

This homework is due by the beginning of class on Mon, May 5th. Note that there are two pages to the homework.

**Part I:**

Read section 12.4 in the book (pages 338-343).

Do: Problems 12.17, 12.18 (Hint: Both are invalid)

**Part II:**

Show that the following arguments are invalid by producing a counterexample. To do this, create a grid and add shapes to it.

Argument 1

P1.  $\forall x \forall y ((\text{Square}(x) \wedge \text{Square}(y)) \rightarrow \text{SameRow}(x,y))$

P2.  $\exists x \exists y ((\text{Filled}(x) \wedge \text{Filled}(y) \wedge \text{SameRow}(x,y))$

Conc.  $\forall x \forall y ((\text{Square}(x) \wedge \text{Filled}(y)) \rightarrow \text{SameRow}(x,y))$

Argument 2

P1.  $\forall x (\text{Square}(x) \rightarrow \forall y (\text{Circle}(y) \rightarrow \text{LeftOf}(x,y))$

P2.  $\forall x (\text{Square}(x) \rightarrow \exists y (\text{Circle}(y) \wedge \text{SameRow}(x,y))$

Conc.  $\forall x \forall y (\text{LeftOf}(x,y) \rightarrow (\text{Square}(x) \wedge \text{Circle}(y))$

Argument 3

P1.  $\exists x (\text{Square}(x) \wedge \forall y (\text{Circle}(y) \rightarrow \text{LeftOf}(x,y))$

P2.  $\forall x ((\text{Square}(x) \wedge \text{Filled}(x)) \rightarrow \exists y (\text{Circle}(y) \wedge \text{SameCol}(x,y))$

P3.  $\exists x (\text{Square}(x) \wedge \text{Filled}(x))$

Conc.  $\forall x (\text{Circle}(x) \rightarrow \exists y (\text{Filled}(y) \wedge \text{SameCol}(x,y)))$

**Part III:**

**Diagrams:**

Determine which of these sentences are true on which of these diagrams. For example, a 4x7 grid of 28 true/false answers is one way to answer this. It might help to think about students passing tests.

1.  $\exists x (\text{S}(x) \wedge \forall y (\text{T}(y) \rightarrow \text{P}(x,y)))$

2.  $\exists x (\text{T}(x) \wedge \forall y (\text{S}(y) \rightarrow \text{P}(y,x)))$

3.  $\forall x (\text{S}(x) \rightarrow \exists y (\text{T}(y) \wedge \text{P}(x,y)))$

4.  $\forall x (\text{T}(x) \rightarrow \exists y (\text{S}(y) \wedge \neg \text{P}(y,x)))$

5.  $\exists x (\text{T}(x) \wedge \forall y (\text{S}(y) \rightarrow \neg \text{P}(y,x)))$

6.  $\forall x (\text{T}(x) \rightarrow \exists y \exists z (\text{S}(y) \wedge \text{S}(z) \wedge \text{P}(y,x) \wedge \neg \text{P}(z,x)))$

7.  $\exists x \exists y (\text{T}(x) \wedge \text{T}(y) \wedge \forall z (\text{S}(z) \rightarrow (\text{P}(z,x) \vee \text{P}(z,y))))$

**Diagrams to use for Part III:**

Diagram 1

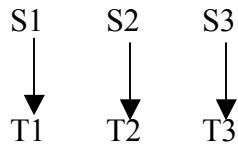


Diagram 2

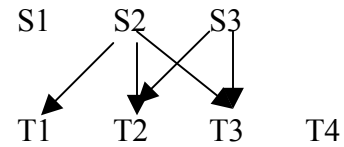


Diagram 3

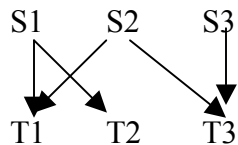
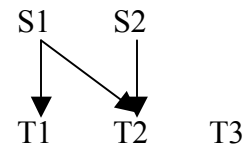


Diagram 4



**Part IV:**

**Diagrams as Models:**

Show that each of the following arguments is invalid by producing a countermodel. In each problem, you should produce a single diagram where each of the premises is true but the conclusion is false. So produce three diagrams for this part. It might help to think about teachers attending meetings.

Argument 1

- P1.  $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y)))$
- P2.  $\forall x(M(x) \rightarrow \exists y(T(y) \wedge A(y,x)))$
- Conc.  $\exists x(M(x) \wedge \forall y(T(y) \rightarrow A(y,x)))$

Argument 2

- P1.  $\exists x(M(x) \wedge \forall y(T(y) \rightarrow A(y,x)))$
- P2.  $\exists x(T(x) \wedge \forall y(M(y) \rightarrow A(x,y)))$
- Conc.  $\forall x \forall y((T(x) \wedge M(y)) \rightarrow A(x,y))$  (this final 'y' was changed from 'z' in a previous version)

Argument 3

- P1.  $\exists x(M(x) \wedge \forall y(T(y) \rightarrow \neg A(y,x)))$
- P2.  $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y)))$
- Conc.  $\exists x(M(x) \wedge \forall y(T(y) \rightarrow A(y,x)))$