Puzzle

You meet A and B in the land of knights and knaves.

A says "I am a knight if and only if B is also a knight."

B says "A and I are of different kinds."

Who is what?

Building and Using Truth Tables

Monday, 3 February

Truth Tables

- Example: truth table for ¬(P∧Q)
  First, give truth conditions of the atomic sentences:

P	Q	¬ (P ∧ Q)	
T	T	T T	
T	F	T F	
F	T	F T	
F	F	F F	

Truth Tables

- Example: truth table for ¬(P∧Q)
  Then assign truth conditions of the combinations:

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T	F	T **F** F	
F	T	F **F** T	
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TRUTH TABLES

- Example: truth table for $\neg(P \wedge Q)$
  Then assign truth conditions of the combinations:

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T	F	T **F** F	
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F	T		**T** F **T** **F** T
F	F		**T** F **T** **T** F

Truth Tables

- Example: joint truth table for ¬(P∧Q) and (¬P∨¬Q)
This shows that the two sentences are equivalent.

P	Q	¬ (P ∧ Q)	(¬ P ∨ ¬ Q)
T	T	**F** T **T** T	**F** T **F** **F** T
T	F	**T** T **F** F	**F** T **T** **T** F
F	T	**T** F **F** T	**T** F **T** **F** T
F	F	**T** F **F** F	**T** F **T** **T** F

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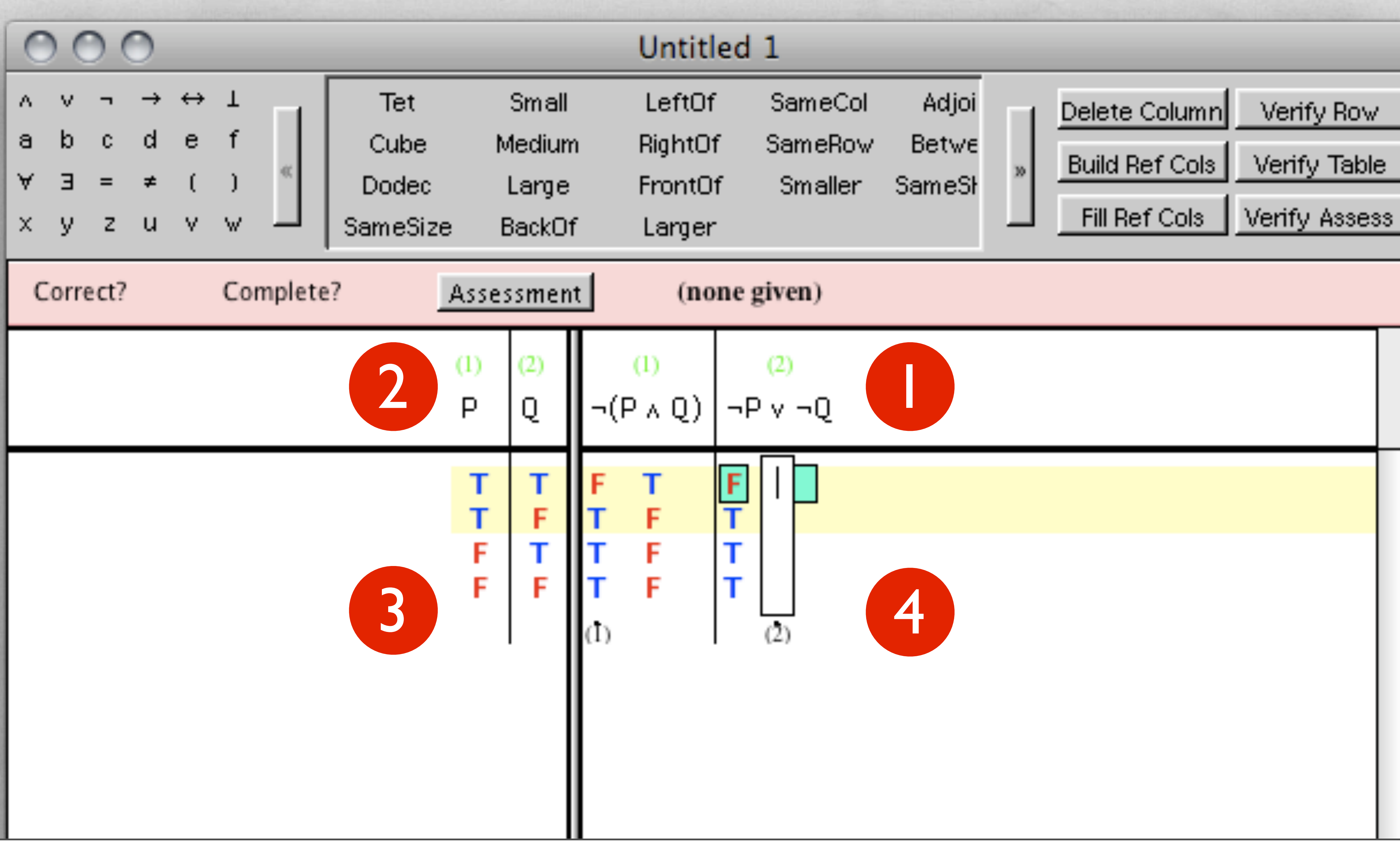
T	F	**T** T **F** F	**F** T **T** **T** F
F	T	**T** F **F** T	**T** F **T** **F** T
F	F	**T** F **F** F	**T** F **T** **T** F

Truth Tables

We will construct truth tables in Boole.



TRUTH TABLES

- Example: truth table for $(A \wedge \neg B) \vee (A \wedge \neg B)$

Truth Tables

- Example: truth table for (A ∧ ¬B) ∨ (A ∧ ¬B)

A	B	(A ∧ ¬ B) ∨ (¬ A ∧ B)
T	T	
T	F	
F	T	
F	F	

TRUTH TABLES

- Example: truth table for (A ∧ ¬B) ∨ (A ∧ ¬B)

A	B	(A ∧ ¬ B) ∨ (¬ A ∧ B)
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T	F	T F
F	T	F T
F	F	T T

Truth Tables

- Example: truth table for $(A \wedge \neg B) \vee (A \wedge \neg B)$

A	B	$(A \wedge \neg B) \vee (\neg A \wedge B)$
T	T	F F F F
T	F	T T F F
F	T	F F T T
F	F	F T T F

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- Example: truth table for (A ∧ ¬B) ∨ (A ∧ ¬B)

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T	F	T T F F
F	T	F F T T
F	F	F T T F

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T	T	F F **F** F F
T	F	T T **T** F F
F	T	F F T T
F	F	F T T F

Truth Tables

- Example: truth table for $(A \land \neg B) \lor (A \land \neg B)$

A	B	$(A \land \neg B) \lor (\neg A \land B)$				
T	T	F	F	**F**	F	F
T	F	T	T	**T**	F	F
F	T	F	F	**T**	T	T
F	F	F	T		T	F

Truth Tables

- Example: truth table for $(A \wedge \neg B) \vee (A \wedge \neg B)$

A	B	$(A \wedge \neg B) \vee (\neg A \wedge B)$
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T	F	T T **T** F F
F	T	F F **T** T T
F	F	F T **F** T F

Truth Tables

- Notice that in rows 2 and 3 (but not 1 and 4) exactly one of A and B are true

A	B	(A ∧ ¬ B) ∨ (¬ A ∧ B)
T	T	F F **F** F F
T	F	T T **T** F F
F	T	F F **T** T T
F	F	F T **F** T F

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T	T	F F **F** F F
T	F	T T **T** F F
F	T	F F **T** T T
F	F	F T **F** T F

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- Notice that in rows 2 and 3 (but not 1 and 4) exactly one of A and B are true

A	B	$(A \wedge \neg B) \vee (\neg A \wedge B)$	$\neg(A \leftrightarrow B)$
T	T	F F **F** F F	T
T	F	T T **T** F F	F
F	T	F F **T** T T	F
F	F	F T **F** T F	T

Truth Tables

- Notice that in rows 2 and 3 (but not 1 and 4) exactly one of A and B are true

A	B	(A ∧ ¬ B) ∨ (¬ A ∧ B)	¬(A ↔ B)
T	T	F F **F** F F	**F** T
T	F	T T **T** F F	**T** F
F	T	F F **T** T T	**T** F
F	F	F T **F** T F	**F** T

Truth Tables
-with three atoms-

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- When there are two atomic sentences involved (say P and Q) there are four possibilities
- But if there are three atomic sentences involved, there are eight possibilities.

Truth Tables -with three atoms-

- When there are two atomic sentences involved (say P and Q) there are four possibilities
- But if there are three atomic sentences involved, there are eight possibilities.
    - In general, X atoms means 2^X possibilities

Examples with three atoms

A	B	C	$A \lor (B \land C)$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

Examples with three atoms

A	B	C	A ∨ (B ∧ C)
T	T	T	T
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

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T	T	T	T
T	T	F	F
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

Examples with three atoms

A	B	C	$A \vee (B \wedge C)$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

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A	B	C	A ∨ (B ∧ C)
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T	T	F	F
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F	T	F	F
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EXAMPLES WITH THREE ATOMS

A	B	C	$A \vee (B \wedge C)$
T	T	T	**T** T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
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T	T	T	**T** T
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F	F	T	**F** F
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- Example: truth table for $(P \rightarrow Q)$

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- Example: truth table for $(\neg P \rightarrow \neg Q)$

Truth Tables with Conditionals

- Example: truth table for (¬P → ¬Q)

P	Q	(¬P → ¬Q)
T	T	
T	F	
F	T	
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- Joint truth table for P → Q and (¬P → ¬Q)
  These are not equivalent

P	Q	P → Q	(¬P → ¬Q)
T	T	**T**	F **T** F
T	F	**F**	F **T** T
F	T	**T**	T **F** F
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T	T	
T	F	
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Truth Tables with Conditionals

- Truth table for $(Cube(a) \land Cube(b)) \to Cube(a)$
  This sentence is a **Tautology**

Cube(a)	Cube(b)	(Cube(a) ∧ Cube(b)) → Cube(a)
T	T	T **T** T
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Tautological versus Logical

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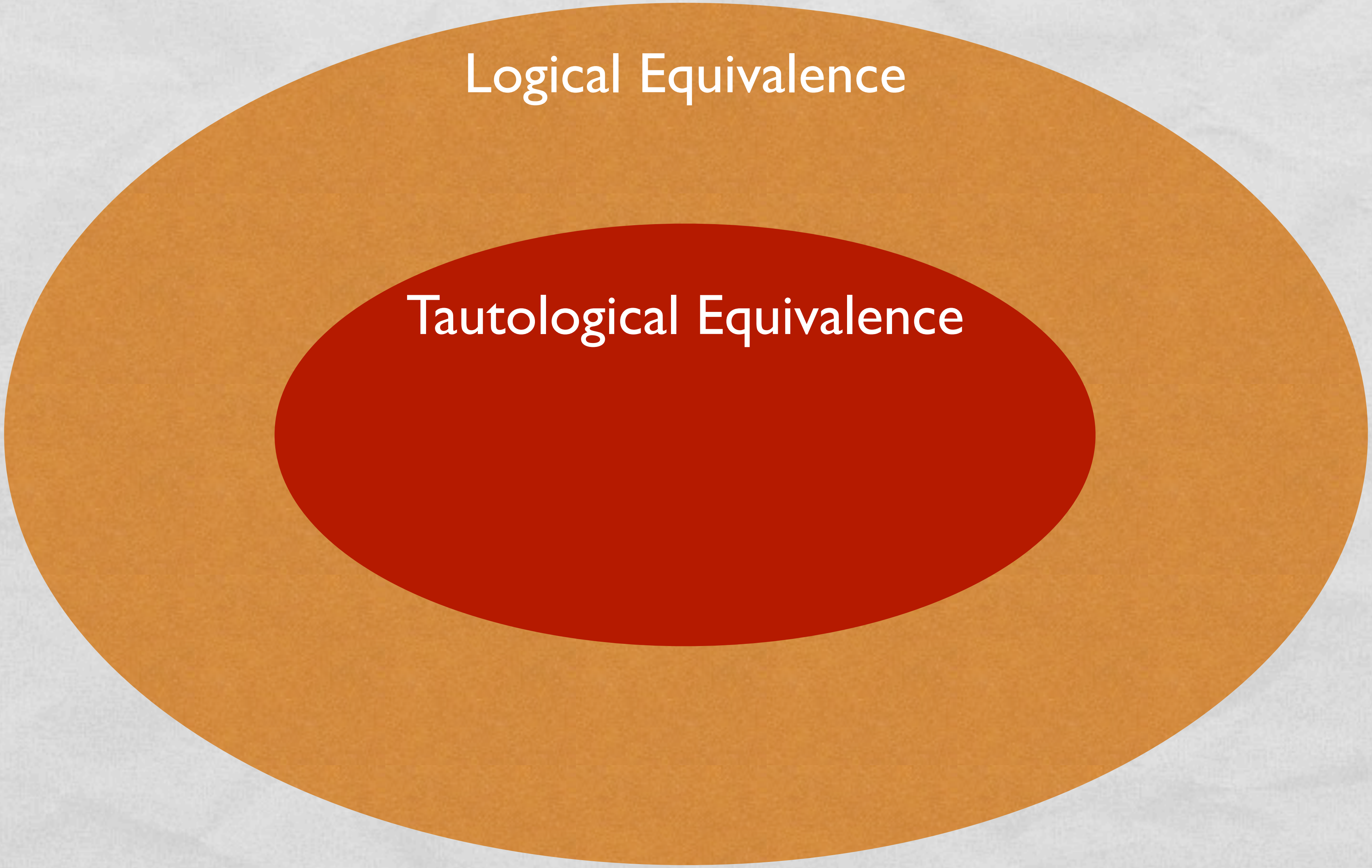
- Two sentences are tautologically equivalent if they have matching truth tables, i.e., the same truth values for all combinations of atomic sentences' truth values.
- Tautological equivalence results simply from the meanings of the truth-functional connectives.
  (Ex: DeMorgan's Laws, double negation, contraposition)

Tautological versus Logical



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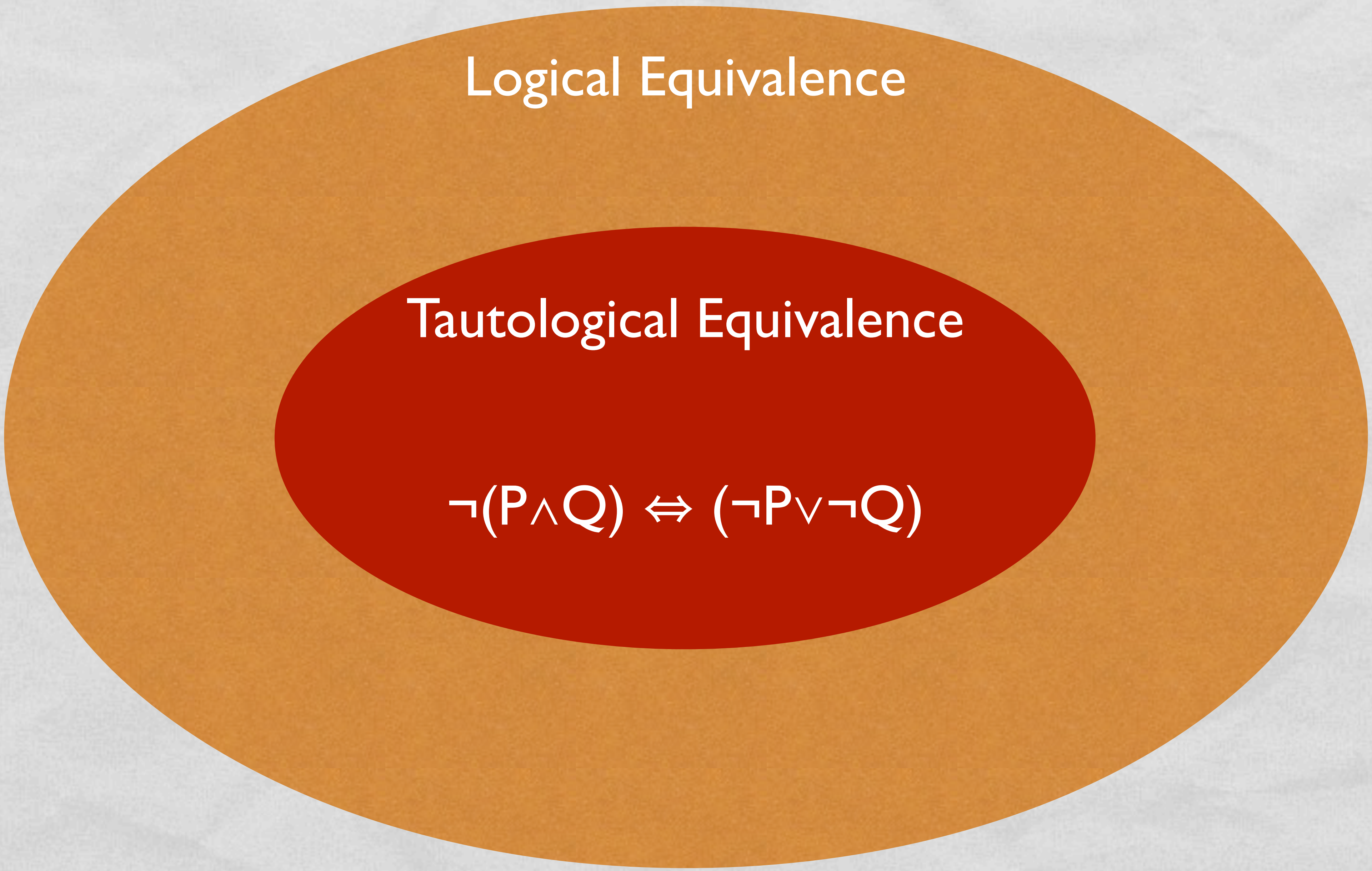

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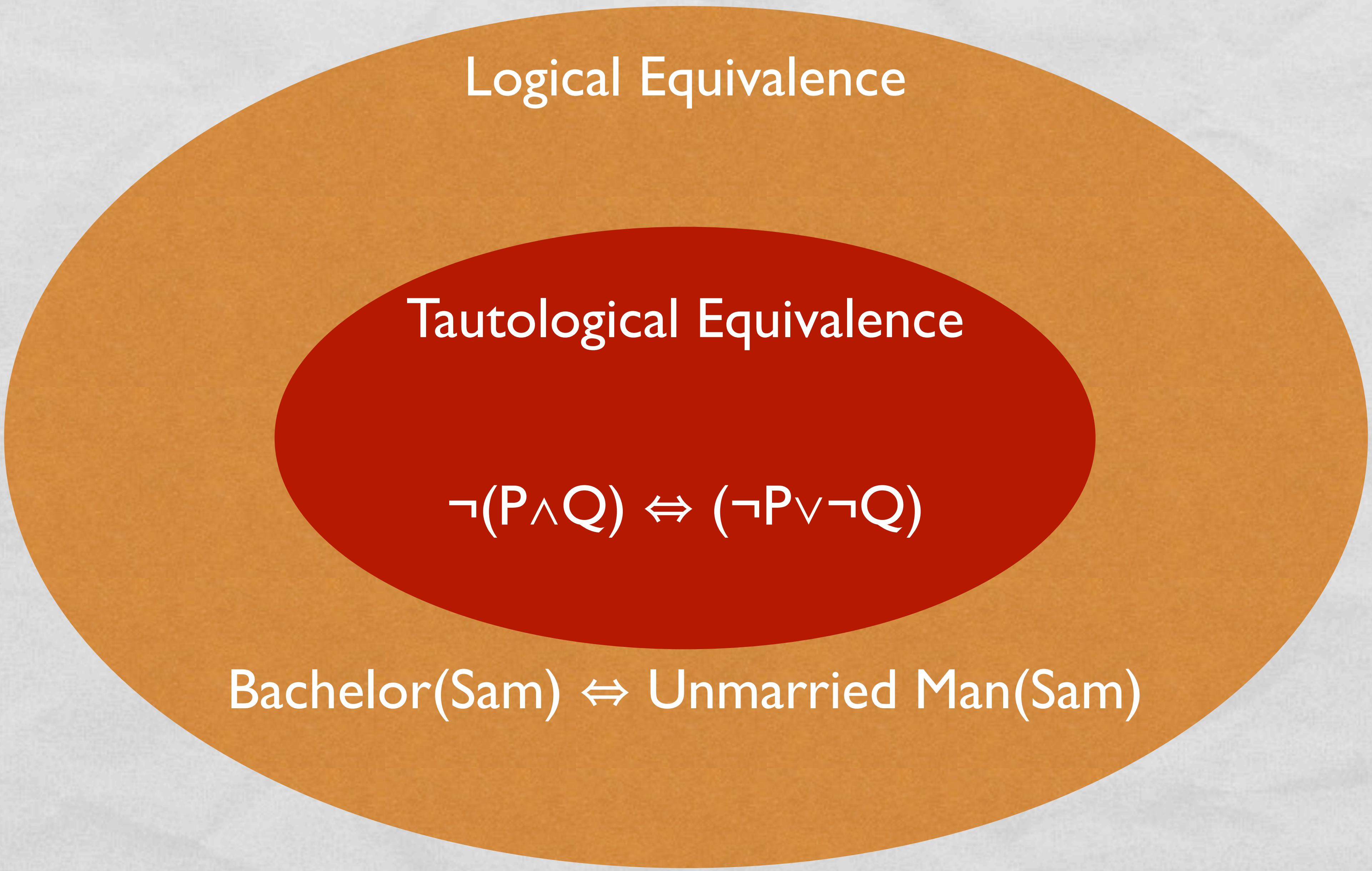

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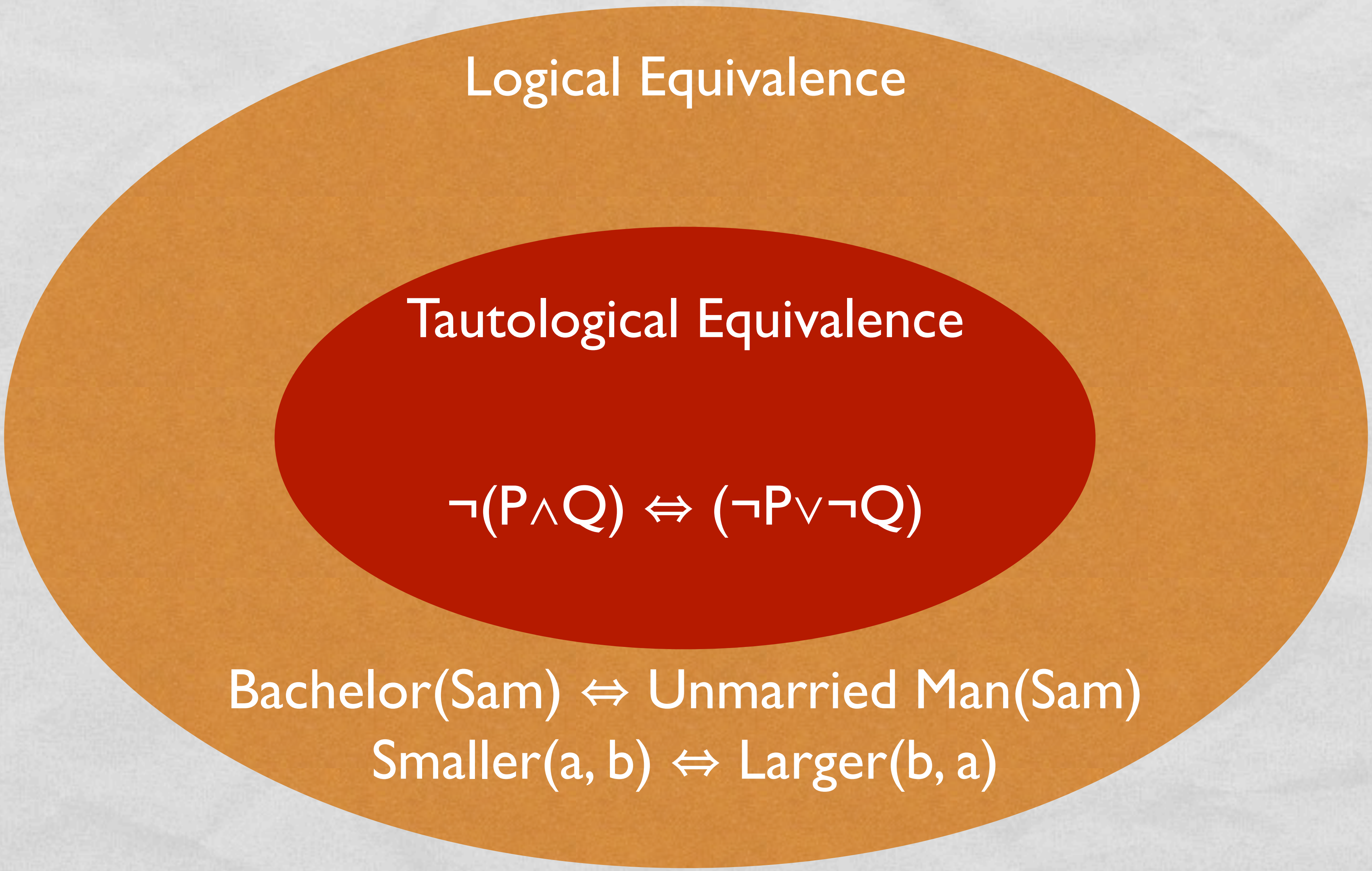

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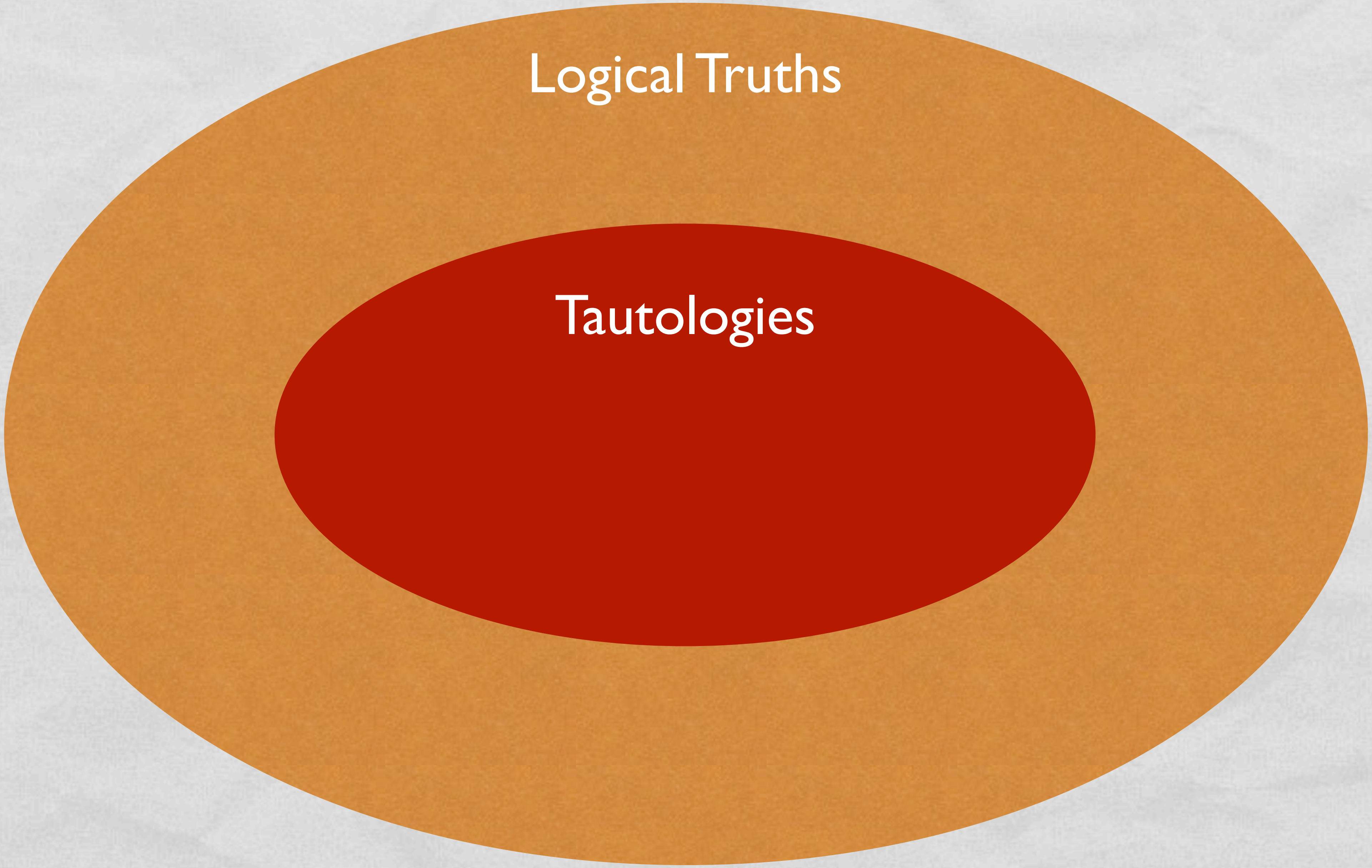
- A sentence is a logical truth iff it is logically necessary, i.e., is a logical consequence of any set of sentences. (It is impossible for a logical truth to be false.)
- A sentence is a tautology iff every row of its truth table assigns TRUE to that sentence.
- Tautologies are true in virtue of the meanings of the truth-functional connectives alone. (Ex: $P \lor \neg P$)

Tautological versus Logical



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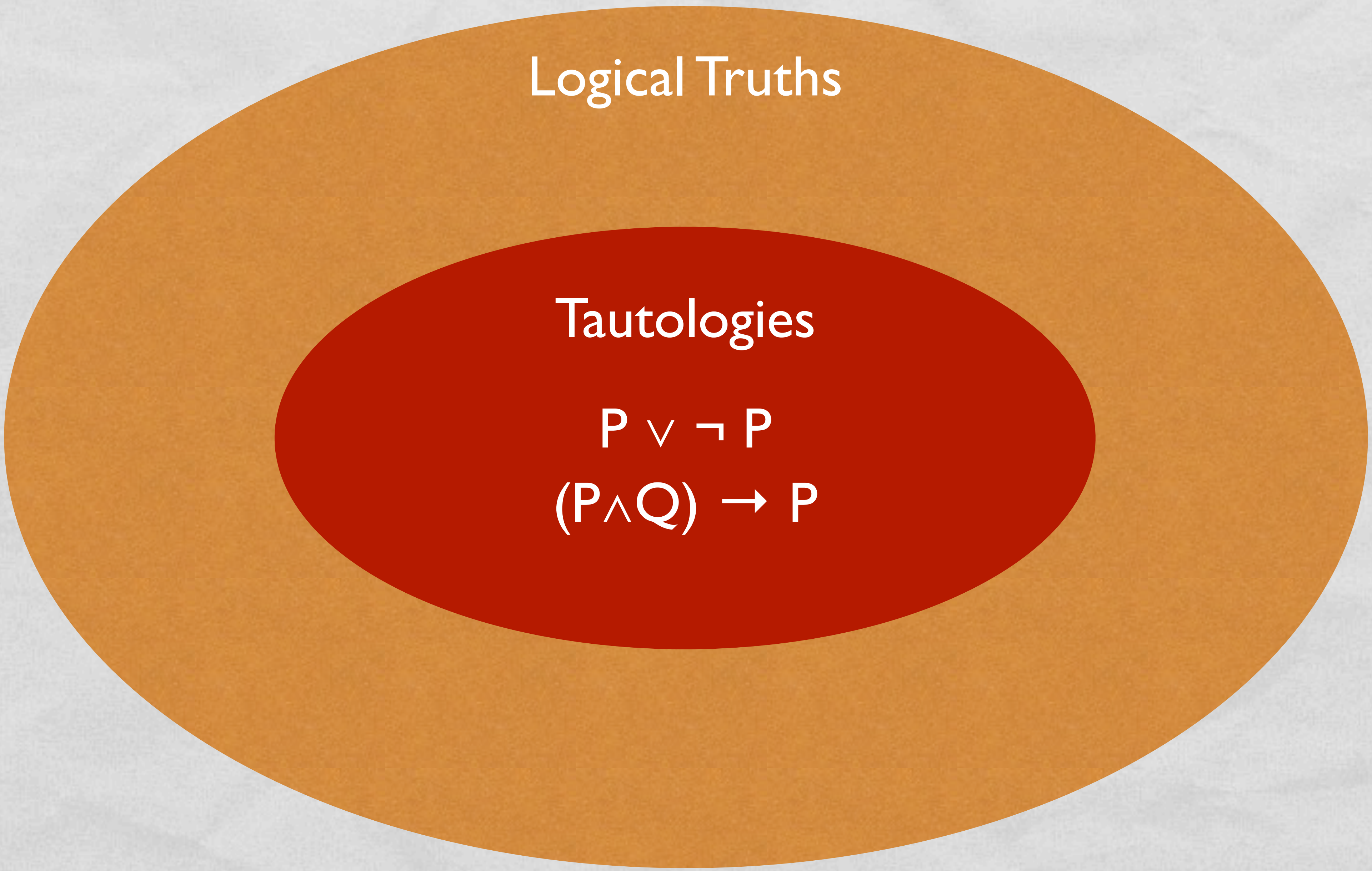

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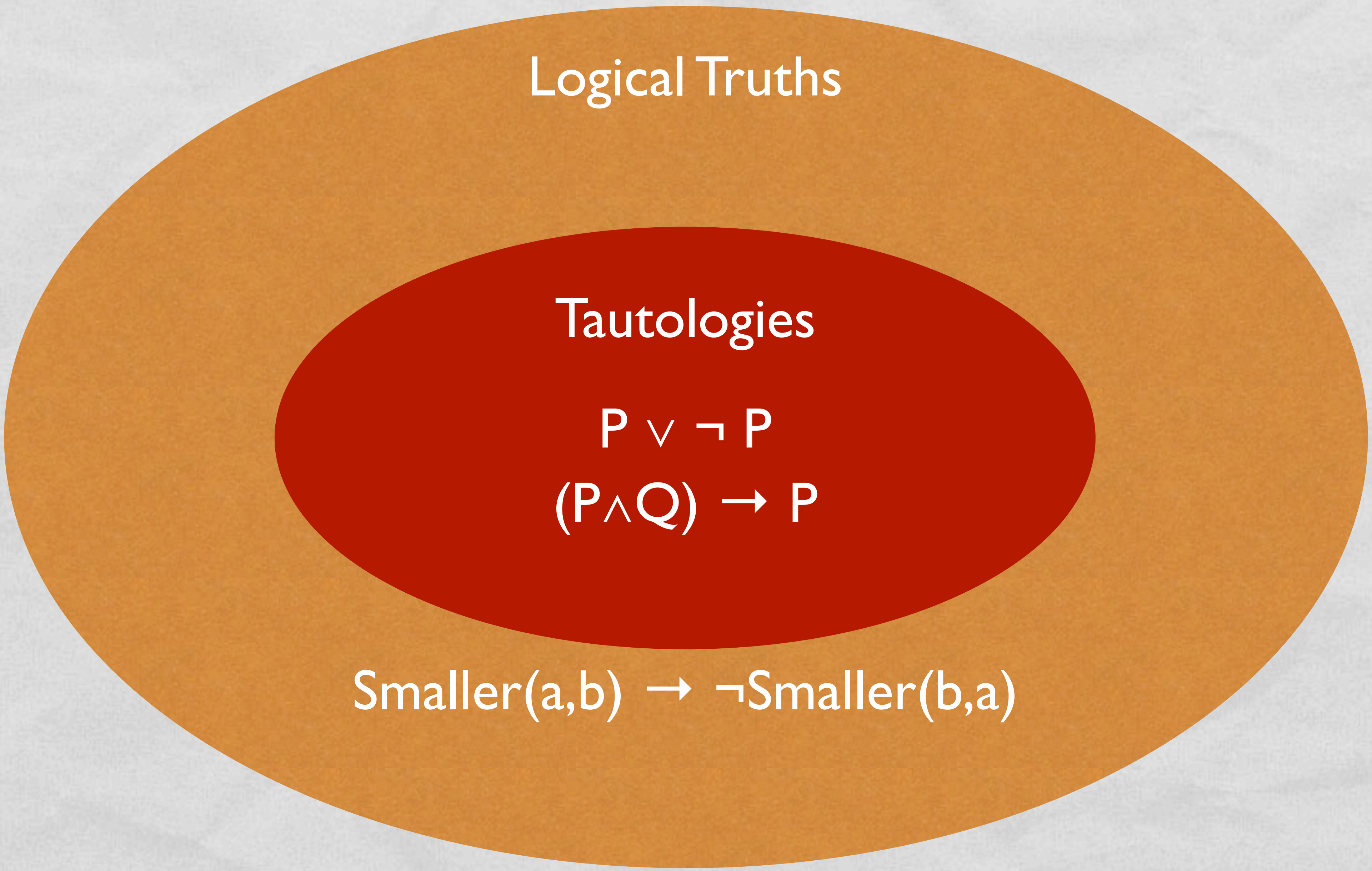

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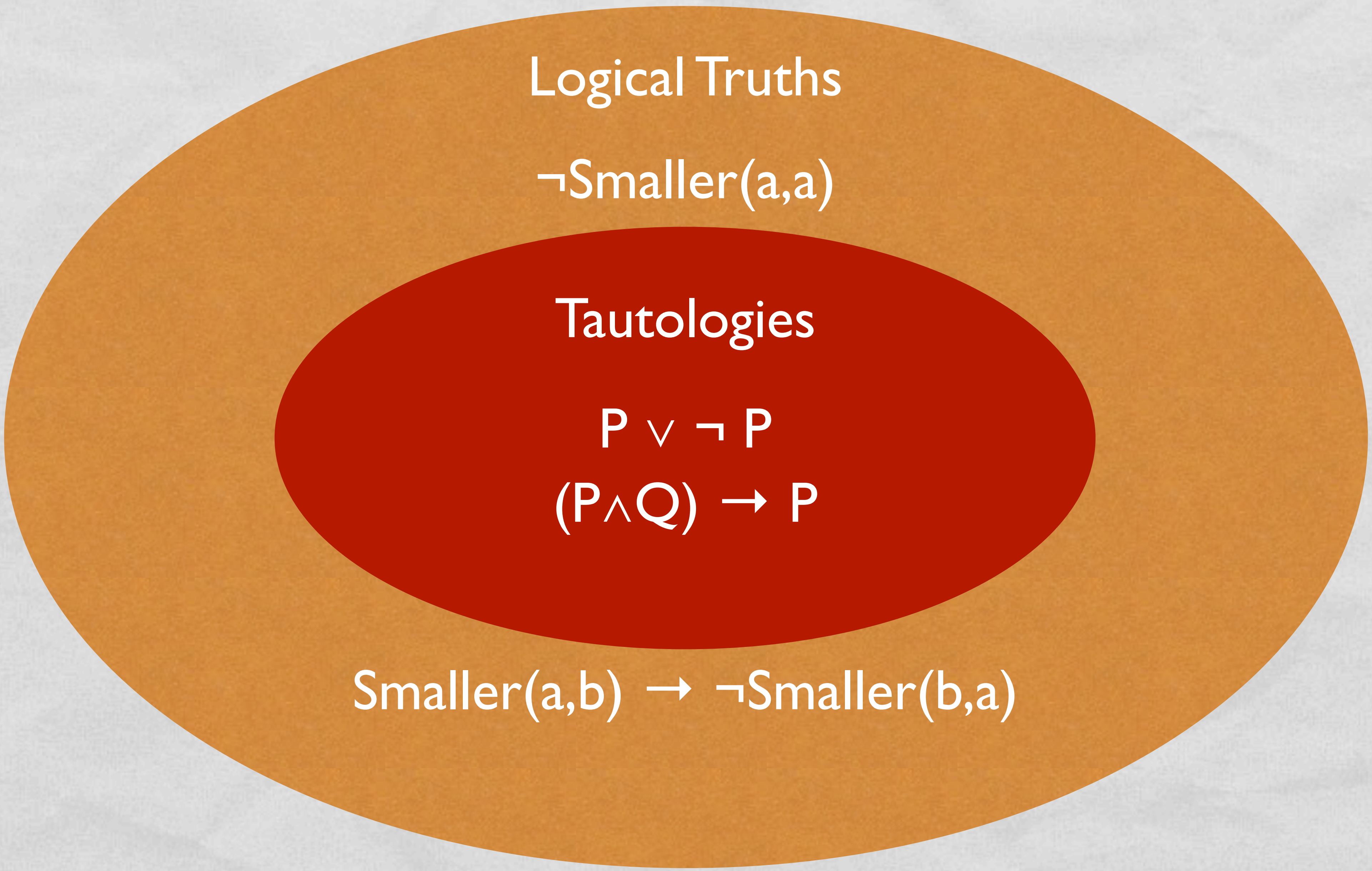

Logical and Tautological Consequence

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- A sentence S is a logical consequence of a set of sentences $P_1 \ldots P_n$ iff whenever $P_1 \ldots P_n$ are true, S is also true.

Logical and Tautological Consequence

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- Note: The Ps and Qs might be complex sentences.

Logical and Tautological Consequence

- Example:

1) A

2) $A \rightarrow B$

3) $\neg B \vee C$

4) Conclusion: C

Logical and Tautological Consequence

A	B	C	A	A→B	¬B ∨ C	C
T	T	T	T	T	F **T**	T
T	T	F	T	T	F **F**	F
T	F	T	T	F	T **T**	T
T	F	F	T	F	T **T**	F
F	T	T	F	T	F **T**	T
F	T	F	F	T	F **F**	F
F	F	T	F	T	T **T**	T
F	F	F	F	T	T **T**	F

No row is T, T, T, F

Logical and Tautological Consequence

A	B	C	A	A→B	B ∨ C	C
T	T	T	T	T	**T**	T
T	T	F	T	T	**T**	F
T	F	T	T	F	**T**	T
T	F	F	T	F	**F**	F
F	T	T	F	T	**T**	T
F	T	F	F	T	**T**	F
F	F	T	F	T	**T**	T
F	F	F	F	T	**T**	F

Second row is T, T, T, F
So NOT valid

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- $(P_1 \wedge P_2 \wedge \ldots \wedge P_n) \rightarrow Q$ is a logical truth iff Q is a logical consequence of $P_1, P_2 \ldots P_n$.

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- Recall: $A \leftrightarrow B \Leftrightarrow (A \rightarrow B) \wedge (B \rightarrow A)$.
- Similarly, A is logically equivalent to B iff A is a logical consequence of B and B is a logical consequence of A.