Philosophy 2310 -- Assignment #6
Supplement

We saw in chapter 4 how to produce full truth tables for sentences. Producing a full truth-table is an effective method for solving any problem in sentential logic, however, it is impractically long with many types of problems. For example, showing whether an argument which involves five sentence letters is valid or not would require producing a table with thirty-two lines. This is obviously impractical and time-consuming.

The "short table" method is a way of systematically searching for the type of row on the table that you are interested in. If we are trying to determine whether or not an argument is valid, we want to know whether there is any row of the table that makes all of the premises true and the conclusion false. If we assume that there is such a row and then ask ourselves what it has to look like, we may be able to quickly discover such a TVA. If we discover a contradiction by assuming that there is such a TVA, then we know that the argument is valid.

EXAMPLE 1.

Is the sequent $P \rightarrow Q, \neg P \vdash \neg Q$ valid or invalid?

Note that if the sequent is invalid, then there is a TVA which makes all of the premises true and the conclusion false. In order to make premise 2 true, we would have to make P false. And in order to make the conclusion false, we would have to make Q true. So if there is an invalidating assignment, P: F and Q: T has to be it. We check to make sure that the first premise is true on this assignment and it is. Therefore, this argument is invalid.

EXAMPLE 2:

$P \rightarrow Q, P \wedge \neg R \vdash R \vee S$

Note that in order to make the second premise true, we have to make P true and R false. Since we made P true, and we want to make $P \rightarrow Q$ true, we also have to make Q true. Now we have to make the conclusion false. The only way for a disjunction to be false is for both disjuncts to be false. We have already made R false, so now we just have to make S false and we will have our invalidating assignment. This argument is invalid since P: T, Q: T, R: F, S: F, makes each of the premises true and the conclusion false. Thus the argument is invalid.

EXAMPLE 3:

$P \rightarrow Q, \neg R \vee \neg Q \vdash P \rightarrow \neg R$

The conclusion is a conditional, so the only way to make it false is to make its antecedent true and its consequent false. To do this, we must make P true and R true. Since we made P true, the first premise forces us to make Q true but now with both R and Q true, we have made the second premise false. So there can't be any invalidating assignment so this argument is valid.

STRATEGY:

When trying to find an invalidating assignment, you should first assign values to letters that you are forced to assign rather than guessing one of many possibilities. For example, to make $P \wedge Q$ true, you must make P true and Q true. But to make it false, you have multiple options. You could make P false or you could make Q false or both.

Facts that are particularly helpful to remember about the table for these types of problems are:

Quite obviously there is only one assignment to atomic sentences or to negations that will yield the desired value. For example, to make P true we have to assign P: T and to make $\neg(P \wedge Q)$ true we have to make $P \wedge Q$ false.

For binary connectives, remember:

There is only one way to make a conditional false (T, F)
There is only one way to make a conjunction true (T, T)
There is only one way to make a disjunction false (F, F)

Here are more difficult examples:

EXAMPLE 4

$\neg(Q \wedge R),\ U \rightarrow S \ \vdash\ (P \wedge \neg Q) \vee (P \rightarrow S)$

As always, we are attempting to find a way to make all of the premises true and the conclusion false. Since premise 1 is a negation, the only way to make it true is to make $Q \wedge R$ false. There are multiple ways to make $Q \wedge R$ false so we move on to the next premise. There are multiple ways of making $U \rightarrow S$ true so move on to the conclusion. The conclusion is a disjunction, so to make it false we have to make each disjunct false. The second disjunct is $P \rightarrow S$ so to make it false we have to make P true and S false. Since S is false, the second premise makes U false. Now since P is true, and $P \wedge \neg Q$ is false,

Assignments we need:

$\neg(Q \wedge R)$ true (premise)
$U \rightarrow S$ true (premise)
$(P \wedge \neg Q) \vee (P \rightarrow S)$ false (conclusion)
$Q \wedge R$ false (from $\neg(Q \wedge R)$ true)
$P \wedge \neg Q$ false (from conclusion)
$P \rightarrow S$ false (from conclusion)
P true (from $P \rightarrow S$ false)
S false (from $P \rightarrow S$ false)
U false (from $U \rightarrow S$ and $\neg S$)
Q true (from $\neg(P \wedge \neg Q)$ and P)
R false (from $\neg(Q \wedge R)$ and Q)

we have to make ¬Q false and so Q true. Now since we already made Q∧R false, with Q true we have to make R false.

This assignment P: T, Q: T, R: F, S: F, U: F makes each of the premises true and the conclusion false. Therefore this argument is invalid.

EXAMPLE 5:

S∨(P∨¬R), R→ ¬Q ⊢ (P→Q)→(R→S)

We start by noting that since the goal is a conditional there is only one way to make it false. We have to make P→Q true and make R→S false. In order to make R→S false, we have to make R true and S false. R→S false, we have to make R true and S false. Now since we made R true, we have to make ¬Q true and so make Q false. Now, to make the first premise true, since we made S false, we have to make P∨¬R true. Since we made R true, (and so ¬R false) we have to make P true. Now since we have to make P→Q true (from the conclusion) and we made P true, we have to make Q true. But we already argued that Q had to be false. This is a contradiction. So there is no TVA that makes all three premises true and the conclusion false.

Assignments we need:

S∨(P∨¬R) true (premise)
R→ ¬Q true (premise)
(P→Q)→(R→S) false (conclusion)
P→Q true (from conclusion)
R→S false (from conclusion)
R true (from R→S false)
S false (from R→S false)
Q false (from R→ ¬Q and R)
P∨¬R true (premise 1 and ¬S)
P true (from P∨¬R and R)
Q true (from P→Q and P)

Sometimes when producing a short table, you have to guess at an assignment and then check to see if it works. If you are guessing, remember that if it doesn't work as an invalidating assignment, you have to go back and check to see if there is another invalidating assignment.

EXAMPLE 6:

P∨S, S→ ¬R, Q∨R ⊢ P∧Q

There is no obvious place to start with this problem. Lets just start with the conclusion since there isn't any reason to start anywhere else. To make P∧Q false, we have to either make P false or make Q false. Let's try making P false. If we do that, then we have to make S true (first premise) then ¬R true so R false (second premise) so Q true (third premise.) Now we have P: F, Q: T, R: F, S: T as an invalidating assignment. There are other invalidating assignments. For example, if we had started by making Q false, we would have come up with: P: T, Q: F, R: T, S: F. This assignment would have also worked.

EXAMPLE 7:

$Q \vee P, \neg R \rightarrow P \vdash (P \rightarrow (Q \wedge R)) \rightarrow (Q \wedge R)$

Since the goal is a conditional, start by making $P \rightarrow (Q \wedge R)$ true and $Q \wedge R$ false. But now we have no obvious way to proceed. It doesn't matter what we do here. Since we have to make $Q \wedge R$ false, I will start by trying to make Q false. Now by premise 1 we have to make P true so then by $P \rightarrow (Q \wedge R)$ we would have to make $Q \wedge R$ true and so Q true which contradicts where we started from. So if there is an invalidating assignment, it has to make Q true. Now since I haven't covered every possibility, I go back to check and see whether we can find an invalidating assignment by making R false. If R is false, then P is true (premise 2) again making $Q \wedge R$ true contradicting our assumption that R was false. Now, since we have to make $Q \wedge R$ false but we can't make Q false and we can't make R false, we know that this argument is in fact valid. It is very important that we didn't simply stop after we trying assuming Q was false and then failed to find an invalidating assignment. For example, if the last part of the goal was $Q \wedge S$, I might have started the same way, but that argument is invalid whereas this one is valid.