Phil 2310 Assignment 10
Fall 2014

This homework is due by the beginning of class on Mon, May 5th. Note that there are two pages to the homework.

Part I:
Read section 12.4 in the book (pages 338-343).
Do: Problems 12.17, 12.18 (Hint: Both are invalid)

12.17: To make the conclusion false, you need a medium non-tet. Because of P2 and P3, it will need to be a dodec. If you stopped there (a picture with just one medium dodec) you would be done, but you could also add more as long as you kept all the premises true.

12.18: To make the conclusion false, you need a tet and a cube where the tet is not to the left of the cube. By P2 the cube is small and by P3 the tet is large. If you put the cube to the left of the tet then you violate P1. So they have to be in the same column.

Part II:
Show that the following arguments are invalid by producing a counterexample. To do this, create a grid and add shapes to it.

Argument 1
P1. $\forall x \forall y((\text{Square}(x) \wedge \text{Square}(y)) \rightarrow \text{SameRow}(x,y))$
P2. $\exists x \exists y((\text{Filled}(x) \wedge \text{Filled}(y) \wedge \text{SameRow}(x,y))$
Conc. $\forall x \forall y((\text{Square}(x) \wedge \text{Filled}(y)) \rightarrow \text{SameRow}(x,y))$

For P2, put two filled things in the same row. To make the conclusion false, have a square in a different row from some filled thing. To keep P1 true, just make sure the filled thing isn't a square. So two filled squares in the same row and a filled circle in a different row would work fine.

Argument 2
P1. $\forall x(\text{Square}(x) \rightarrow \forall y\ (\text{Circle}(y) \rightarrow \text{LeftOf}(x,y))$
P2. $\forall x(\text{Square}(x) \rightarrow \exists y\ (\text{Circle}(y) \wedge \text{SameRow}(x,y))$
Conc. $\forall x \forall y(\text{LeftOf}(x,y) \rightarrow (\text{Square}(x) \wedge \text{Circle}(y))$

One kind of answer that makes the two premises true is to not have any squares. To make the conclusion false, you need one thing to the left of another – two circles will do. If you do have a square, your counterexample for the conclusion would have to be two squares. Then make sure you have a circle to the right of them both in the same row.

Argument 3
P1. $\exists x(\text{Square}(x) \wedge \forall y\ (\text{Circle}(y) \rightarrow \text{LeftOf}(x,y))$
P2. $\forall x((\text{Square}(x) \wedge \text{Filled}(x)) \rightarrow \exists y(\text{Circle}(y) \wedge \text{SameCol}(x,y))$

P3. $\exists x(\text{Square}(x) \wedge \text{Filled}(x))$
Conc. $\forall x(\text{Circle}(x) \rightarrow \exists y(\text{Filled}(y) \wedge \text{SameCol}(x,y)))$

To make P3 true, you need a filled square. By P2, add a circle in the same column. By P1, add a square to the left. To falsify the conclusion, add another circle – say to the right of everything else.

Part III:
Diagrams:
Determine which of these sentences are true on which of these diagrams. For example, a 4x7 grid of 28 true/false answers is one way to answer this. It might help to think about students passing tests. The answers are just the T/F answers – English translations are not required. They are just here to help you understand the answers.

1. $\exists x(S(x) \wedge \forall y(T(y) \rightarrow P(x,y)))$ F, F, F, F
   There is a student who passed every test
2. $\exists x(T(x) \wedge \forall y(S(y) \rightarrow P(y,x)))$ F, F, F, T
   There is a test that every student passed
3. $\forall x(S(x) \rightarrow \exists y(T(y) \wedge P(x,y)))$ T, F, T, T
   Every student passed at least one test
4. $\forall x(T(x) \rightarrow \exists y(S(y) \wedge \neg P(y,x)))$ T, T, T, F
   For every test there is at least one student who didn't pass
5. $\exists x(T(x) \wedge \forall y(S(y) \rightarrow \neg P(y,x)))$ F, T, F, T
   There is a test that no students passed
6. $\forall x(T(x) \rightarrow \exists y \exists z(S(y) \wedge S(z) \wedge P(y,x) \wedge \neg P(z,x)))$ T, F, T, F
   For every test, there is a pair of students one of whom passed and the other who didn't.
7. $\exists x \exists y(T(x) \wedge T(y) \wedge \forall z(S(z) \rightarrow (P(z,x) \vee P(z,y))))$ F, F, T, T
   There is a pair of tests such that every student passed at least one of them

Diagrams to use for Part III:

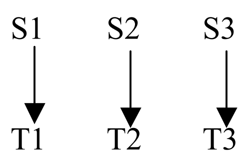
Diagram 1


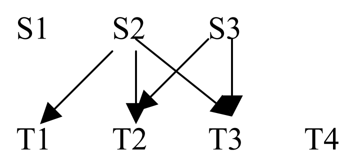
Diagram 2


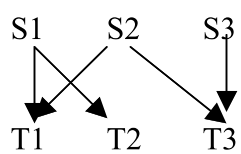
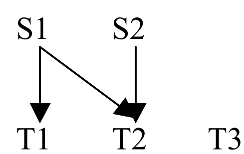
Diagram 3


Diagram 4


Part IV:

Diagrams as Models:

Show that each of the following arguments is invalid by producing a countermodel. In each problem, you should produce a single diagram where each of the premises is true but the conclusion is false. So produce three diagrams for this part. It might help to think about teachers attending meetings.

Argument 1

P1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y)))$

P2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge A(y,x)))$

Conc. $\exists x(M(x) \wedge \forall y(T(y) \rightarrow A(y,x)))$

T1 to M1, T2 to M2

Argument 2

P1. $\exists x(M(x) \wedge \forall y(T(y) \rightarrow A(y,x)))$

P2. $\exists x(T(x) \wedge \forall y(M(y) \rightarrow A(x,y)))$

Conc. $\forall x \forall y((T(x) \wedge M(y)) \rightarrow A(x,y))$

T1 to M1, T1 to M2, T2 to M1

Argument 3

P1. $\exists x(M(x) \wedge \forall y(T(y) \rightarrow \neg A(y,x)))$

P2. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y)))$

Conc. $\exists x(M(x) \wedge \forall y(T(y) \rightarrow A(y,x)))$

Nothing to M1, T1 to M2, T2 to M3