Puzzle

On a special island populated by knights and knaves, the natives understand English perfectly, but they only answer questions in their own language. "Bal" and "Da" mean "Yes" and "No", but you don't know which is which.

You asked a native "Does Bal mean 'Yes'", and he said 'Bal'. Can you infer what "Bal" means? Was the speaker a knight or a knave?

Puzzle Answer

It is not possible to tell what "Bal" means, but we can tell that the speaker must be a knight.

Suppose "Bal" means yes. Then "Bal" is the truthful answer to the question whether "Bal" means yes. So in this case, the speaker is a knight.

Suppose "Bal" means no. Then "No" is the truthful English answer to the question whether "Bal" means yes, therefore "Bal" is the truthful native answer to the question. So again, the speaker is a knight.

Multiple Quanitifiers

Monday, 25 October

PROOFS WITH MULTIPLE QUANTIFIERS

1. $\forall x \forall y(R(x,y) \rightarrow S(y,x))$
2. $\forall x \forall y(S(x,y) \rightarrow T(y,x))$

$\forall x \forall y(R(x,y) \rightarrow T(x,y))$

1. $\forall x \forall y(R(x,y) \to S(y,x))$

2. $\forall x \forall y(S(x,y) \to T(y,x))$

3. a

4. b

5. $R(a,b)$

6. $\forall y(R(a,y) \to S(y,a)$	$\forall$ Elim 1
7. $R(a,b) \to S(b,a)$	$\forall$ Elim 6
8. $S(b,a)$	$\to$ Elim 5,7
9. $\forall y(S(b,y) \to T(y,b))$	$\forall$ Elim 2
10. $S(b,a) \to T(a,b))$	$\forall$ Elim 9
$T(a,b))$	
$R(a,b) \to T(a,b)$	$\to$ Intro
$\forall y(R(a,y) \to T(a,y))$	$\forall$ Intro
$\forall x \forall y(R(x,y) \to T(x,y))$	$\forall$ Intro

1.	$\forall x \forall y (R(x,y) \rightarrow S(y,x))$	
2.	$\forall x \forall y (S(x,y) \rightarrow T(y,x))$	
3.	$\boxed{a}$	
4.	$\boxed{b}$	
5.	$R(a,b)$	
6.	$\forall y (R(a,y) \rightarrow S(y,a))$	$\forall$ Elim 1
7.	$R(a,b) \rightarrow S(b,a)$	$\forall$ Elim 6
8.	$S(b,a)$	$\rightarrow$ Elim 5,7
9.	$\forall y (S(b,y) \rightarrow T(y,b))$	$\forall$ Elim 2
10.	$S(b,a) \rightarrow T(a,b))$	$\forall$ Elim 9
11.	$T(a,b))$	$\rightarrow$ Elim 8,10
12.	$R(a,b) \rightarrow T(a,b)$	$\rightarrow$ Intro 5-11
13.	$\forall y (R(a,y) \rightarrow T(a,y))$	$\forall$ Intro 4-12
14.	$\forall x \forall y (R(x,y) \rightarrow T(x,y))$	$\forall$ Intro 3-13

1. $\forall x(A(x) \rightarrow \forall y(B(y) \rightarrow D(x,y)))$

Everyone on Team A defeated everyone on Team B

$\forall x \forall y[(A(x) \land B(y)) \rightarrow D(x,y)]$

$\forall x \forall y[(B(x) \land A(y)) \rightarrow D(y,x)]$

Everyone on Team B was defeated by everyone on Team A

$\forall x(B(x) \rightarrow \forall y(A(y) \rightarrow D(y,x)))$

1. $\forall x(A(x) \rightarrow \forall y(B(y) \rightarrow D(x,y)))$

2. $\boxed{b}$

3. $B(b)$

4. $\boxed{a}$

5. $A(a)$

6. $A(a) \rightarrow \forall y(B(y) \rightarrow D(a,y))$ $\forall$ Elim 1

7. $\forall y(B(y) \rightarrow D(a,y))$ $\rightarrow$ Elim 5,6

8. $B(b) \rightarrow D(a,b)$ $\forall$ Elim 7

$D(a,b)$

$A(a) \rightarrow D(a,b)$ $\rightarrow$ Intro

$\forall y(A(y) \rightarrow D(y,b))$ $\forall$ Intro

$B(b) \rightarrow \forall y(A(y) \rightarrow D(y,b))$ $\rightarrow$ Intro

$\forall x(B(x) \rightarrow \forall y(A(y) \rightarrow D(y,x)))$ $\forall$ Intro

1.	$\forall x(A(x) \to \forall y(B(y) \to D(x,y)))$	
2.	$\boxed{b}$	
3.	$B(b)$	
4.	$\boxed{a}$	
5.	$A(a)$	
6.	$A(a) \to \forall y(B(y) \to D(a,y))$	$\forall$ Elim 1
7.	$\forall y(B(y) \to D(a,y))$	$\to$ Elim 5,6
8.	$B(b) \to D(a,b)$	$\forall$ Elim 7
9.	$D(a,b)$	$\to$ Elim 3,8
10.	$A(a) \to D(a,b)$	$\to$ Intro 5-9
11.	$\forall y(A(y) \to D(y,b))$	$\forall$ Intro 10
12.	$B(b) \to \forall y(A(y) \to D(y,b))$	$\to$ Intro 3-11
13.	$\forall x(B(x) \to \forall y(A(y) \to D(y,x)))$	$\forall$ Intro 12

1. $\forall x \forall y(R(x,y) \vee R(y,x))$

2. $\boxed{a}$

3. $\forall y(R(a,y) \vee R(y,a))$ $\forall$ Elim 1

4. $R(a,a) \vee R(a,a)$ $\forall$ Elim 2

$R(a,a)$

$\forall x\, R(x,x)$ $\forall$ Intro

1. $\forall x \forall y(R(x,y) \vee R(y,x))$

2. a

3. $\forall y(R(a,y) \vee R(y,a))$ — $\forall$ Elim 1

4. $R(a,a) \vee R(a,a)$ — $\forall$ Elim 2

5. $R(a,a)$ — Taut Con

6. $\forall x\, R(x,x)$ — $\forall$ Intro 5

1. $\forall x \forall y(R(x,y) \rightarrow S(y,x))$

2. $\exists x \exists y(\neg S(x,y) \wedge Q(x,y))$

3. [a] $\exists y(\neg S(a,y) \wedge Q(a,y))$

4. [b] $\neg S(a,b) \wedge Q(a,b)$

5. $R(b,a) \rightarrow S(a,b)$ — $\forall$ Elim 1 x2 [:x>b :y>a]

6. $\neg R(b,a)$ — Taut Con 4,5

7. $\neg R(b,a) \wedge Q(a,b)$ — Taut Con 4,6

8. $\exists y(\neg R(b,y) \wedge Q(y,b))$ — $\exists$ Intro 7

$\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$

$\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ — $\exists$ Elim

$\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ — $\exists$ Elim

1. $\forall x \forall y(R(x,y) \rightarrow S(y,x))$

2. $\exists x \exists y(\neg S(x,y) \wedge Q(x,y))$

3. $\boxed{a}$ $\exists y(\neg S(a,y) \wedge Q(a,y))$

4. $\boxed{b}$ $\neg S(a,b) \wedge Q(a,b)$

5. $R(b,a) \rightarrow S(a,b)$ — $\forall$ Elim 1 x2 [:x>b :y>a]

6. $\neg R(b,a)$ — Taut Con 4,5

7. $\neg R(b,a) \wedge Q(a,b)$ — Taut Con 4,6

8. $\exists y(\neg R(b,y) \wedge Q(y,b))$ — $\exists$ Intro 7

9. $\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ — $\exists$ Intro 8

10. $\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ — $\exists$ Elim 3,4-9

11. $\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ — $\exists$ Elim 2, 3-10

1. $\exists x \forall y(\exists z\, R(y,z) \rightarrow R(y,x))$

2. $\forall x \exists y\, R(x,y)$

3. $\boxed{a}$ $\forall y(\exists z\, R(y,z) \rightarrow R(y,a))$

4. $\boxed{b}$

5. $\exists z\, R(b,z) \rightarrow R(b,a)$ — $\forall$ Elim 3

6. $\exists y\, R(b,y)$ — $\forall$ Elim 2

7. $\boxed{c}$ $R(b,c)$

8. $\exists z\, R(b,z)$ — $\exists$ Intro 7

$\exists z\, R(b,z)$

$R(b,a)$ — $\rightarrow$ Elim

$\forall y\, R(y,a)$ — $\forall$ Intro

$\exists x \forall y\, R(y,x)$ — $\exists$ Intro

$\exists x \forall y\, R(y,x)$ — $\exists$ Elim

1.	$\exists x \forall y(\exists z\, R(y,z) \rightarrow R(y,x))$	
2.	$\forall x \exists y\, R(x,y)$	
3.	[a] $\forall y(\exists z\, R(y,z) \rightarrow R(y,a))$	
4.	[b]	
5.	$\exists z\, R(b,z) \rightarrow R(b,a)$	$\forall$ Elim 3
6.	$\exists y\, R(b,y)$	$\forall$ Elim 2
7.	[c] $R(b,c)$	
8.	$\exists z\, R(b,z)$	$\exists$ Intro 7
9.	$\exists z\, R(b,z)$	$\exists$ Elim 6,7-8
10.	$R(b,a)$	$\rightarrow$ Elim 5,9
11.	$\forall y\, R(y,a)$	$\forall$ Intro 10
12.	$\exists x \forall y\, R(y,x)$	$\exists$ Intro 11
13.	$\exists x \forall y\, R(y,x)$	$\exists$ Elim 1,3-12