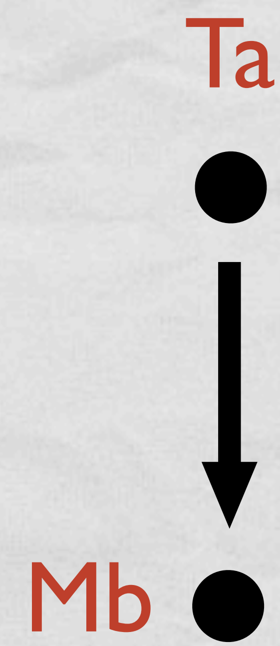
7. [b] $M(b) \wedge A(a,b)$

8. $M(b) \rightarrow \exists y(T(y) \wedge \neg A(y,b))$ — $\forall$ Elim 2

9. $\exists y(T(y) \wedge \neg A(y,b))$ — Taut Con 7,8

10. [c] $T(c) \wedge \neg A(c,b)$

$\exists x \exists y(x \neq y \wedge M(x) \wedge M(y))$ — $\exists$ Elim + $\exists$ Elim + $\exists$ Elim



1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y))$

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3. $\exists x\, T(x)$

---

4. [a] $T(a)$

---

5. $T(a) \rightarrow \exists y(M(y) \wedge A(a,y)$ — $\forall$ Elim 1

6. $\exists y(M(y) \wedge A(a,y)$ — $\rightarrow$ Elim 4,5

7. [b] $M(b) \wedge A(a,b)$

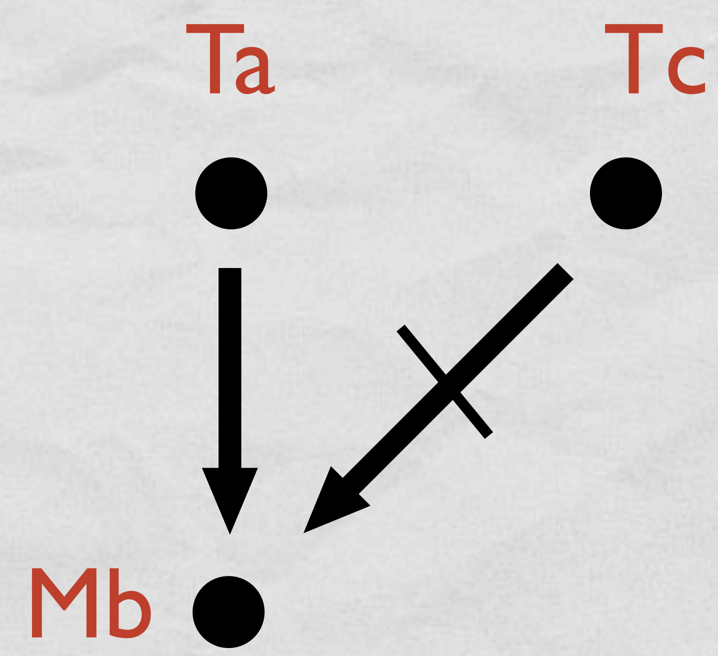
---

8. $M(b) \rightarrow \exists y(T(y) \wedge \neg A(y,b)$ — $\forall$ Elim 2

9. $\exists y(T(y) \wedge \neg A(y,b)$ — Taut Con 7,8

10. [c] $T(c) \wedge \neg A(c,b)$

$\exists x \exists y(x \neq y \wedge M(x) \wedge M(y))$ — $\exists$ Elim + $\exists$ Elim + $\exists$ Elim



1. $\forall x(T(x) \rightarrow \exists y(M(y) \land A(x,y))$

2. $\forall x(M(x) \rightarrow \exists y(T(y) \land \neg A(y,x))$

3. $\exists x\, T(x)$

4. [a] $T(a)$

5. $T(a) \rightarrow \exists y(M(y) \land A(a,y)$ — $\forall$ Elim 1

6. $\exists y(M(y) \land A(a,y)$ — $\rightarrow$ Elim 4,5

7. [b] $M(b) \land A(a,b)$

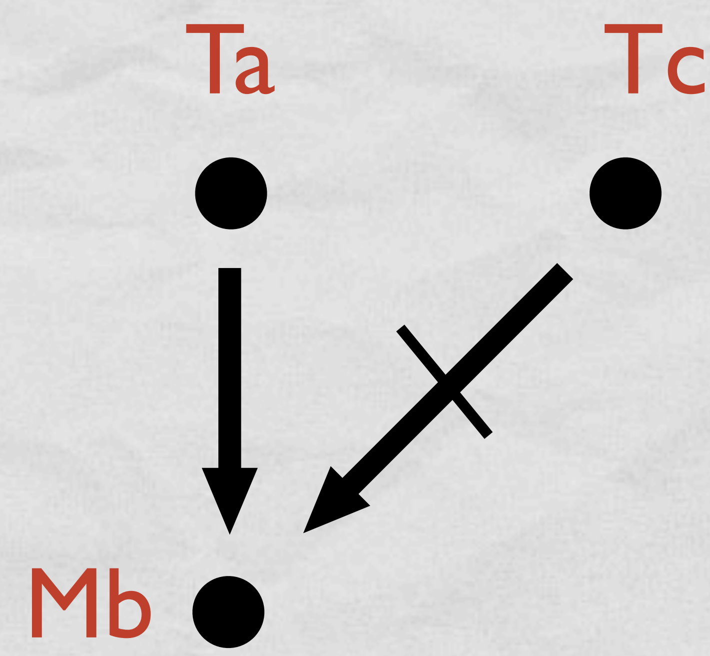
8. $M(b) \rightarrow \exists y(T(y) \land \neg A(y,b)$ — $\forall$ Elim 2

9. $\exists y(T(y) \land \neg A(y,b)$ — Taut Con 7,8

10. [c] $T(c) \land \neg A(c,b)$

11. $T(c) \rightarrow \exists y(M(y) \land A(c,y)$ — $\forall$ Elim 1

$\exists x \exists y(x \neq y \land M(x) \land M(y))$ — $\exists$ Elim + $\exists$ Elim + $\exists$ Elim



1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y))$

2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge \neg A(y,x))$

3. $\exists x\, T(x)$

4. [a] $T(a)$

5. $T(a) \rightarrow \exists y(M(y) \wedge A(a,y)$ — $\forall$ Elim 1

6. $\exists y(M(y) \wedge A(a,y)$ — $\rightarrow$ Elim 4,5

7. [b] $M(b) \wedge A(a,b)$

8. $M(b) \rightarrow \exists y(T(y) \wedge \neg A(y,b)$ — $\forall$ Elim 2

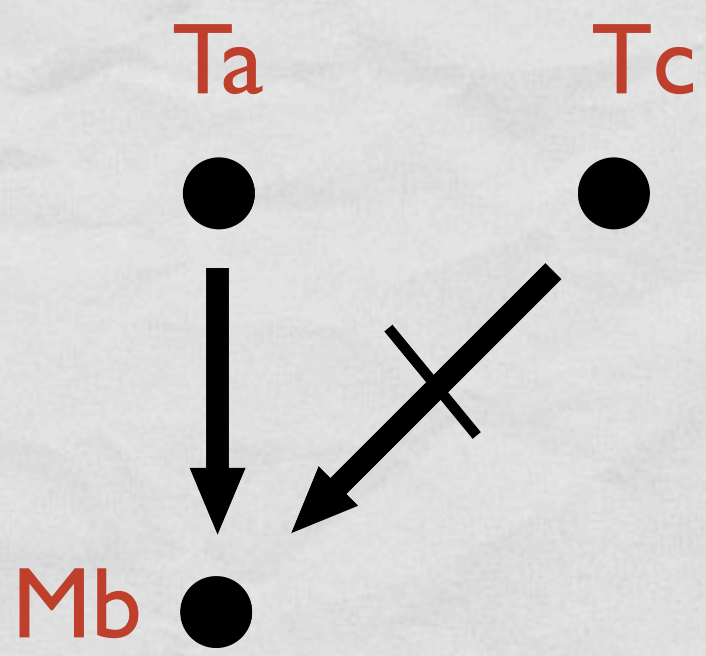
9. $\exists y(T(y) \wedge \neg A(y,b)$ — Taut Con 7,8

10. [c] $T(c) \wedge \neg A(c,b)$

11. $T(c) \rightarrow \exists y(M(y) \wedge A(c,y)$ — $\forall$ Elim 1

12. $\exists y(M(y) \wedge A(c,y)$ — Taut Con 10,11

$\exists x \exists y(x \neq y \wedge M(x) \wedge M(y))$ — $\exists$ Elim + $\exists$ Elim + $\exists$ Elim



1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y)))$

2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge \neg A(y,x)))$

3. $\exists x\, T(x)$

---

4. [a] $T(a)$

---

5. $T(a) \rightarrow \exists y(M(y) \wedge A(a,y))$ — $\forall$ Elim 1

6. $\exists y(M(y) \wedge A(a,y))$ — $\rightarrow$ Elim 4,5

7. [b] $M(b) \wedge A(a,b)$

---

8. $M(b) \rightarrow \exists y(T(y) \wedge \neg A(y,b))$ — $\forall$ Elim 2

9. $\exists y(T(y) \wedge \neg A(y,b))$ — Taut Con 7,8

10. [c] $T(c) \wedge \neg A(c,b)$

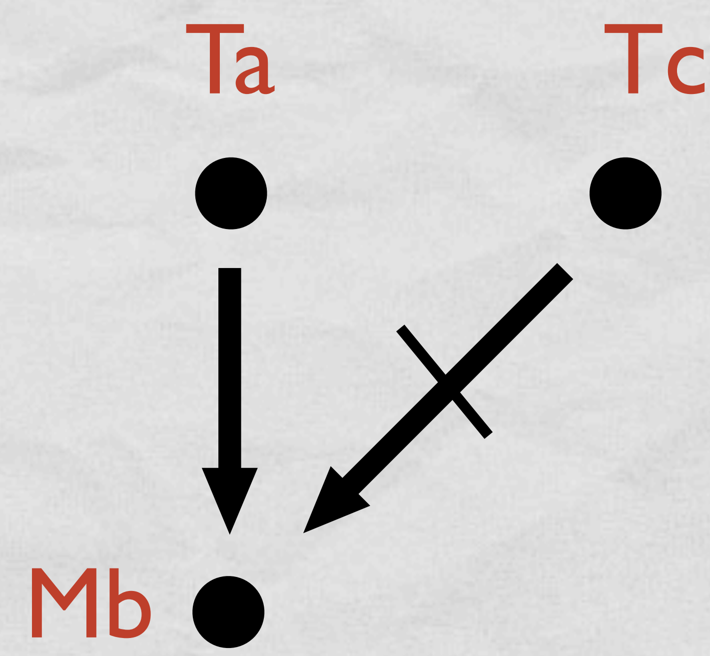
---

11. $T(c) \rightarrow \exists y(M(y) \wedge A(c,y))$ — $\forall$ Elim 1

12. $\exists y(M(y) \wedge A(c,y))$ — Taut Con 10,11

13. [b] $M(d) \wedge A(c,d)$

$\exists x \exists y(x \neq y \wedge M(x) \wedge M(y))$ — $\exists$ Elim + $\exists$ Elim + $\exists$ Elim + $\exists$ Elim



1. $\forall x(T(x) \rightarrow \exists y(M(y) \land A(x,y))$

2. $\forall x(M(x) \rightarrow \exists y(T(y) \land \neg A(y,x))$

3. $\exists x\, T(x)$

4. [a] $T(a)$

5. $T(a) \rightarrow \exists y(M(y) \land A(a,y)$ — $\forall$ Elim 1

6. $\exists y(M(y) \land A(a,y)$ — $\rightarrow$ Elim 4,5

7. [b] $M(b) \land A(a,b)$

8. $M(b) \rightarrow \exists y(T(y) \land \neg A(y,b)$ — $\forall$ Elim 2

9. $\exists y(T(y) \land \neg A(y,b)$ — Taut Con 7,8

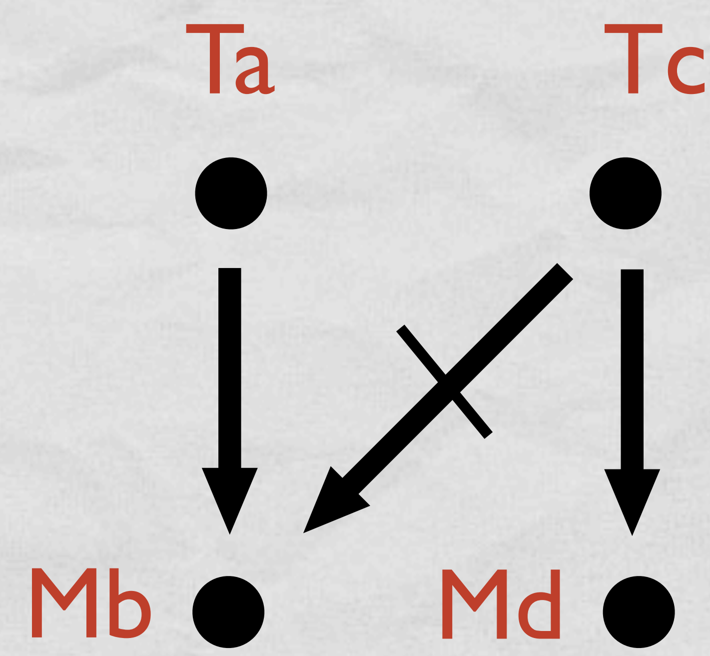
10. [c] $T(c) \land \neg A(c,b)$

11. $T(c) \rightarrow \exists y(M(y) \land A(c,y)$ — $\forall$ Elim 1

12. $\exists y(M(y) \land A(c,y)$ — Taut Con 10,11

13. [b] $M(d) \land A(c,d)$

$\exists x \exists y(x \neq y \land M(x) \land M(y))$ — $\exists$ Elim + $\exists$ Elim + $\exists$ Elim + $\exists$ Elim



1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y))$

2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge \neg A(y,x))$

3. $\exists x\, T(x)$

4. [a] $T(a)$

5. $T(a) \rightarrow \exists y(M(y) \wedge A(a,y)$ — $\forall$ Elim 1

6. $\exists y(M(y) \wedge A(a,y)$ — $\rightarrow$ Elim 4,5

7. [b] $M(b) \wedge A(a,b)$

8. $M(b) \rightarrow \exists y(T(y) \wedge \neg A(y,b)$ — $\forall$ Elim 2

9. $\exists y(T(y) \wedge \neg A(y,b)$ — Taut Con 7,8

10. [c] $T(c) \wedge \neg A(c,b)$

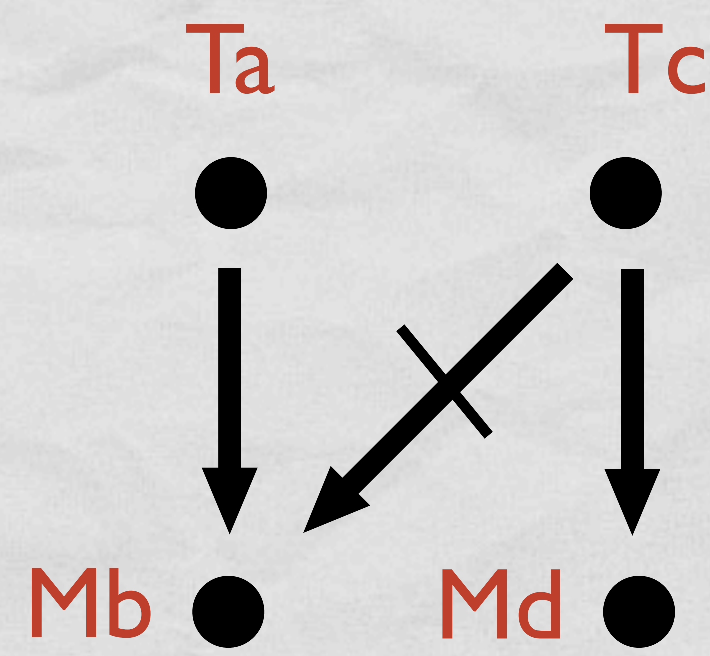
11. $T(c) \rightarrow \exists y(M(y) \wedge A(c,y)$ — $\forall$ Elim 1

12. $\exists y(M(y) \wedge A(c,y)$ — Taut Con 10,11

13. [b] $M(d) \wedge A(c,d)$

14. $b \neq d$ — NI 10,13 FO con

$\exists x \exists y(x \neq y \wedge M(x) \wedge M(y))$ — $\exists$ Elim + $\exists$ Elim + $\exists$ Elim + $\exists$ Elim



1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y)))$

2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge \neg A(y,x)))$

3. $\exists x\, T(x)$

4. [a] $T(a)$

5. $T(a) \rightarrow \exists y(M(y) \wedge A(a,y))$ — ∀ Elim 1

6. $\exists y(M(y) \wedge A(a,y))$ — → Elim 4,5

7. [b] $M(b) \wedge A(a,b)$

8. $M(b) \rightarrow \exists y(T(y) \wedge \neg A(y,b))$ — ∀ Elim 2

9. $\exists y(T(y) \wedge \neg A(y,b))$ — Taut Con 7,8

10. [c] $T(c) \wedge \neg A(c,b)$

11. $T(c) \rightarrow \exists y(M(y) \wedge A(c,y))$ — ∀ Elim 1

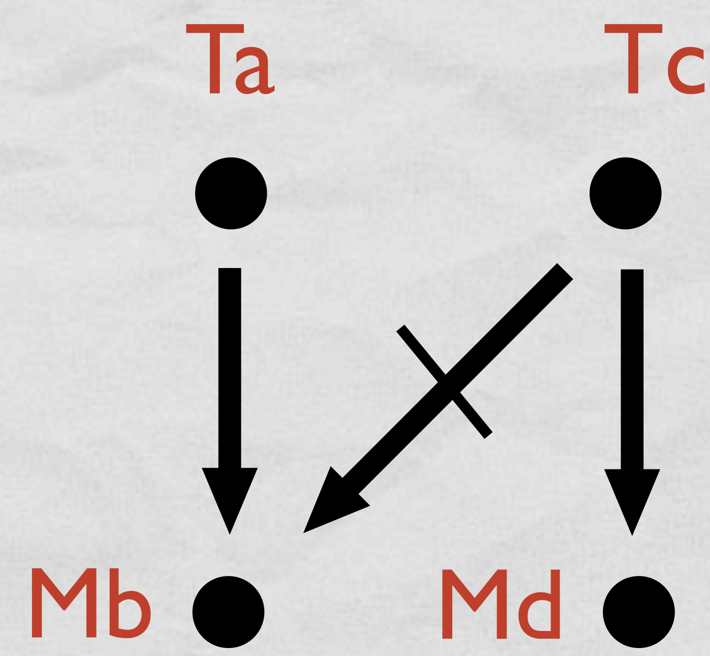
12. $\exists y(M(y) \wedge A(c,y))$ — Taut Con 10,11

13. [b] $M(d) \wedge A(c,d)$

14. $b \neq d$ — NI 10,13 FO con

15. $b \neq d \wedge M(b) \wedge M(d)$ — Taut Con 7,13, 15

$\exists x \exists y(x \neq y \wedge M(x) \wedge M(y))$ — ∃ Elim + ∃ Elim + ∃ Elim + ∃ Elim



1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y))$

2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge \neg A(y,x))$

3. $\exists x\, T(x)$

4. [a] $T(a)$

5. $T(a) \rightarrow \exists y(M(y) \wedge A(a,y)$ — $\forall$ Elim 1

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7. [b] $M(b) \wedge A(a,b)$

8. $M(b) \rightarrow \exists y(T(y) \wedge \neg A(y,b)$ — $\forall$ Elim 2

9. $\exists y(T(y) \wedge \neg A(y,b)$ — Taut Con 7,8

10. [c] $T(c) \wedge \neg A(c,b)$

11. $T(c) \rightarrow \exists y(M(y) \wedge A(c,y)$ — $\forall$ Elim 1

12. $\exists y(M(y) \wedge A(c,y)$ — Taut Con 10,11

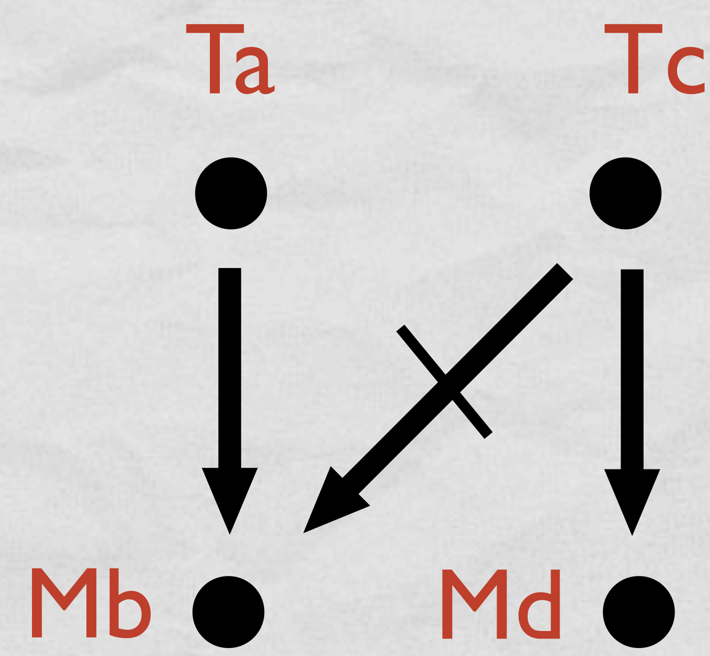
13. [b] $M(d) \wedge A(c,d)$

14. $b \neq d$ — NI 10,13 FO con

15. $b \neq d \wedge M(b) \wedge M(d)$ — Taut Con 7,13, 15

16. $\exists x \exists y(x \neq y \wedge M(x) \wedge M(y))$ — $\exists$ Intro 15 x2

$\exists x \exists y(x \neq y \wedge M(x) \wedge M(y))$ — $\exists$ Elim + $\exists$ Elim + $\exists$ Elim + $\exists$ Elim

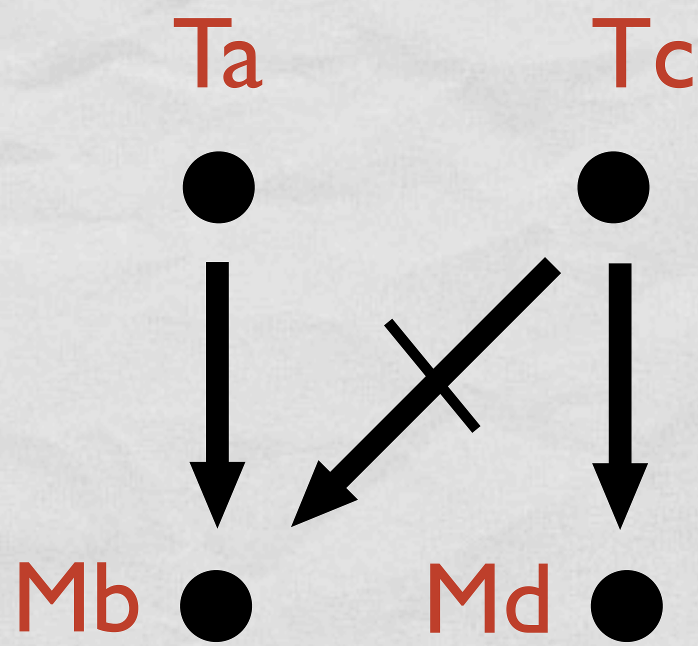


1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y))$

2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge \neg A(y,x))$

3. $\exists x\, T(x)$

$\vdash \exists x \exists y(x \neq y \wedge M(x) \wedge M(y))$

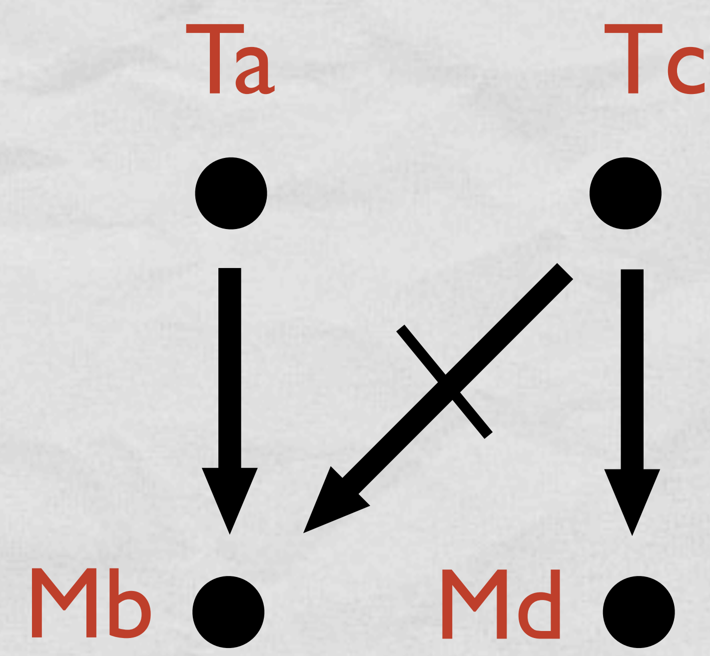


1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y))$

2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge \neg A(y,x))$

3. $\exists x\, T(x)$

$$\vdash \exists x \exists y(x \neq y \wedge M(x) \wedge M(y))$$



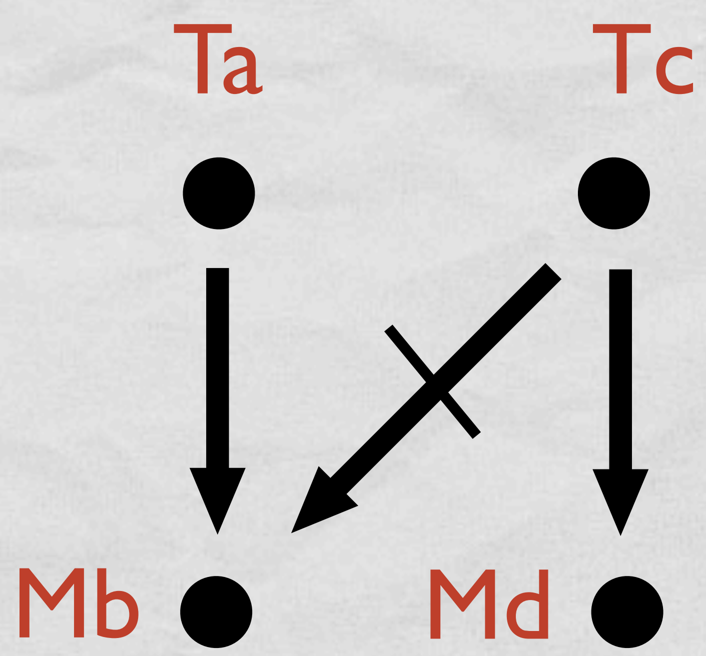
Since this diagram makes all these premises true, anything not true on this diagram doesn't follow from the premises

1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y))$

2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge \neg A(y,x))$

3. $\exists x\, T(x)$

$$\vdash \exists x \exists y(x \neq y \wedge M(x) \wedge M(y))$$



Since this diagram makes all these premises true, anything not true on this diagram doesn't follow from the premises

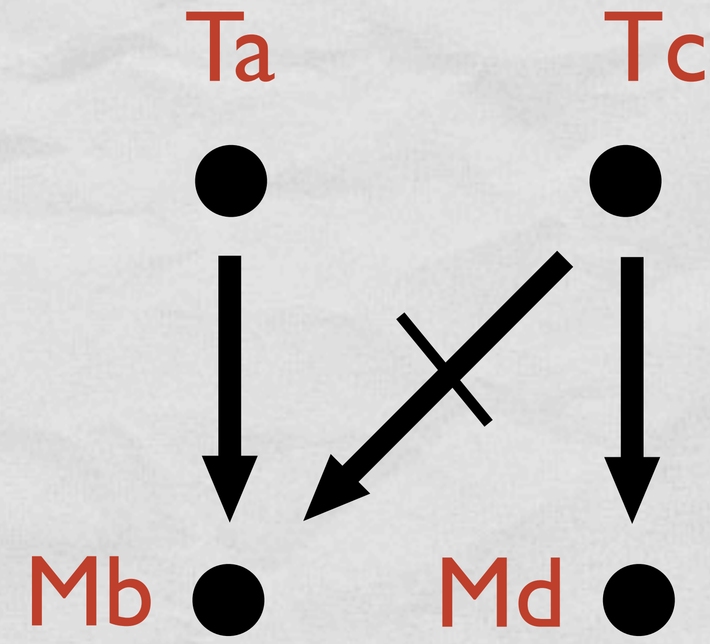
$$\nvdash \exists x \exists y \exists z(x \neq y \wedge y \neq z \wedge x \neq z \wedge M(x) \wedge M(y) \wedge M(z))$$

1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y))$

2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge \neg A(y,x))$

3. $\exists x\, T(x)$

$$\vdash \exists x \exists y(x \neq y \wedge M(x) \wedge M(y))$$



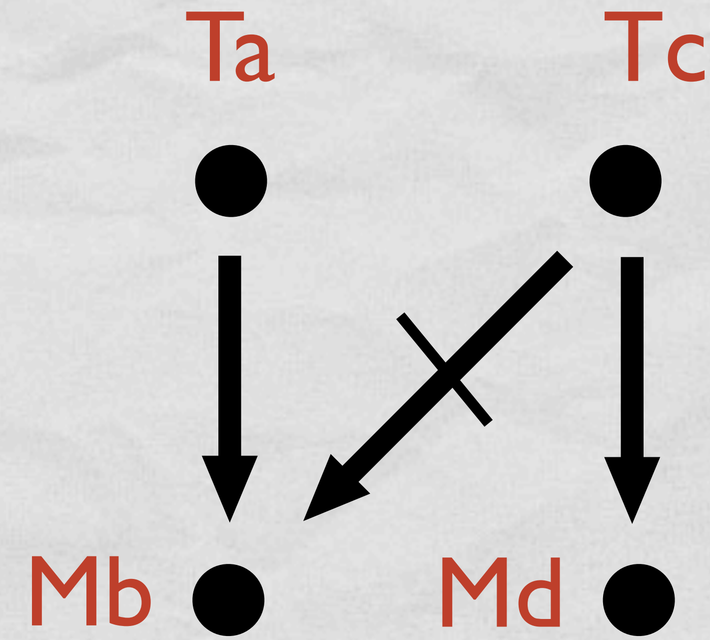
Since this diagram makes all these premises true, anything <u>not</u> true on this diagram doesn't follow from the premises

$$\nvdash \exists x \exists y \exists z(x \neq y \wedge y \neq z \wedge x \neq z \wedge M(x) \wedge M(y) \wedge M(z))$$

$$\nvdash \exists x \exists y \exists z(x \neq y \wedge y \neq z \wedge x \neq z \wedge T(x) \wedge T(y) \wedge T(z))$$

1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y))$
2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge \neg A(y,x))$
3. $\exists x\, T(x)$

$$\vdash \exists x \exists y(x \neq y \wedge M(x) \wedge M(y))$$



Since this diagram makes all these premises true, anything not true on this diagram doesn't follow from the premises

$$\nvdash \exists x \exists y \exists z(x \neq y \wedge y \neq z \wedge x \neq z \wedge M(x) \wedge M(y) \wedge M(z))$$

$$\nvdash \exists x \exists y \exists z(x \neq y \wedge y \neq z \wedge x \neq z \wedge T(x) \wedge T(y) \wedge T(z))$$

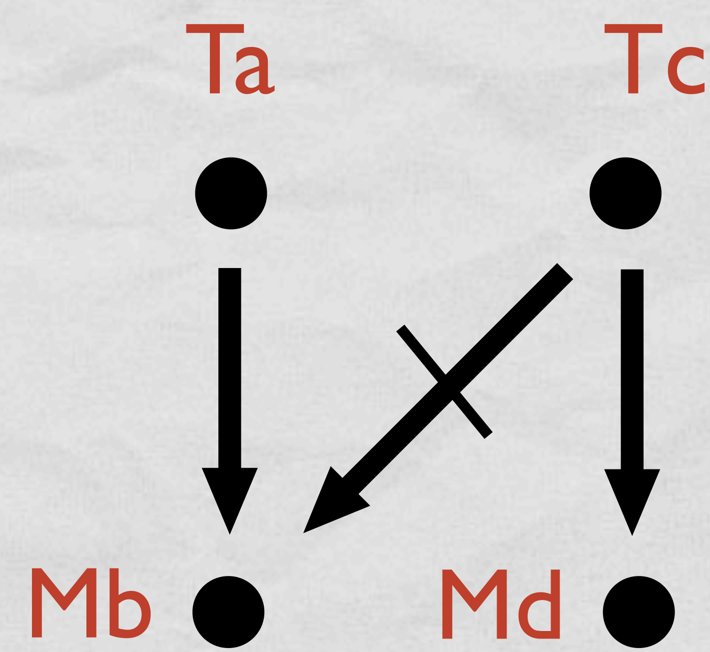
$$\nvdash \forall x \forall y((x \neq y \wedge T(x) \wedge T(y)) \rightarrow \exists z(M(z) \wedge A(x,z) \wedge A(y,z)))$$

1. $\forall x(T(x) \to \exists y(M(y) \wedge A(x,y))$

2. $\forall x(M(x) \to \exists y(T(y) \wedge \neg A(y,x))$

3. $\exists x\, T(x)$

$$\vdash \exists x \exists y(x \neq y \wedge M(x) \wedge M(y))$$



Since this diagram makes all these premises true, anything not true on this diagram doesn't follow from the premises

$$\nvdash \exists x \exists y \exists z(x \neq y \wedge y \neq z \wedge x \neq z \wedge M(x) \wedge M(y) \wedge M(z))$$

$$\nvdash \exists x \exists y \exists z(x \neq y \wedge y \neq z \wedge x \neq z \wedge T(x) \wedge T(y) \wedge T(z))$$

$$\nvdash \forall x \forall y((x \neq y \wedge T(x) \wedge T(y)) \to \exists z(M(z) \wedge A(x,z) \wedge A(y,z)))$$

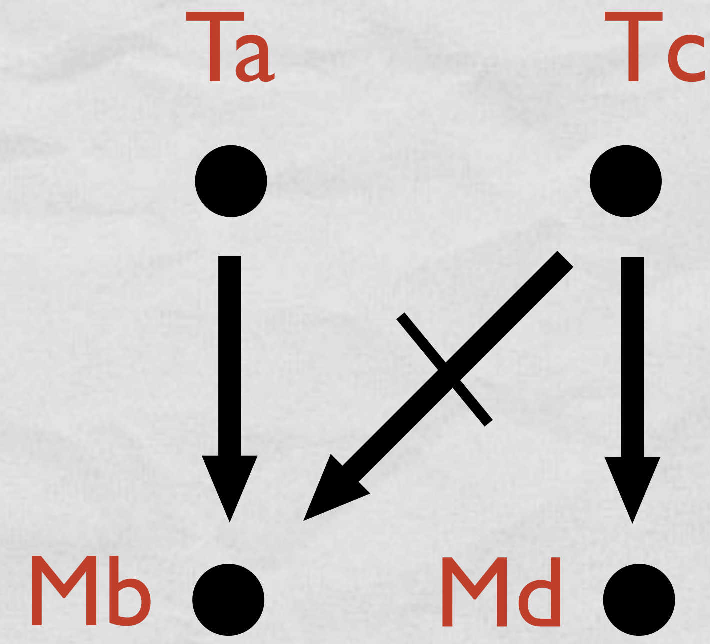
Anything we were forced to do with the diagram we can prove

1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y))$

2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge \neg A(y,x))$

3. $\exists x\, T(x)$

$$\vdash \exists x \exists y(x \neq y \wedge M(x) \wedge M(y))$$



Since this diagram makes all these premises true, anything <u>not</u> true on this diagram doesn't follow from the premises

$$\nvdash \exists x \exists y \exists z(x \neq y \wedge y \neq z \wedge x \neq z \wedge M(x) \wedge M(y) \wedge M(z))$$

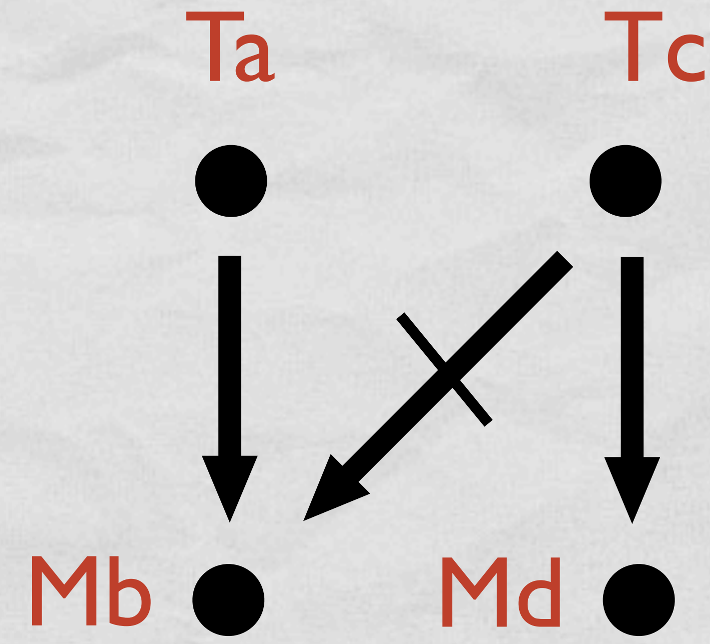
$$\nvdash \exists x \exists y \exists z(x \neq y \wedge y \neq z \wedge x \neq z \wedge T(x) \wedge T(y) \wedge T(z))$$

$$\nvdash \forall x \forall y((x \neq y \wedge T(x) \wedge T(y)) \rightarrow \exists z(M(z) \wedge A(x,z) \wedge A(y,z)))$$

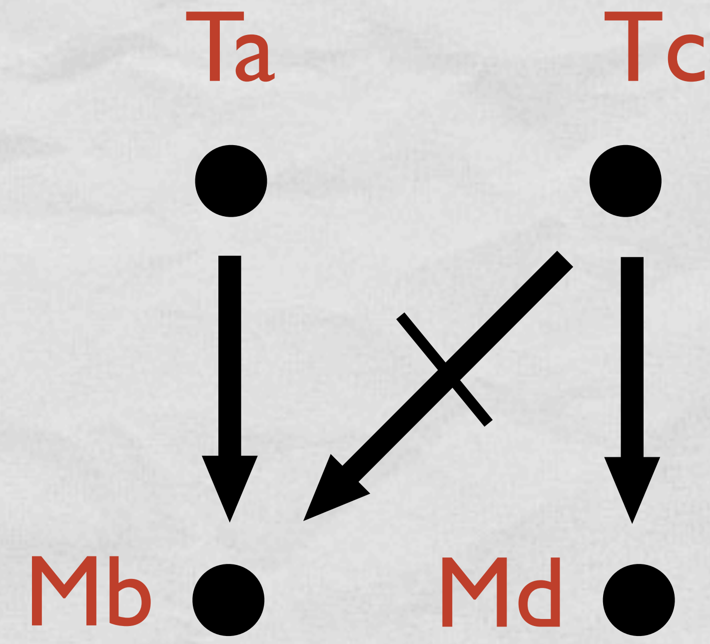
Anything we were <u>forced</u> to do with the diagram we <u>can</u> prove

$$\vdash \exists x \exists y(x \neq y \wedge T(x) \wedge T(y))$$

1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y))$
2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge \neg A(y,x))$
3. $\exists x\, T(x)$



1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y))$
2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge \neg A(y,x))$
3. $\exists x\, T(x)$

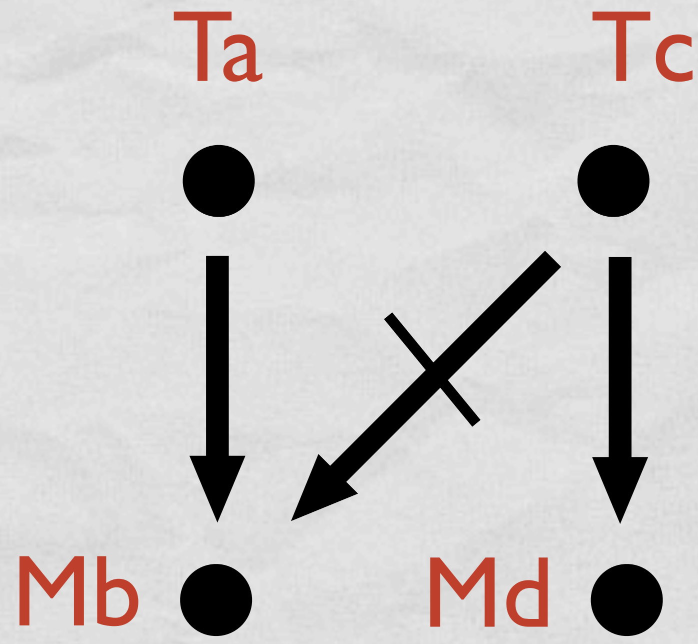


What else can we prove from these premises?

1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y)))$

2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge \neg A(y,x)))$

3. $\exists x\, T(x)$



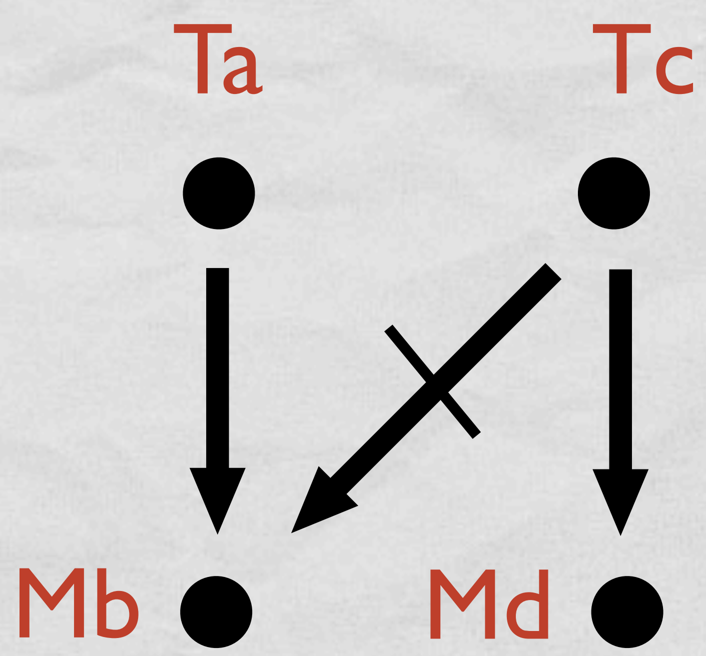
What else can we prove from these premises?

$$\vdash \forall x(T(x) \rightarrow \exists y(M(y) \wedge \neg A(x,y))$$?

1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y)))$

2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge \neg A(y,x)))$

3. $\exists x\, T(x)$



What else can we prove from these premises?

$$\vdash \forall x(T(x) \rightarrow \exists y(M(y) \wedge \neg A(x,y))) \quad ?$$

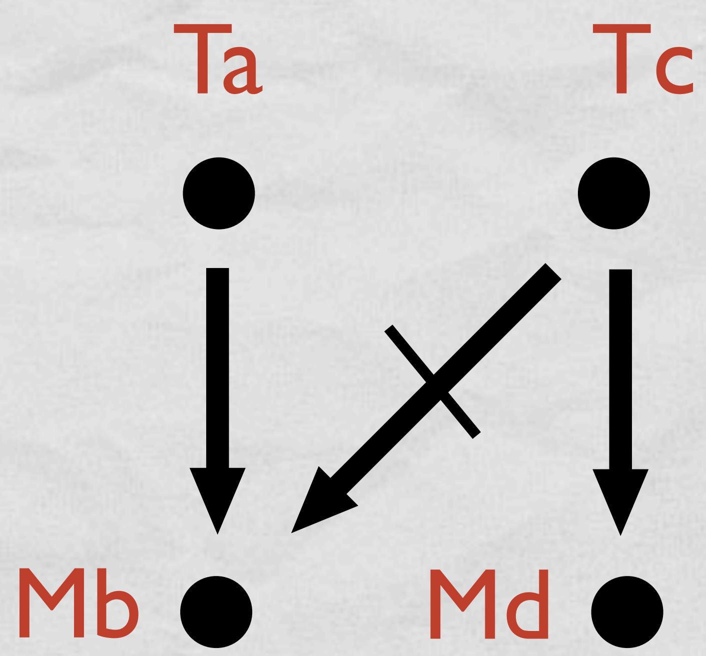
If we were sure there were only two teachers, we could safely add ¬R(a,d) to our diagram by P2 making '?' true. So

$$1\text{-}3 \vdash \forall x \forall y \forall z((T(x) \wedge T(y) \wedge T(z)) \rightarrow x=y \vee y=z \vee x=z) \rightarrow \forall x(T(x) \rightarrow \exists y(M(y) \wedge \neg A(x,y)))$$

1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y))$

2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge \neg A(y,x))$

3. $\exists x\, T(x)$



What else can we prove from these premises?

$$\vdash \forall x(T(x) \rightarrow \exists y(M(y) \wedge \neg A(x,y)) \quad ?$$

If we were sure there were only two teachers, we could safely add ¬R(a,d) to our diagram by P2 making '?' true. So

$$1\text{-}3 \vdash \forall x\forall y\forall z((T(x) \wedge T(y) \wedge T(z)) \rightarrow x=y \vee y=z \vee x=z) \rightarrow \forall x(T(x) \rightarrow \exists y(M(y) \wedge \neg A(x,y))$$

But $\nvdash \forall x\forall y\forall z((T(x) \wedge T(y) \wedge T(z)) \rightarrow x=y \vee y=z \vee x=z)$

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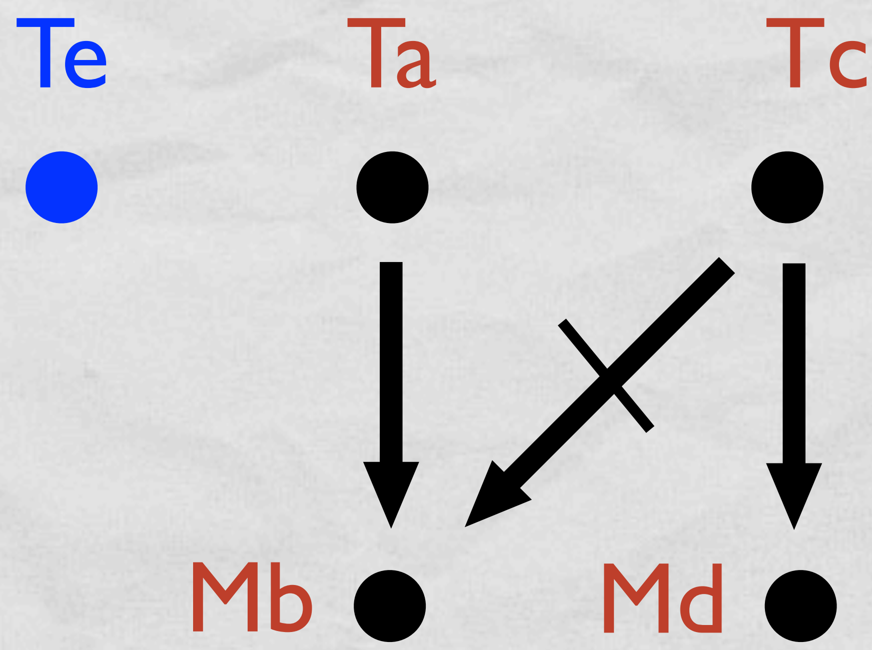

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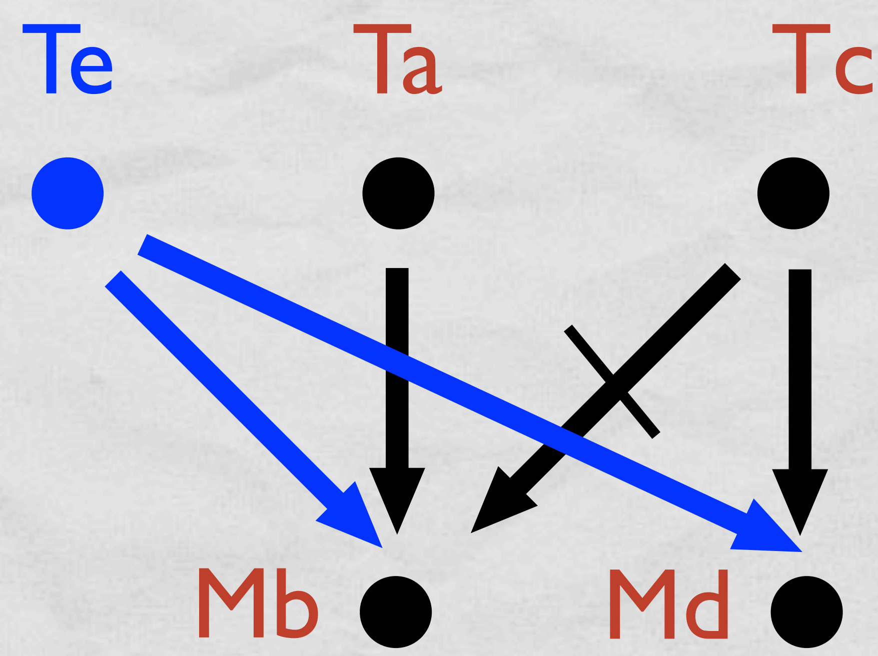

What else can we prove from these premises?

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If we were sure there were only two teachers, we could safely add ¬R(a,d) to our diagram by P2 making '?' true. So

$$\text{1-3} \vdash \forall x \forall y \forall z((T(x) \wedge T(y) \wedge T(z)) \rightarrow x=y \vee y=z \vee x=z) \rightarrow \forall x(T(x) \rightarrow \exists y(M(y) \wedge \neg A(x,y))$$

But $\nvdash \forall x \forall y \forall z((T(x) \wedge T(y) \wedge T(z)) \rightarrow x=y \vee y=z \vee x=z)$