THE HARDEST LOGIC PUZZLE EVER

Three gods A, B, and C are called, in some order, True, False, and Random. True always speaks truly, False always speaks falsely, but whether Random speaks truly or falsely is a completely random matter. Your task is to determine the identities of A, B, and C by asking three yes/no questions; each question must be put to exactly one god. The gods understand English, but will answer all questions in their own language, in which the words for *yes* and *no* are 'da' and 'ja', in some order. You do not know which word means which.

Solution - break it into steps

First step (really, last...)

If you know you are talking to a knight (who will answer ‘Bal’ or ‘Da’) how can you determine X?

If you know you are talking to a knave (who will answer ‘Bal’ or ‘Da’) how can you determine X?

If you know you are talking to a normal (who will answer ‘Bal’ or ‘Da’) what <u>can</u> you determine?

SOLUTION - BREAK IT INTO STEPS

First step (really, last...)

If you know you are talking to a knight (who will answer 'Bal' or 'Da') how can you determine X?

Does "'Bal' means yes" have the same truth value as X?

The knight will answer 'Bal' iff X is true.

Solution - break it into steps

First step (really, last...)

If you know you are talking to a knave (who will answer ‘Bal’ or ‘Da’) how can you determine X?

Same question: Does “‘Bal’ means yes” have the same truth value as X?

The knave will answer ‘Bal’ iff X is false.

USING AND BUILDING DIAGRAMS

Monday, 8 November

Diagrams

- Since a diagram is an interpretation, if any diagram can make all the premises of an argument true but the conclusion false, that argument is invalid.
- Diagrams can also be used as ‘guides’ to what can be proved from a set of premises. If you are forced to add something to a diagram, then you could prove that it follows (and sometimes the diagram helps you figure out how).

DIAGRAMS AS COUNTERMODELS

P1. $\forall x \exists y\ R(x,y)$

Conc. $\forall x \exists y\ R(y,x)$

Valid?

One strategy: Can in be falsified with one thing?
How about two? Three? .....

Falsify the conclusion: $\neg \forall x \exists y\ R(y,x)$

$\Leftrightarrow \exists x \forall y\ \neg R(y,x)$

We need a point like this - let's call it 'a'.

Diagrams as Countermodels

P1. $\forall x \exists y \; R(x,y)$

¬ Conc: $\exists x \forall y \; \neg R(y,x)$

Can we make both true?

But this makes P1 false

P1: Everything has to point somewhere

We can’t add R(a,a) - ‘a’ is supposed to be the one that nothing points to (from the conclusion)

So we need another point

Diagrams as Countermodels

P1. $\forall x \exists y\, R(x,y)$

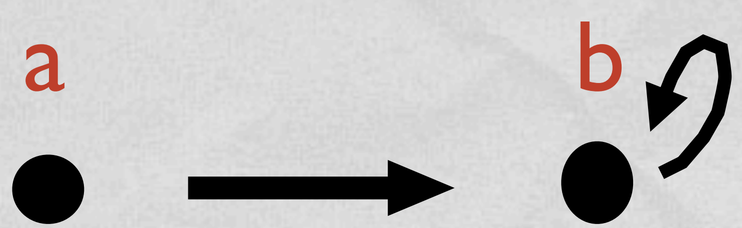
$\neg$ Conc: $\exists x \forall y\, \neg R(y,x)$

Can we make both true?

Problem: Now b needs to point somewhere.

It can't point to a.

The argument is invalid



Diagrams as Countermodels

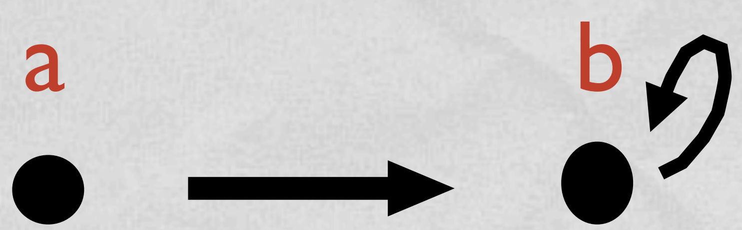
P1. $\forall x \exists y\ R(x,y)$

P2. $\exists x \forall y\ \neg R(y,x)$

Conc: $\exists x \exists y\ (x \neq y)$

On the other hand, we do know that this is valid

We were forced to add a second point in order to make the first two sentences true.



Diagrams as Countermodels

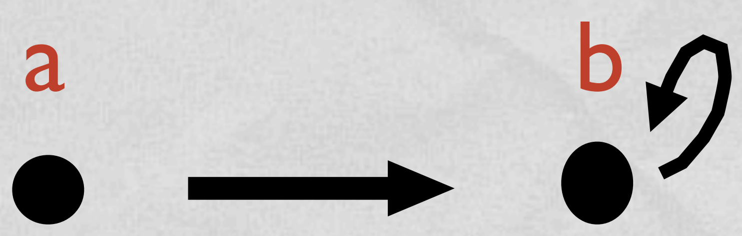
P1. $\forall x \exists y\ R(x,y)$

P2. $\exists x \forall y\ \neg R(y,x)$

Conc: $\exists x\ R(x,x)$

What about this?

b did have to point somewhere.
But we weren't <u>forced</u> to add R(b,b)



DIAGRAMS AS COUNTERMODELS

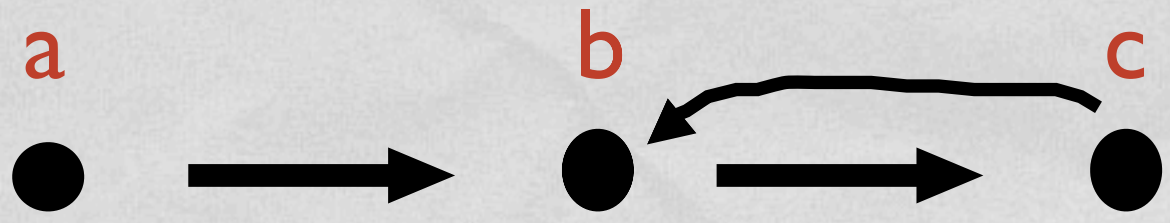
P1. $\forall x \exists y\ R(x,y)$

P2. $\exists x \forall y\ \neg R(y,x)$

Conc: $\exists x\ R(x,x)$

Now c has to point somewhere

So this argument is also invalid



Diagrams for Proofs

P1. $\forall x \exists y\, R(x,y)$

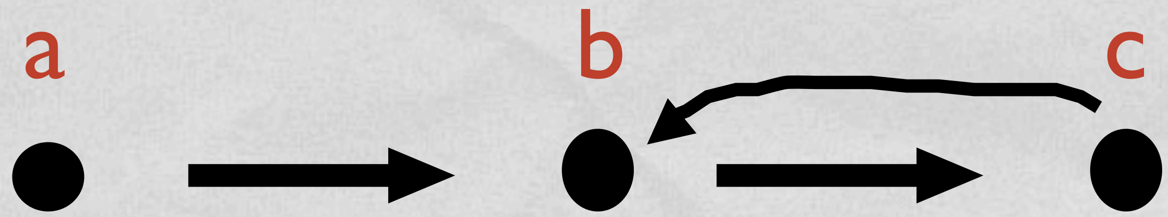
P2. $\exists x \forall y\, \neg R(y,x)$

P3. $\forall x\, \neg R(x,x)$

Conc: $\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z)$

We can prove this

Think about how we generated the diagram



1. $\forall x \exists y\, R(x,y)$

2. $\exists x \forall y\, \neg R(y,x)$

3. $\forall x\, \neg R(x,x)$

4. $\boxed{a}$ $\forall y\, \neg R(y,a)$

5. $\exists y\, R(a,y)$ — $\forall$ Elim 1

6. $\boxed{b}$ $R(a,b)$

7. $\neg R(a,a)$ — $\forall$ Elim 3

8. $a \neq b$ — NI 6,7 — FO con

9. $\exists y\, R(b,y)$ — $\forall$ Elim 1

10. $\boxed{c}$ $R(b,c)$

$\exists x \exists y \exists z(x \neq y \wedge y \neq z \wedge x \neq z)$

$\exists x \exists y \exists z(x \neq y \wedge y \neq z \wedge x \neq z)$ — $\exists$ Elim

$\exists x \exists y \exists z(x \neq y \wedge y \neq z \wedge x \neq z)$ — $\exists$ Elim

$\exists x \exists y \exists z(x \neq y \wedge y \neq z \wedge x \neq z)$ — $\exists$ Elim

6. $\boxed{b}$ $R(a,b)$

7. $\neg R(a,a)$ — $\forall$ Elim 3

8. $a \neq b$ — NI 6,7 — FO con

9. $\exists y\, R(b,y)$ — $\forall$ Elim 1

10. $\boxed{c}$ $R(b,c)$

11. $\neg R(b,b)$ — $\forall$ Elim 3

12. $b \neq c$ — NI 10,11 — FO con

13. $\neg R(b,a)$ — $\forall$ Elim 4

14. $a \neq c$ — NI 10,13 — FO con

15. $a \neq b \wedge b \neq c \wedge a \neq c$ — Taut Con 8,12,14

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z)$

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z)$ — $\exists$ Elim

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z)$ — $\exists$ Elim

$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z)$ — $\exists$ Elim

6. [b] R(a,b)

7. ¬R(a,a) — ∀ Elim 3

8. a≠b — NI 6,7 — FO con

9. ∃y R(b,y) — ∀ Elim 1

10. [c] R(b,c)

11. ¬R(b,b) — ∀ Elim 3

12. b≠c — NI 10,11 — FO con

13. ¬R(b,a) — ∀ Elim 4

14. a≠c — NI 10,13 — FO con

15. a≠b ∧ b≠c ∧ a≠c — Taut Con 8,12,14

16. ∃x∃y∃z(x≠y ∧ y≠z ∧ x≠z) — ∃ Intro x3 15

17. ∃x∃y∃z(x≠y ∧ y≠z ∧ x≠z) — ∃ Elim 9,10-16

18. ∃x∃y∃z(x≠y ∧ y≠z ∧ x≠z) — ∃ Elim 5,6-17

19. ∃x∃y∃z(x≠y ∧ y≠z ∧ x≠z) — ∃ Elim 2, 4-18