Puzzle

There are three defendants – A, B, and C – and the following facts are known:

1. If A is innocent, then both B and C are guilty.
2. If A is guilty, then B is also guilty.
3. If C is guilty, then B is innocent.

Note that you do not know how many of these defendants are guilty. It may be 0, 1, 2, or all 3.

Who is innocent and who is guilty?

Biconditionals and Truth Tables

Friday, 31 January

More Translation Examples

- If ***a*** is a Tet, then so are ***b*** and ***c***
  - $Tet(a) \rightarrow (Tet(b) \wedge Tet(c))$
- Neither ***a*** nor ***b*** are large unless ***c*** is
  - $\neg Large(c) \rightarrow \neg(Large(a) \vee Large(b))$
  - $\neg Large(c) \rightarrow (\neg Large(a) \wedge \neg Large(b))$
  - $(Large(a) \vee Large(b)) \rightarrow Large(c)$

More Translation Examples

- ***a*** is a Tet only if at least one of ***b*** and ***c*** is a Tet
  - $Tet(a) \rightarrow (Tet(b) \vee Tet(c))$
- ***a*** is a Tet if exactly one of ***b*** and ***c*** is a Tet
  - $((Tet(b) \wedge \neg Tet(c)) \vee (\neg Tet(b) \wedge Tet(c))) \rightarrow Tet(a)$

Translations

- ***a*** is a Tet if ***b*** is and also only if ***b*** is.
  - $(Tet(b) \rightarrow Tet(a)) \wedge (Tet(a) \rightarrow Tet(b))$
- ***a*** is a Tet if ***b*** is but ***a*** is not a Tet unless ***b*** is.
  - $(Tet(b) \rightarrow Tet(a)) \wedge (\neg Tet(b) \rightarrow \neg Tet(a))$
- ***a*** is a Tet if and only if (exactly when, just in case) ***b*** is
  - $Tet(a) \leftrightarrow Tet(b)$

The Biconditional

- Another new connective: the biconditional (↔).
- If A and B are sentences, then A ↔ B is a sentence.
- A sentence of the form P ↔ Q is true iff P and Q have the same truth value.
- Generally, the English expression used to express the biconditional is "if and only if". Our book also uses "just in case".

The Biconditional

- Truth table for the biconditional:

A	B	$A \leftrightarrow B$
TRUE	TRUE	TRUE
TRUE	FALSE	FALSE
FALSE	TRUE	FALSE
FALSE	FALSE	TRUE

- $A \leftrightarrow B$ is logically equivalent to $(A \rightarrow B) \wedge (B \rightarrow A)$

The Biconditional

- $A \leftrightarrow B$ is logically equivalent to $(A \rightarrow B) \wedge (B \rightarrow A)$
- $A \leftrightarrow B$ is logically equivalent to $(A \rightarrow B) \wedge (\neg A \rightarrow \neg B)$
- $A \leftrightarrow B$ is logically equivalent to $(A \wedge B) \vee (\neg A \wedge \neg B)$

The Biconditional

- $A \leftrightarrow B$ just means that A and B have the same truth value (it could just be a coincidence)
- $\neg(A \leftrightarrow B)$ means that A and B have different truth values
- So $\neg(A \leftrightarrow B)$ is logically equivalent to $\neg A \leftrightarrow B$ which is logically equivalent to $A \leftrightarrow \neg B$. (Which means exactly one of A and B)

Equivalence

- We saw some equivalences already:
  - Neither A nor B
    - $\neg(A \lor B)$ is equivalent to $\neg A \land \neg B$
  - Not both A and B
    - $\neg(A \land B)$ is equivalent to $\neg A \lor \neg B$
    - These two equivalences are called "DeMorgan's Laws"

Equivalence

- We denote FOL equivalences using the symbol ⇔, e.g., ¬¬P ⇔ P

- When two sentences are logically equivalent, they have the same truth conditions, i.e., are true in the same circumstances.

- DeMorgan'sLaws:
  ¬(P∧Q) ⇔ (¬P∨¬Q)   ¬(P∨Q) ⇔ (¬P∧¬Q)

Equivalence

- There are LOTS of equivalences. Some more obvious than others.
- Either Alice or Bill went to the party
  - $P(a) \vee P(b)$
  - $P(b) \vee P(a)$
  - $\neg\neg P(a) \vee \neg\neg\neg\neg P(b)$
  - $\neg[\neg P(a) \wedge \neg P(b)]$
  - $[\neg P(a) \rightarrow P(b)] \wedge [\neg P(b) \rightarrow P(a)]$
  - $[\neg P(a) \rightarrow P(b)]$

Truth Tables

- Truth tables show how the truth value of a complex sentence depends on the truth values of its components in all possible cases.
- They also help us keep track of relationships that exist between the truth values of different sentences.
- So, for example, we can use truth tables to show logical equivalence.
- Two sentences are logically equivalent if they have the same truth values in all possible circumstances.

Truth Tables

- Example: truth table for ¬(P∧Q)
  First, give truth conditions of the atomic sentences:

P	Q	¬ (P ∧ Q)	
T	T	T T	
T	F	T F	
F	T	F T	
F	F	F F	

Truth Tables

- Example: truth table for ¬(P∧Q)
  Then assign truth conditions of the combinations:

P	Q	¬ (P ∧ Q)	
T	T	**F** T **T** T	
T	F	**T** T **F** F	
F	T	**T** F **F** T	
F	F	**T** F **F** F	

Truth Tables

- Example: truth table for (¬P∨¬Q)
  First, give truth conditions of the atomic sentences:

P	Q		(¬ P ∨ ¬ Q)
T	T		T T
T	F		T F
F	T		F T
F	F		F F

Truth Tables

- Example: truth table for (¬P∨¬Q)
  Then assign truth conditions of the combinations:

P	Q		(¬ P ∨ ¬ Q)
T	T		**F** T **F** **F** T
T	F		**F** T **T** **T** F
F	T		**T** F **T** **F** T
F	F		**T** F **T** **T** F

TRUTH TABLES

- Example: joint truth table for ¬(P∧Q) and (¬P∨¬Q)
  This shows that the two sentences are equivalent.

P	Q	¬ (P ∧ Q)	(¬ P ∨ ¬ Q)
T	T	**F** T **T** T	**F** T **F** **F** T
T	F	**T** T **F** F	**F** T **T** **T** F
F	T	**T** F **F** T	**T** F **T** **F** T
F	F	**T** F **F** F	**T** F **T** **T** F

TRUTH TABLES

We will construct truth tables in Boole.