Puzzle

On a special island populated by knights and knaves, the natives understand English perfectly, but they only answer questions in their own language. "Bal" and "Da" mean "Yes" and "No", but you don't know which is which.

1) In one question, find out what "Bal" means

2) Ask a question that you know any speaker will answer "Bal"

Puzzle Answer

1) Are you a knight? Both knights and knaves say "yes" so they say "Bal" if and only if "Bal" means "yes"

2) Are you a knight if and only if "Bal" means "yes"?

If knight and Bal=yes, then true so says yes=Bal
If knight and Bal=no, then false so says no=Bal
If knave and Bal=yes, then false so says yes=Bal
If knave and Bal=no, then true so says no=Bal

Testing Validity with Tarski's World

Wednesday, 27 October

1. $\forall x \forall y \forall z([R(x,y) \land R(x,z)] \rightarrow R(y,z))$

2. $\forall x\ R(x,x)$

3. [a]

4. [b]

5. $R(a,b)$

6. $R(x,b) \land R(x,a)] \rightarrow R(b,a)$ $\forall$ Elim x3

Want R(b,a) here

So make x = a

We can pick any x we want here

$R(b,a))$

$R(a,b) \rightarrow R(b,a)$ $\rightarrow$ Intro

$\forall y(R(a,y) \rightarrow R(y,a))$ $\forall$ Intro

$\forall x \forall y(R(x,y) \rightarrow R(y,x))$ $\forall$ Intro

1. $\forall x \forall y \forall z([R(x,y) \land R(x,z)] \rightarrow R(y,z))$

2. $\forall x\ R(x,x)$

3. a

4. b

5. $R(a,b)$

6. $R(a,b) \land R(a,a)] \rightarrow R(b,a)$ — $\forall$ Elim x3

7. $R(a,a)$ — $\forall$ Elim 2

8. $R(b,a)$ — Taut Con 5,6,7

$R(b,a))$

$R(a,b) \rightarrow R(b,a)$ — $\rightarrow$ Intro

$\forall y(R(a,y) \rightarrow R(y,a))$ — $\forall$ Intro

$\forall x \forall y(R(x,y) \rightarrow R(y,x))$ — $\forall$ Intro

1. $\forall x \forall y \forall z([R(x,y) \wedge R(x,z)] \rightarrow R(y,z))$
2. $\forall x\ R(x,x)$

3. $\boxed{a}$

4. $\boxed{b}$

5. $R(a,b)$

6. $R(a,b) \wedge R(a,a)] \rightarrow R(b,a)$ — $\forall$ Elim x3

7. $R(a,a)$ — $\forall$ Elim 2

8. $R(b,a)$ — Taut Con 5,6,7

9. $R(a,b) \rightarrow R(b,a)$ — $\rightarrow$ Intro 5-8

10. $\forall y(R(a,y) \rightarrow R(y,a))$ — $\forall$ Intro 4-9

11. $\forall x \forall y(R(x,y) \rightarrow R(y,x))$ — $\forall$ Intro 5-10

INFORMAL SEMANTICS

- An argument is invalid, if there is a way for the premises to be true and the conclusion false.
- If every TVA that makes the premises true also makes the conclusion true, it is t-f valid (and so *really* valid) but if not, it is not necessarily invalid.
- Example: $\forall x\ P(x)$ therefore $\exists y\ P(y)$ is not t-f valid but it is FO valid.
- FO valid means that every interpretation that makes the premises true also makes the conclusion true.

Informal Semantics

- FO invalid means that there is some interpretation that makes all the premises true and the conclusion false. An interpretation gives the meaning of the constants, functions, and predicates and gives a domain (so we know what 'for all x' means.
- $\exists x\, P(x)$ does not FO entail $\forall x\, P(x)$
- Domain: Natural numbers {0,1,2 .... }, P(x) = x is even
- Alternate interpretation: Domain: All people, P(x) = x is male.

Tarski's World Interpretations

- Tarski's world can illustrate some interpretations. [But not all! If Tarski's world can't falsify it, it doesn't mean FO valid]
- Example: $\exists x$ Cube(x) does not FO entail $\forall x$ Cube(x)
- Domain: Objects in the picture on the screen, Cube(x) = x is a cube.
- If you can make the premises true and conclusion false in a Tarski world, then the argument is *really* invalid. If you can make a "suitable translation" that shows invalidity, it is FO invalid.

Tarski's World "Suitable Translations"

- $\exists x\, P(x)$ does not FO entail $\forall x\, P(x)$
- Domain: Objects in the picture on the screen, P(x) = x is a cube.
- $\exists x$ SelfIdentical(x) does not FO entail $\forall x$ SelfIdentical(x) but it does *really* entail it since the latter is a necessary truth.
- FO validity completely ignores the meaning of the predicates. LPL talks about "non-sense" predicates. I like "P", "Q", "R", etc. Replace predicates with arbitrary letters like this to test FO validity.

Testing Validity using Tarski's World

$\forall x(\text{Cube}(x) \lor \text{Tet}(x))$

$\exists x(\text{Small}(x) \land \neg\text{Cube}(x))$

$\forall x(\text{Tet}(x) \rightarrow \text{Small}(x))$

Valid or not?

Try to make premises true and conclusion false. If you succeed, it is definitely not valid.

Testing Validity using Tarski's World

$\forall x(P(x) \vee Q(x))$
$\forall x(P(x) \rightarrow \neg R(x))$
$\exists x\, R(x)$

$\exists x(Q(x) \wedge S(x))$

Valid or not?

Lets try Px = x is a cube
Qx = x is a dodec
Rx = x is a tet
Sx = x is small

Px = x is a cube
Qx = x is a dodec
Rx = x is large
Sx = x is small

Testing Validity using Tarski's World

| SameRow(a,b)
|---
| SameRow(b,a)

Valid or not?

Can't make T, F in Tarski's World. But this clearly depends on the meaning of SameRow. S(a,b) therefore S(b,a) is not FO valid.

What if "SameRow(x,y)" meant RightOf(x,y)?

TESTING VALIDITY USING TARSKI'S WORLD

$\forall x(\text{Cube}(x) \lor x=a)$

$\exists x\ \text{Small}(x)$

Valid or not?

$\forall x(x=a \rightarrow (\text{Dodec}(x) \lor \text{Small}(x)))$

Translations with Identity

$\forall x(Cube(x) \lor x=a) \Leftrightarrow \forall x(\neg Cube(x) \rightarrow x=a)$

If like this then you are =a

a is the only non-cube (if there is one)

$\forall x(x=a \rightarrow (Dodec(x) \lor Small(x)))$

If you are =a then you are like this

a is either a dodec or is small

Translations with Identity

$\forall x(x{=}a \rightarrow P(x)) \quad \Leftrightarrow \quad P(a)$

$\exists x(x{=}a \land P(x)) \quad \Leftrightarrow \quad P(a)$

therefore

$\vdash \forall x(x{=}a \rightarrow P(x)) \leftrightarrow \exists x(x{=}a \land P(x))$

1. $\forall x(x=a \rightarrow P(x))$

2. $a=a \rightarrow P(a)$ $\forall$ Elim 1

3. $a=a$ = Intro

4. $a=a \wedge P(a)$ Taut Con 2,3

$\exists x(x=a \wedge P(x))$

$\exists x(x=a \wedge P(x))$

$\forall x(x=a \rightarrow P(x))$

$\forall x(x=a \rightarrow P(x)) \leftrightarrow \exists x(x=a \wedge P(x))$ $\leftrightarrow$ Intro

1. $\forall x(x=a \rightarrow P(x))$

2. $a=a \rightarrow P(a)$ — $\forall$ Elim 1

3. $a=a$ — $=$ Intro

4. $a=a \wedge P(a)$ — Taut Con 2,3

5. $\exists x(x=a \wedge P(x))$ — $\exists$ Intro 4

6. $\exists x(x=a \wedge P(x))$

$\forall x(x=a \rightarrow P(x))$

$\forall x(x=a \rightarrow P(x)) \leftrightarrow \exists x(x=a \wedge P(x))$ — $\leftrightarrow$ Intro

6. ∃x(x=a ∧ P(x))

7. [b] b=a ∧ P(b)

8. [c]

9. c=a

10. b=c ∧ P(c) = Elim 7,9

P(c)

c=a → P(c) → Intro

∀x(x=a → P(x)) ∀ Intro

∀x(x=a → P(x)) ∃ Elim

∀x(x=a → P(x)) ↔ ∃x(x=a ∧ P(x)) ↔ Intro

6. $\exists x(x=a \land P(x))$

7. [b] $b=a \land P(b)$

8. [c]

9. $c=a$

10. $b=c \land P(c)$ = Elim 7,9

11. $P(c)$ $\land$ Elim 10

12. $c=a \rightarrow P(c)$ $\rightarrow$ Intro 9-11

13. $\forall x(x=a \rightarrow P(x))$ $\forall$ Intro 8-12

14. $\forall x(x=a \rightarrow P(x))$ $\exists$ Elim 6,7-13

15. $\forall x(x=a \rightarrow P(x)) \leftrightarrow \exists x(x=a \land P(x))$ $\leftrightarrow$ Intro 1-5, 6-14