

This homework is due by the beginning of class on Fri, March 14th.

**Part I:** Practicing Taut Con

Show that each of the following arguments is valid by constructing a proof in  $\mathcal{F}$ . You should write out each proof on a piece of paper and put it in your TA's folder in class. You could also write your proof in Fitch and then simply print it out. If you do that, you must click 'show step numbers' and also 'verify proof' before you print it.

You may use any rules of  $\mathcal{F}_T$  plus you can use Taut Con for any step that I consider to be sufficiently obvious. This will be a judgment call so err on the side of caution (and use other rules). If it is something we explicitly mentioned in class, that is okay. Below are other steps that are okay uses of Taut Con. Some of these will be helpful for the problems and the problems are written to get you to use some of these.

Modus Tollens  
 $P \rightarrow Q, \neg Q \vdash \neg P$

Disjunctive Syllogism  
 $P \vee Q, \neg P \vdash Q$

Biconditionals  
 $P \leftrightarrow Q, \neg P \vdash \neg Q$   
 $P \leftrightarrow Q \Leftrightarrow \neg P \leftrightarrow \neg Q$   
 $\neg(P \leftrightarrow Q) \Leftrightarrow \neg P \leftrightarrow Q$

Conditionals  
 $\neg P \vee Q \Leftrightarrow P \rightarrow Q$   
 $P \wedge \neg Q \Leftrightarrow \neg(P \rightarrow Q)$

DeMorgan's Laws  
 $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$   
 $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$

1.  $P \vee Q, Q \rightarrow R \vdash \neg R \rightarrow P$
2.  $P \vee Q \vdash (\neg Q \rightarrow \neg P) \rightarrow Q$
3.  $P \rightarrow Q, \neg P \rightarrow R \vdash Q \vee R$
4.  $\neg(P \wedge Q), \neg(\neg P \wedge Q) \vdash \neg Q$
5.  $(P \rightarrow Q) \rightarrow P, P \rightarrow R \vdash R$
6.  $(P \rightarrow Q) \rightarrow Q \vdash (Q \rightarrow P) \rightarrow P$
7.  $P \leftrightarrow (Q \leftrightarrow R) \vdash (\neg Q \wedge \neg P) \rightarrow R$

**Part II.** In each of the following cases, determine whether the argument is valid by either giving an invalidating assignment, or by giving some argument that there is none.

1.  $R \rightarrow \neg P, Q \vee R \vdash R \rightarrow P$
2.  $(Q \rightarrow R) \rightarrow S, (U \vee R) \wedge Q \vdash (U \vee Q) \rightarrow S$
3.  $((Q \rightarrow R) \rightarrow R) \vee P, P \rightarrow (Q \wedge \neg Q) \vdash Q \vee R$
4.  $P \rightarrow \neg P, \neg R \rightarrow R \vdash P \wedge (\neg R \wedge S)$
5.  $(P \rightarrow Q) \rightarrow R, (R \wedge S) \rightarrow U \vdash (\neg U \wedge \neg Q) \rightarrow \neg S$
6.  $\neg(P \rightarrow Q), R \wedge (Q \vee S) \vdash (R \wedge U) \vee (P \wedge \neg U)$