

# PUZZLE

Suppose that in the country of Knights and Knaves you meet three individuals, A, B, and C. You discover that at least one of them is a Knight and at least one of them is a Knave.

A says “B or C is a Knight” and  
B says “A or C is a Knight.”

Which of them are Knights and which are Knaves?

# THE BOOLEAN CONNECTIVES

Monday, 27 January

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- Note: SameSize( $\neg a$ , b) is NOT a sentence.

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$\neg(\text{Cube}(a) \wedge a=b) \vee (\text{Larger}(b, c) \vee \neg \text{Medium}(b))$

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- Parentheses are used to determine the order of the connectives and disambiguate sentences.

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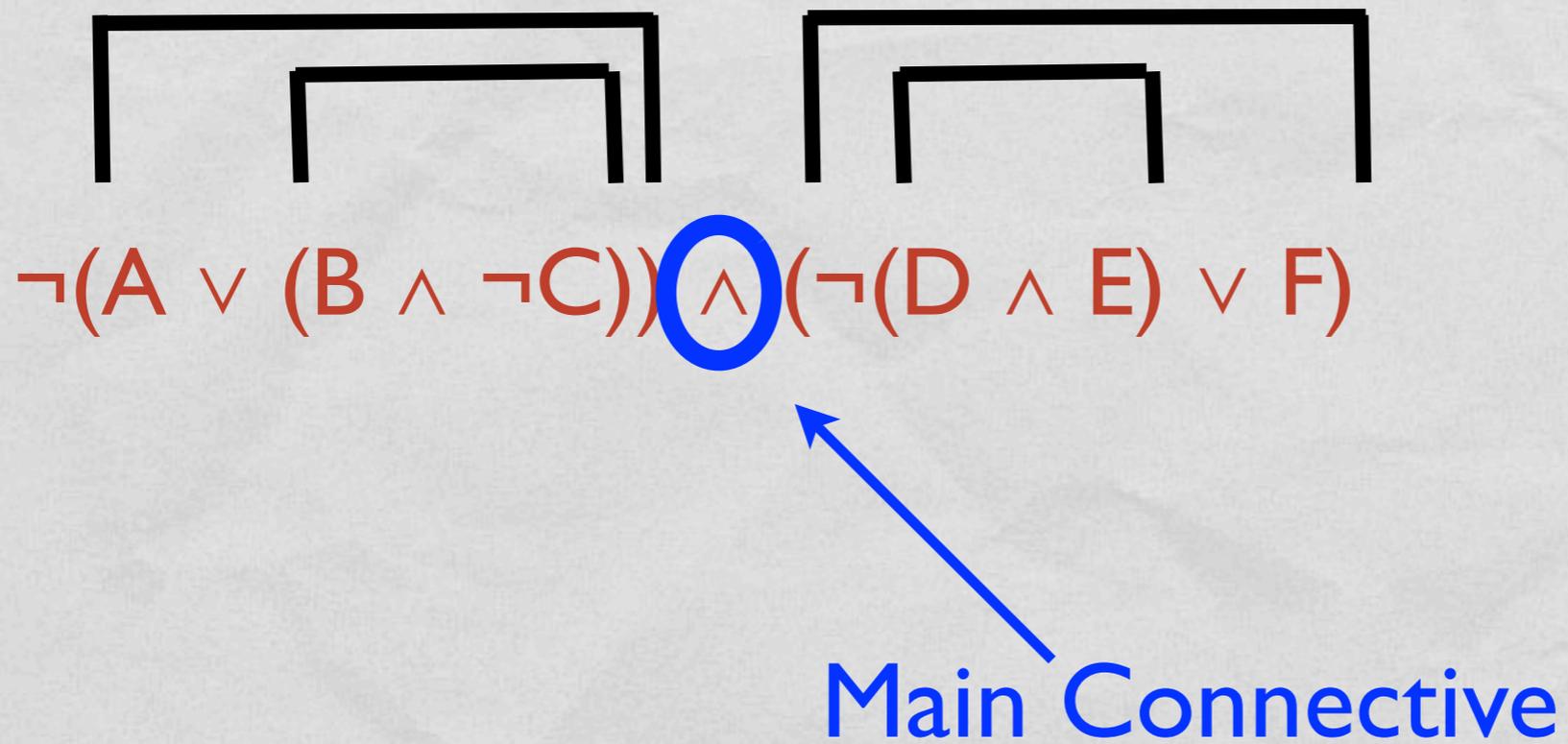
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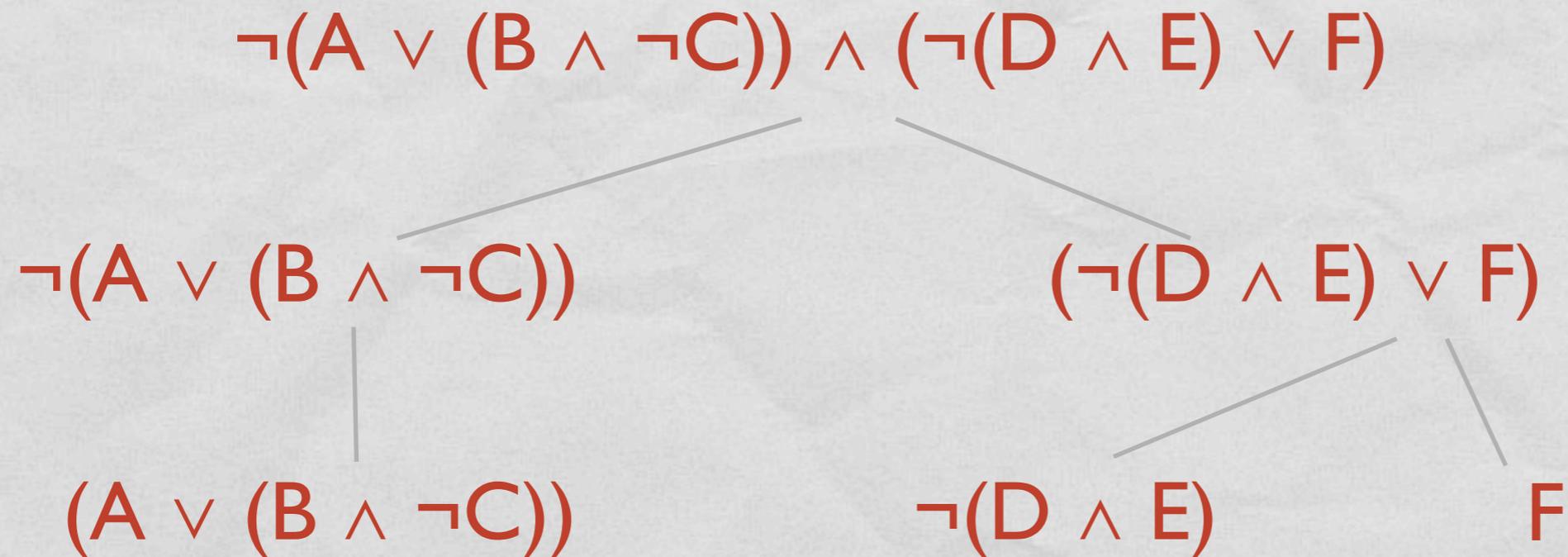
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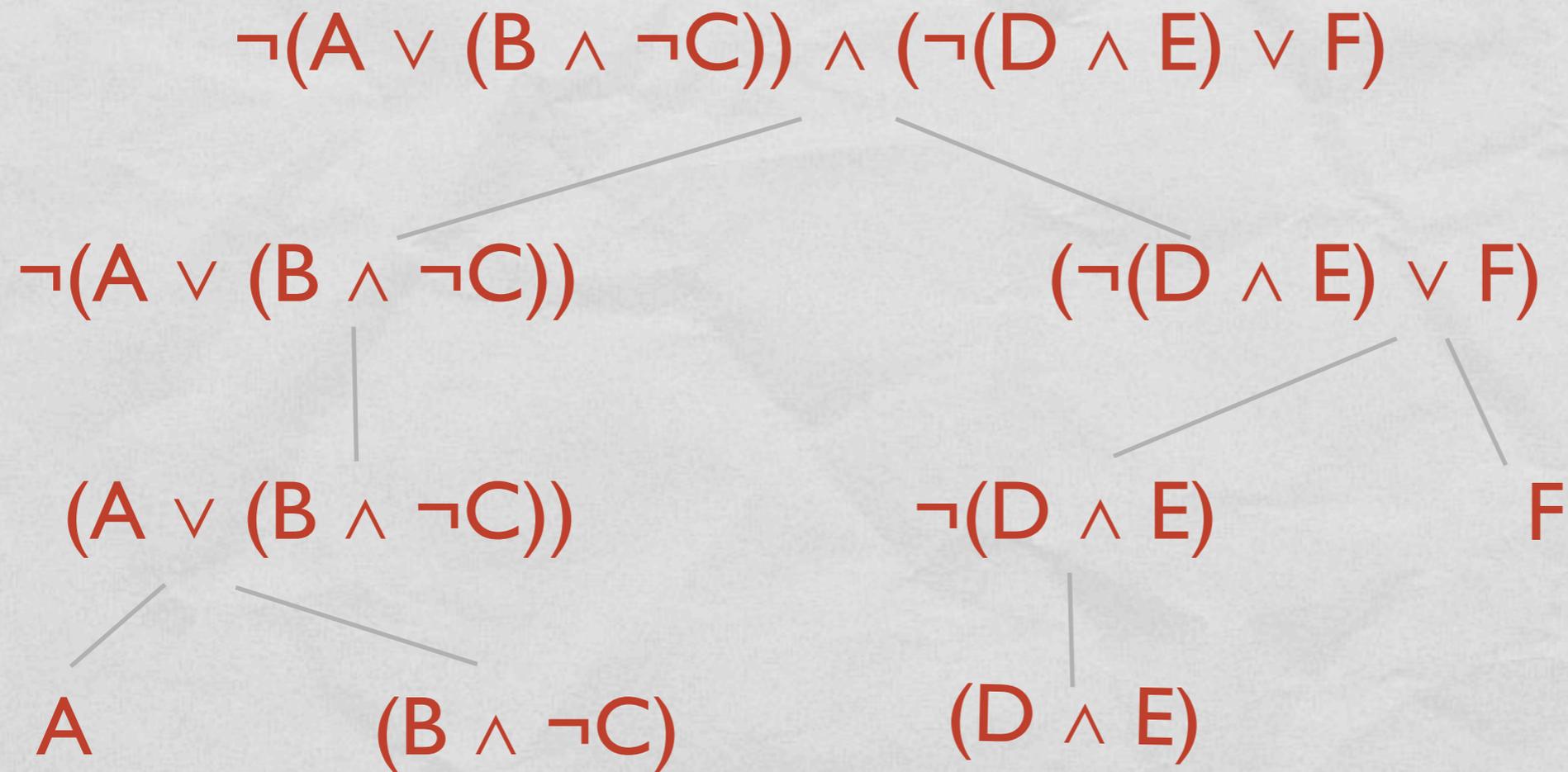
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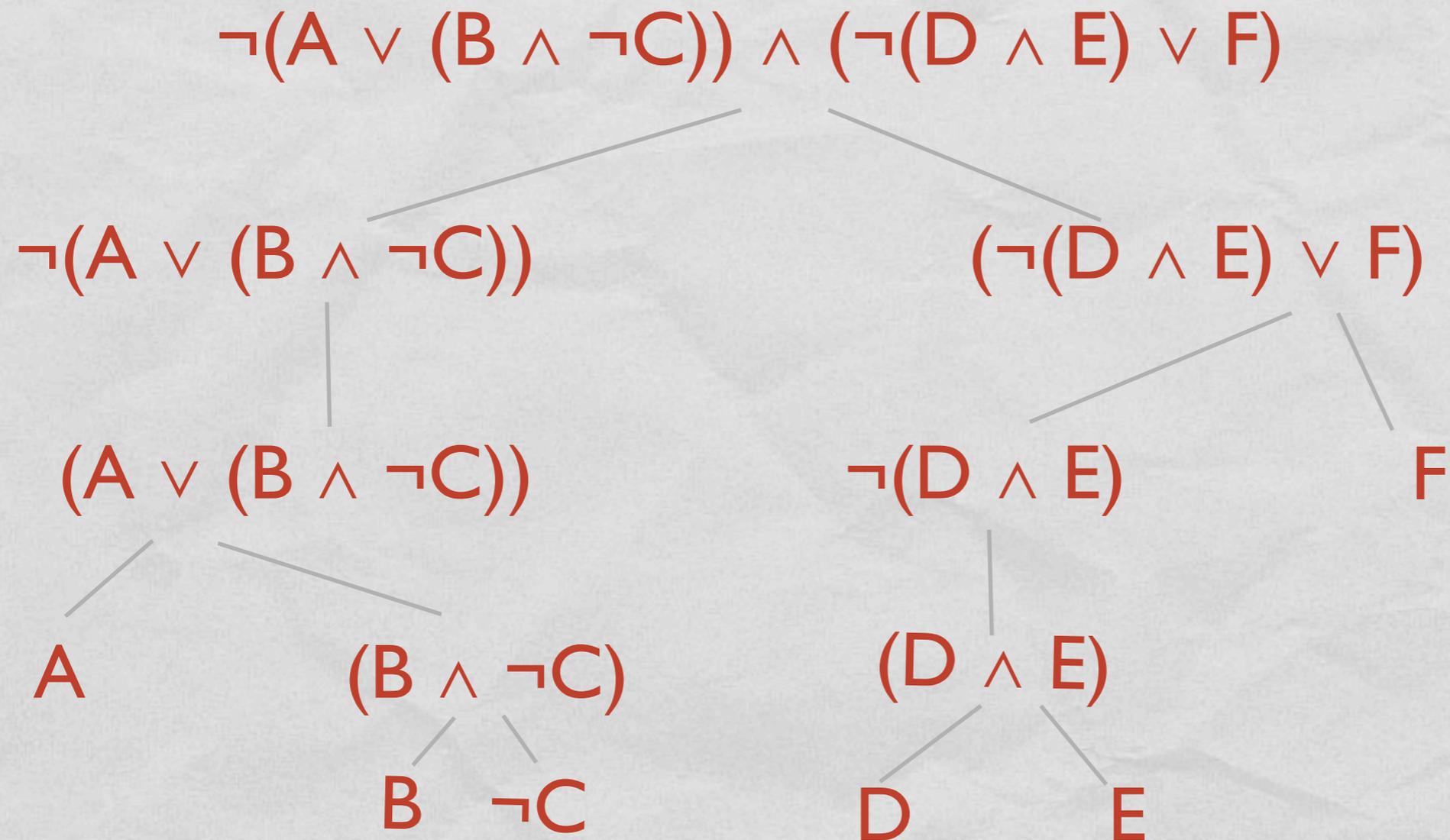
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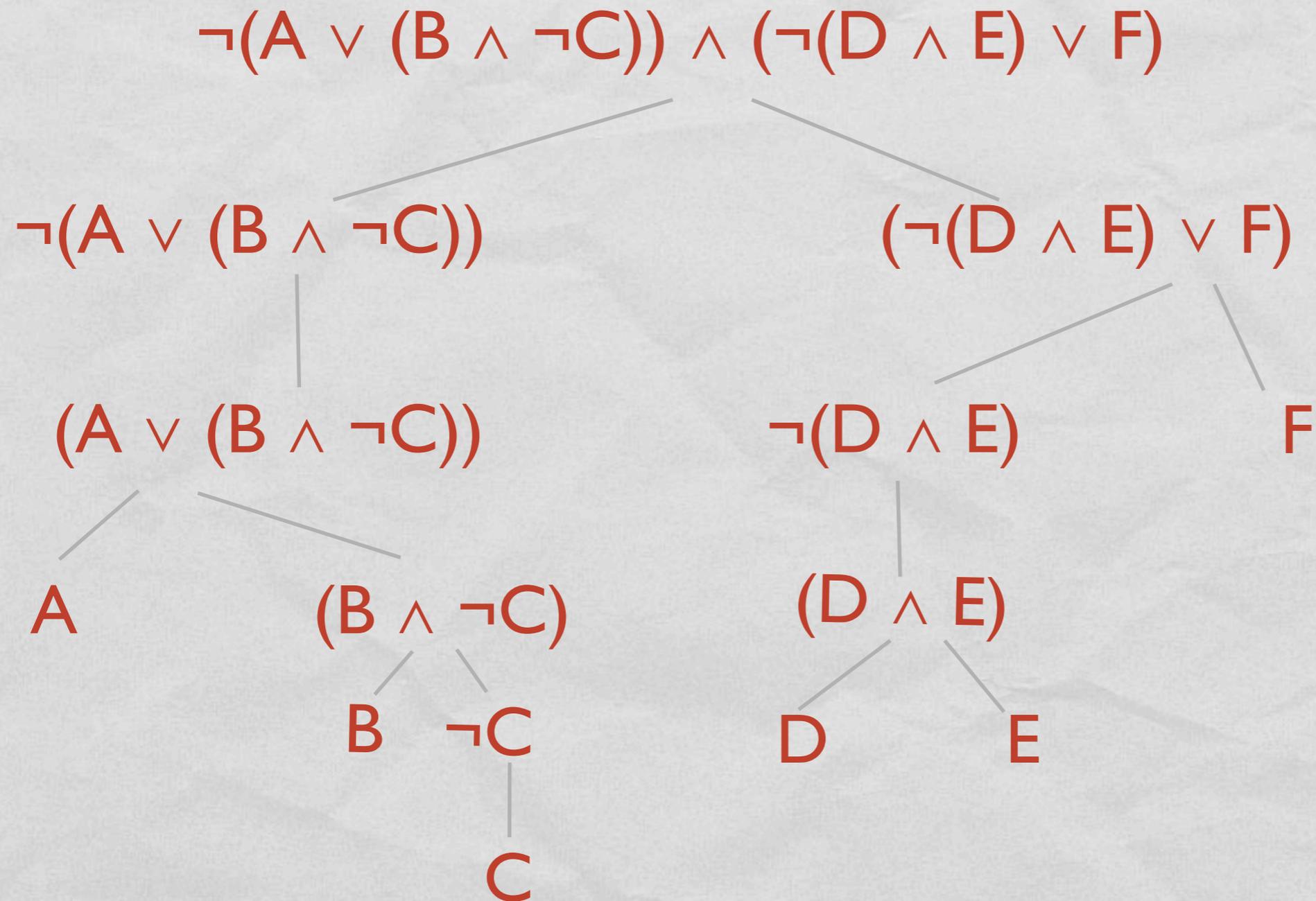
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- 'And' in English can link two names or properties. For example, Sam and Sarah had breakfast; Sam had breakfast and went to the park.  
In FOL, conjunction only links two sentences.
- In English, 'and' is often used to imply causation or a temporal sequence.  
'And' does not have this implication in FOL.

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- In English, 'A or B' is often used to mean that either A is true, or B is true, but not both (exclusive or).

In FOL, exclusive or (exactly one of) could be expressed as  $(A \vee B) \wedge \neg(A \wedge B)$

The simple disjunction  $(A \vee B)$  is always the inclusive or (at least one of and maybe both).

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