Puzzle

It might seem impossible that a human being could count to 10^20 in their lifetime, but what is wrong with the following argument?

1. It is possible for a human to count to 100.
2. If it is possible to count to x, then it is possible to count to x+1.

Therefore 3. It is possible to count to 10^20.

Note that $\forall x(\text{CanCount}(x) \rightarrow \text{CanCount}(x+1)) \Leftrightarrow \neg\exists x(\text{CanCount}(x) \wedge \neg\text{CanCount}(x+1))$

Multiple Quanitifiers

Friday, 22 October

Quantifiers and Conditionals

$\forall x(P \vee Q(x)) \Leftrightarrow P \vee \forall x\, Q(x)$

and

$\exists x(P \vee Q(x)) \Leftrightarrow P \vee \exists x\, Q(x)$

so

$\forall x(P \rightarrow Q(x)) \Leftrightarrow P \rightarrow \forall x Q(x)$

and

$\exists x(P \rightarrow Q(x)) \Leftrightarrow P \rightarrow \exists x\, Q(x)$

but

$\forall x(P(x) \rightarrow Q) \Leftrightarrow \exists x\, P(x) \rightarrow Q$

and

$\exists x(P(x) \rightarrow Q) \Leftrightarrow \forall x\, P(x) \rightarrow Q$

since

$\forall x(\neg P(x) \vee Q) \Leftrightarrow \neg \exists x\, P(x) \vee Q$

and

$\exists x(\neg P(x) \vee Q) \Leftrightarrow \neg \forall x\, P(x) \vee Q$

Quantifiers and Conditionals

If anyone goes to the party, then Bob will be happy

$\exists x\ AtParty(x) \rightarrow Happy(bob)$

It is true of everyone that if they go to the party, then Bob will be happy

$\forall x\ (AtParty(x) \rightarrow Happy(bob))$

If Anyone Goes...

1. ∃x AtParty(x) → Happy(bob)
   2. a (for ∀ Intro)
      3. AtParty(a) (for → Intro)
      4. ∃x AtParty(x) ∃ Intro 3
      Happy(bob)
   AtParty(a) → Happy(bob) → Intro
∀x(AtParty(x) → Happy(bob) ∀ Intro

If Anyone Goes...

1. ∃x AtParty(x) → Happy(bob)
   2. a (for ∀ Intro)
      3. AtParty(a) (for → Intro)
         4. ∃x AtParty(x) ∃ Intro 3
      5. Happy(bob) →Elim 1,4
   6. AtParty(a) → Happy(bob) → Intro 3-5
7. ∀x(AtParty(x) → Happy(bob) ∀ Intro 6

Interpretations

An interpretation (world) specifies a domain that the quantifiers range over and the meaning of predicates, constants, and functions

Example:
Domain = all people
A(x): x is on Team A
B(x): x is on Team B
D(x,y): x defeated y (the last time they played chess...)

a: Adam
b: Barbara

Translations

Everyone on Team A defeated Adam

$\forall x(A(x) \rightarrow D(x,a))$

Someone on Team B was defeated by Barbara

$\exists x(B(x) \wedge D(b,x))$

Everyone on Team A defeated someone

$\forall x(A(x) \rightarrow \exists y\, D(x,y))$

Someone on Team B defeated everyone on Team A

$\exists x(B(x) \wedge \forall y(A(y) \rightarrow D(x,y)))$

Translations

If anyone on Team A defeated Adam, Barbara did

$\exists x(A(x) \wedge D(x,a)) \rightarrow D(b,a)$

Someone on Team A other than Barbara defeated Adam

$\exists x(A(x) \wedge x \neq b \wedge D(x,a)) \quad \wedge A(b)$?

Someone on Team A was not defeated by anyone on Team B

$\exists x(A(x) \wedge \neg\exists y(B(y) \wedge D(y,x)))$

No one on Team A defeated everyone on Team B

$\forall x(A(x) \rightarrow \neg\forall y(B(y) \rightarrow D(x,y)))$

Translations

Not everyone on Team A who defeated Adam also defeated Barbara

$$\neg\forall x([A(x) \wedge D(x,a)] \rightarrow D(x,b))$$

Anyone on Team A who defeated anyone at all defeated Barbara

$$\forall x([A(x) \wedge \exists y\, D(x,y)] \rightarrow D(x,b))$$

Only members of Team A defeated Adam

$$\forall x(\neg A(x) \rightarrow \neg D(x,a)) \quad \wedge \quad \exists x(A(x) \wedge D(x,a))\ ?$$

No one on Team A defeated Barbara except those who defeated everyone

$$\forall x[A(x) \wedge D(x,b)] \rightarrow \forall y\, Dxy)$$

Step by Step...

Everyone on Team A who defeated anyone on Team B was defeated by both Barbara and someone on Team C who defeated Adam

$\forall x([A(x) \wedge \exists y(B(y) \wedge D(x,y)] \rightarrow .....$

Now say that this person (x) was defeated by both Barbara and someone on Team C who defeated Adam

$D(b,x) \wedge \exists y([C(x) \wedge D(y,a)] \wedge D(y,x))$

Step by Step...

Everyone on Team A who defeated anyone on Team B was defeated by both Barbara and someone on Team C who defeated Adam

$$\forall x([A(x) \wedge \exists y(B(y) \wedge D(x,y)] \rightarrow$$

$$(D(b,x) \wedge \exists y([C(x) \wedge D(y,a)] \wedge D(y,x))))$$