

Philosophy 2310
Indirect Table Method for finding invalidating assignments.

EXAMPLE 1: $P \vee Q, S \rightarrow R \vdash \neg Q \rightarrow R$

To find out whether or not this argument is valid, we can search for an invalidating assignment. An invalidating assignment makes all of the premises true and the conclusion false. If there is one, it would look like this on the table:

P, Q, R, S	$P \vee Q,$	$S \rightarrow R,$	$\neg Q \rightarrow R$
	T	T	F

This is now an incomplete table. We will attempt to fill in as much detail as possible in order to get the truth-value assignments for P, Q, R, and S that give us this row.

Step 1: Since we know that $\neg Q \rightarrow R$ is F, we know that $\neg Q$ is T and R is F. Now we simply fill those parts of the table in.

P, Q, R, S	$P \vee Q,$	$S \rightarrow R,$	$\neg Q \rightarrow R$
F	T	T F	T F F

Step 2: Since we had to make $\neg Q$ T we have to make Q false:

P, Q, R, S	$P \vee Q,$	$S \rightarrow R,$	$\neg Q \rightarrow R$
F F	T F	T F	T F F

Step 3: Now since we have to make $P \vee Q$ T and we have already made Q F, we have to make P T. Also, since we have to make $S \rightarrow R$ T but we have already made R false, we have to make S F.

P, Q, R, S	$P \vee Q,$	$S \rightarrow R,$	$\neg Q \rightarrow R$
T F F F	T T F	F T F	T F F

I haven't produced a full table, but I did complete this one row of the table. Since I was not forced to make any contradictory assignments (both T and F to the same letter) this sequent is invalid since P: T, Q: F, R: F, S:F is an invalidating assignment.

EXAMPLE 2: $(P \rightarrow Q) \rightarrow Q, (Q \rightarrow R) \rightarrow R, \vdash \neg Q \rightarrow (P \wedge R)$

If there is an invalidating assignment, it looks like this:

P, Q, R,	$(P \rightarrow Q) \rightarrow Q, (Q \rightarrow R) \rightarrow R, \neg Q \rightarrow (P \wedge R)$
	T T F

Step 1: To make the conclusion false, we have to make $\neg Q$ T and $P \wedge R$ false.

P, Q, R,	$(P \rightarrow Q) \rightarrow Q, (Q \rightarrow R) \rightarrow R, \neg Q \rightarrow (P \wedge R)$
	T T T F F

Step 2: To make $\neg Q$ T we need Q false.

P, Q, R,	$(P \rightarrow Q) \rightarrow Q, (Q \rightarrow R) \rightarrow R, \neg Q \rightarrow (P \wedge R)$
F	F T F F T T F F

Step 3: Now to make P1 T, since we have a false consequent (Q) we need a false antecedent ($P \rightarrow Q$). To make $P \rightarrow Q$ false we need P True and Q false.

P, Q, R,	$(P \rightarrow Q) \rightarrow Q, (Q \rightarrow R) \rightarrow R, \neg Q \rightarrow (P \wedge R)$
T F	T F F T F F T T F T F

Step 4: Going back to the conclusion, we knew that we had to make $P \wedge R$ False. But now that we have P true, the only way to do that is to make R false. But adding R false produces the following row on the table:

P, Q, R,	$(P \rightarrow Q) \rightarrow Q, (Q \rightarrow R) \rightarrow R, \neg Q \rightarrow (P \wedge R)$
T F F	T F F T F F T F F F T F T F F

I put the 'F' in premise 2 in bold because this F is supposed to be a T. I have argued that if there is an invalidating assignment it has to be T, F, F since this is the only way to make the conclusion false and Premise 1 true. But this assignment makes the second premise false so there can't be an invalidating assignment. If I were packing all of this into a short answer on a homework, I might write this:

It is VALID. In order to make the conclusion false, we have to make $\neg Q$ True (so Q False) and $(P \wedge R)$ false. Now by P1, since Q is false, $P \rightarrow Q$ has to be false so P has to be true. Now since $P \wedge R$ is false and P is true, R has to be false. But this assignment makes P2 false.

EXAMPLE 3: $((P \rightarrow Q) \rightarrow Q) \rightarrow R, \neg S \rightarrow (U \vee \neg Q) \vdash \neg R \rightarrow S$

I will drop the visual use of the table for the next two examples, but feel free to continue to use a table to help yourself reach the answer.

In this example, to make the conclusion false, we have to make $\neg R$ true (so R false) and S false. Now by premise 1, since R is false, $(P \rightarrow Q) \rightarrow Q$ has to be false. To make this false, $(P \rightarrow Q)$ has to be true and Q has to be false. Since $P \rightarrow Q$ is true and Q is false, P has to be false. Now by premise 2, since S is false, $U \vee \neg Q$ has to be true. In fact I have already done that by making Q false so it is easy to see that no matter what I assign to U, it won't matter. So the argument is invalid. Here is one of the two invalidating assignments:

P: F, Q: F, S: F, R: F, U: T (U could also be false)

EXAMPLE 4: $S \rightarrow T, (\neg R \wedge S) \rightarrow \neg Q \vdash (T \rightarrow Q) \rightarrow (S \rightarrow R)$

To make the conclusion false, we make $T \rightarrow Q$ true and $S \rightarrow R$ false. To make $S \rightarrow R$ false, we have to make S true and R false. Since S is true, P1 forces us to make T true. Now with $T \rightarrow Q$ true and T true, we know Q is true. But now this assignment: R: F, S: T, T: T and Q: T makes premise 2 false so this argument is VALID.