Puzzle

A murder has been committed on the island of knights and knaves. You are sure that either A or B is guilty but not both of them. At trial, they say the following:

A says "I am a guilty knight."

B says "The guilty party is a knave."

Who is guilty?

Reductio Ad Absurdum

Friday, 17 September

Formal Proof Rules (¬)

- ¬ Elimination
  From ¬¬P, we can infer P.

  1. ¬¬(P→(Q↔R))

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  2. P→(Q↔R) ¬ Elim: 1

- ¬ Introduction
  This is our rule that formalizes the proof technique known as indirect proof, or Reductio Ad Absurdum. To prove something, assume it is show and show that this leads to contradiction.

CONTRADICTIONS

- We use the special symbol $\bot$ to represent a contradiction. This sentence is always false - it is false on every row of any table.

- This means that

$$\bot \text{ is tautologically equivalent to } P \land \neg P$$

Boring Reductios

- I know you were a good high school student. If you weren't, you would have never been accepted at Cornell. But here you are.

- I didn't do laundry yesterday. If I did, I wouldn't have this giant pile of dirty laundry in my hamper.

Famous Reductios

- $\sqrt{2}$ must be irrational. If it were rational, it would be equal to p/q where p and q are integers. But... (see chap 4)
- There are an infinite number of prime numbers. If there weren't, there would be a greatest one. Call it P. Now take all the primes less than P and multiply them together and add 1. Call this X. If X is prime, it is bigger, if X is not prime, it has prime factors bigger than P...

NEGATION INTRODUCTION

- ¬ Introduction
  From showing P leads to ⊥, we can infer ¬P.

  1. P
  ...
  j. ⊥

  k. ¬P ¬ Intro: 1-j

- Within a subproof we derive ⊥ from P;
  outside the subproof we conclude ¬P.

Rules Using Contradictions

- ⊥ Introduction
  From P and ¬P, we can infer ⊥.

1. P	
2. ¬P	
3. ⊥	⊥ Intro: 1, 2

Reductio Ad Absurdum

Example:

| P
| ¬(P ∧ Q)
|———
| ¬Q

| 1. P
| 2. ¬(P ∧ Q)
|———
| | 3. Q — for ¬ Intro
| |———
| | 4. P∧Q — ∧ Intro 1,3
| | 5. ⊥ — ⊥ Intro 2,4
| 6. ¬Q — ¬ Elim 3-5

Reductio Ad Absurdum

Example:

$\neg(a=b \vee b=c)$

$a \neq b \wedge b \neq c$

1.	$\neg(a=b \vee b=c)$	
2.	$a=b$	for $\neg$ Intro
3.	$a=b \vee b=c$	$\vee$ Intro 2
4.	$\bot$	$\bot$ Intro 1,3
5.	$a \neq b$	$\neg$ Intro 2-4
6.	$b=c$	for $\neg$ Intro
7.	$a=b \vee b=c$	$\vee$ Intro 6
8.	$\bot$	$\bot$ Intro 1,7
9.	$b \neq c$	$\neg$ Intro 6-8
10.	$a \neq b \wedge b \neq c$	$\wedge$ Intro 5-9

Rules Using Contradictions

- ⊥ Elimination

  From ⊥, we can infer absolutely whatever we want.

1. ⊥		
2. BlueCheese(Moon)		⊥ Elim: 1

- This is helpful when we want to eliminate a disjunct when we know that its negation is true.
- We don't technically need this rule; we could just use ¬ Intro and ¬ Elim.

Rules Using Contradictions

Example: Disjunctive Syllogism

$P \vee Q$
$\neg P$

Q

1. $P \vee Q$
2. $\neg P$

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3. P — for $\vee$ Elim
4. $\bot$ — $\bot$ Intro 2,3
5. Q — $\bot$ Elim 4

6. Q — for $\vee$ Elim

7. Q — $\vee$ Elim 1,3-5,6-6