Surprised?

At the beginning of the class, the professor told her students "I will do something that you don't expect today and you will be surprised." The students waited until the end of class and nothing surprising seemed to happen. At the end of class, Bob said, "Hey, you said you would do something we didn't expect. But I wasn't surprised at all." The professor said "You expected that you would be surprised in class today right?" Bob: "Yes" Prof: "But you weren't. Therefore something happened that you didn't expect. So I surprised you after all."

Quantifiers

Friday, 8 October

Limits of Truth-Functions

a is a cube
b is not a cube

$a \neq b$

This is provable if you add the identity rules

a is a cube
b is not a cube

There are at least two things

This is still not

Limits of Truth-Functions

All men are mortal
Socrates is a man

Socrates is mortal

All men are tall
Not every man is bald

Some tall people aren't bald

No apples are rotten
Some fruits are rotten

Some fruits aren't apples

For any number, there is a larger prime number

There is no largest prime number

None are truth-functionally valid
- We need a stronger logical system

QUANTIFIERS

Two quantifier symbols:

- ∀ means "everything" or "for all".
- ∃ means "something" or "there exists at least one".
- Just these two quantifiers can be used to capture many of the quantifications we want to talk about. For example, all, every, any, none, not all of, some, some are not, at least one, at least two, exactly two, etc.

Example Sentences

- ∀x Cube(x) - Everything is a cube
- ∃x Cube(x) - Something is a cube
- ∀x(Cube(x)∧Small(x)) - Everything is a small cube
- ∃x(Cube(x)∧Small(x)) - Something is a small cube
- ∀x(Cube(x)→Small(x)) - Every cube is small
- ¬∀x(Cube(x)→Small(x)) - Not every cube is small
- ¬∃x(Cube(x)∧Small(x)) - There aren't any small cubes

Example Sentences

Every boy who is taller than at least two girls is loved by every girl who is taller than him.

$$\forall x([Boy(x) \wedge \exists y \exists z(y \neq z \wedge Girl(y) \wedge Girl(z) \wedge Taller(x,y) \wedge Taller(x,z))] \rightarrow \forall w([Girl(w) \wedge Taller(w,x)] \rightarrow Loves(w,x)))$$

Sentences in FOL

Cube(a)

True in a world if *a* is a cube in that world

∀xCube(x)

True in a world if every object in that world is a cube

For every object x, x is a cube

Sentences in FOL

Cube(a)

True in a world if *a* is a cube in that world

∃xCube(x)

True in a world if at least one object in that world is a cube

For some object x, x is a cube

Cube(x) - Not true or false - not even a sentence

Aristotelian Forms

Forms:	Examples:
All Ps are Qs.	All mammals are animals.
Some Ps are Qs.	Some mammals live in water.
No Ps are Qs.	No humans have wings.
Some Ps are not Qs.	Some birds cannot fly.

Aristotelian Forms

All Ps are Qs

All mammals are animals

For any x, if x is a P,
then x is a Q

For any x, P(x) → Q(x)

∀x(P(x) → Q(x))

∀x(Mammal(x) → Animal(x))

Aristotelian Forms

Some Ps are Qs

Some mammals live in water

There is at least one P that is also a Q

There is at least one thing x
such that x is both P and Q

There is at least one thing x
such that $P(x) \wedge Q(x)$

$\exists x(P(x) \wedge Q(x))$

$\exists x(Mammal(x) \wedge LiWa(x))$

Aristotelian Forms

No Ps are Qs

For any x, if x is a P, then x is not a Q

For any x, $P(x) \rightarrow \neg Q(x)$

$\forall x(P(x) \rightarrow \neg Q(x))$

$\neg \exists x(P(x) \wedge Q(x))$

No humans have wings

$\forall x(Human(x) \rightarrow \neg Wings(x))$

$\neg \exists x(Human(x) \wedge Wings(x))$

Aristotelian Forms

Some Ps are not Qs

Some birds can't fly

There is at least one P that is not a Q

There is at least one thing x such that x is P but not Q

There is at least one thing x such that $P(x) \wedge \neg Q(x)$

$\exists x(P(x) \wedge \neg Q(x))$

$\exists x(Bird(x) \wedge \neg Fly(x))$

$\neg \forall x(P(x) \rightarrow Q(x))$

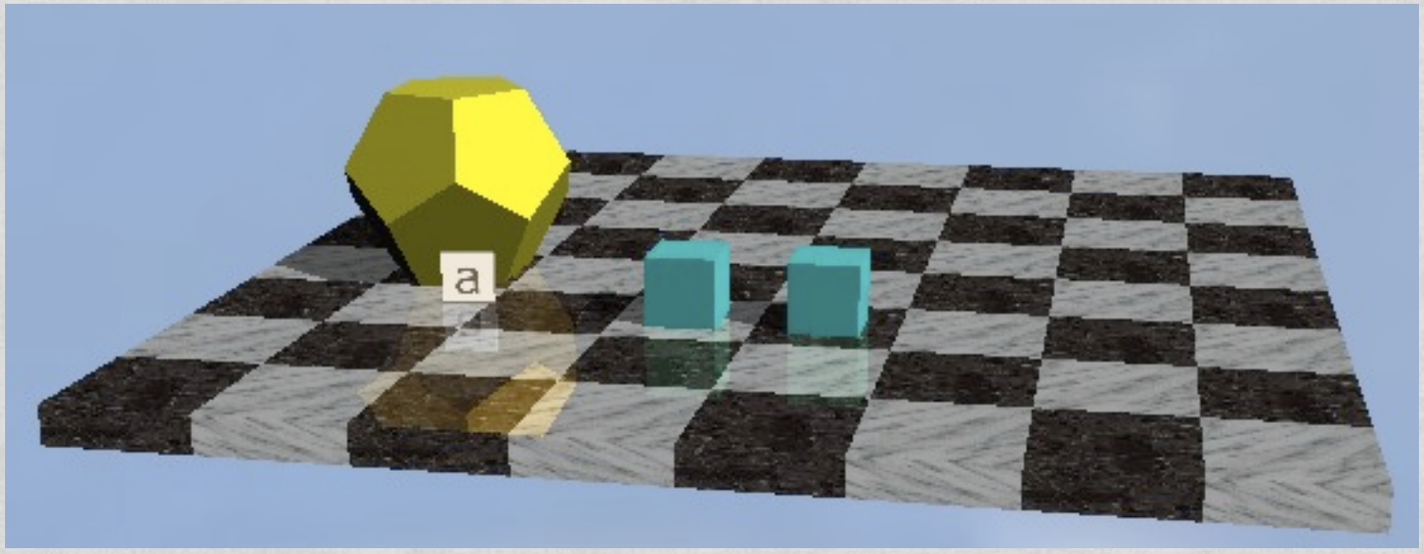
$\neg \forall x(Human(x) \rightarrow Wings(x))$

Aristotelian Forms

Forms:	QL sentence:
All Ps are Qs.	$\forall x(P(x) \rightarrow Q(x))$
Some Ps are Qs.	$\exists x(P(x) \wedge Q(x))$
No Ps are Qs.	$\forall x(P(x) \rightarrow \neg Q(x))$
Some Ps are not Qs.	$\exists x(P(x) \wedge \neg Q(x))$

Satisfaction

- $\forall x$ Cube(x)
- $\exists x$ Cube(x)
- $\forall x(\text{Cube}(x) \rightarrow \text{Small}(x))$
- $\forall x(\text{Cube}(x) \rightarrow \neg\text{Medium}(x))$
- $\forall x(\text{Cube}(x) \wedge \text{Small}(x))$
- $\forall x(\text{Tet}(x) \rightarrow \text{Cube}(x))$
- $\exists x(\text{Cube}(x) \wedge \text{Small}(x))$
- $\exists x(\text{Cube}(x) \rightarrow \text{Large}(x))$



Complex Predicates

Some Ps are Qs

$\exists x(P(x) \wedge Q(x))$

Some Ps that are also Rs are Qs

$\exists x([P(x) \wedge R(x)] \wedge Q(x))$

Some cubes are to the right of *a*

$\exists x(Cubes(x) \wedge RightOf(x,a))$

Some small cubes are to the right of *a*

$\exists x([Small(x) \wedge Cubes(x)] \wedge RightOf(x,a))$

Complex Predicates

All Ps are Qs

$\forall x(P(x) \rightarrow Q(x))$

All Ps that are also Rs are Qs

$\forall x([P(x) \wedge R(x)] \rightarrow Q(x))$

All cubes are to the right of *a*

$\forall x(Cubes(x) \rightarrow RightOf(x,a))$

All small cubes are to the right of *a*

$\forall x([Small(x) \wedge Cubes(x)] \rightarrow RightOf(x,a))$

Complex Predicates

There is a large cube to the left of *b*

$\exists x(L(x) \wedge C(x) \wedge LO(x,b))$

There is a cube to the left of *b* which is in the same row as *c*

$\exists x(C(x) \wedge LO(x,b) \wedge SR(x,c))$

b is in the same row as a large cube

$\exists x(L(x) \wedge C(x) \wedge SR(b,x))$

Complex Predicates

All Ps are Qs

$\forall x(P(x) \rightarrow Q(x))$

All Ps that are also Rs are Qs

$\forall x([P(x) \wedge R(x)] \rightarrow Q(x))$

All cubes are to the right of *a*

$\forall x(Cubes(x) \rightarrow RightOf(x,a))$

All small cubes are to the right of *a*

$\forall x([Small(x) \wedge Cubes(x)] \rightarrow RightOf(x,a))$