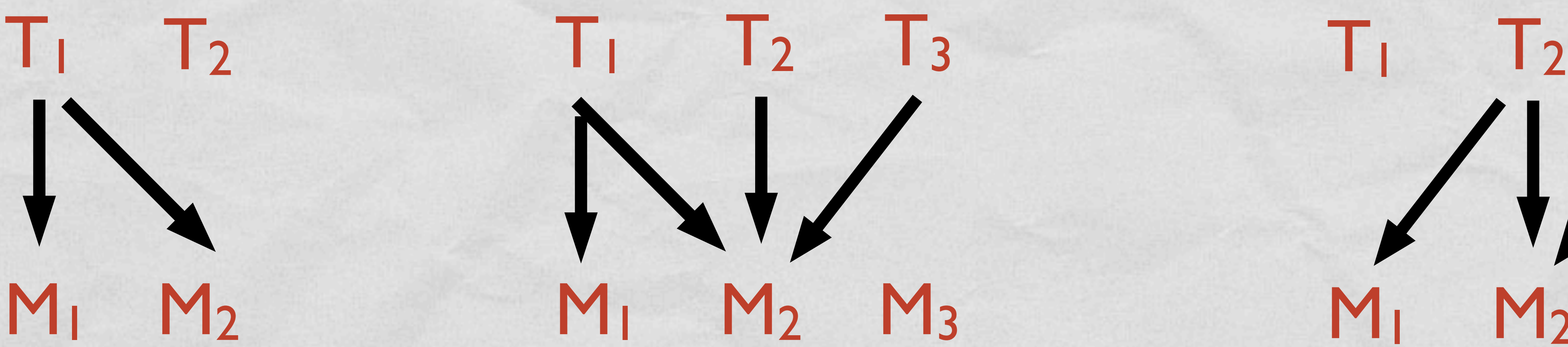
Is the answer to this question “no”?

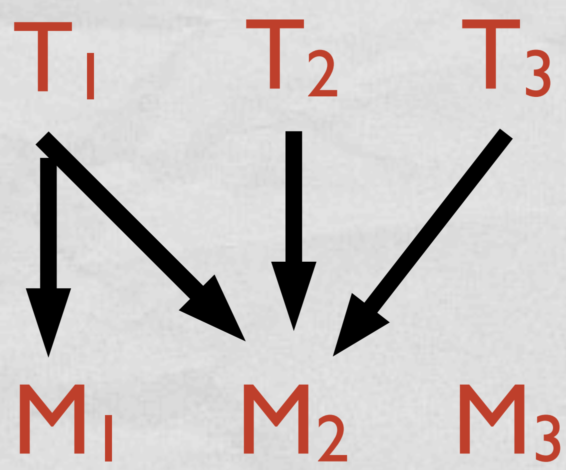
Diagrams and Identity

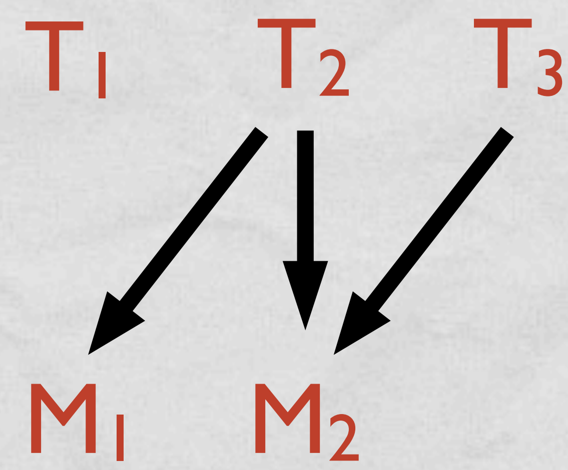
Friday, 1 May

Reading Diagrams



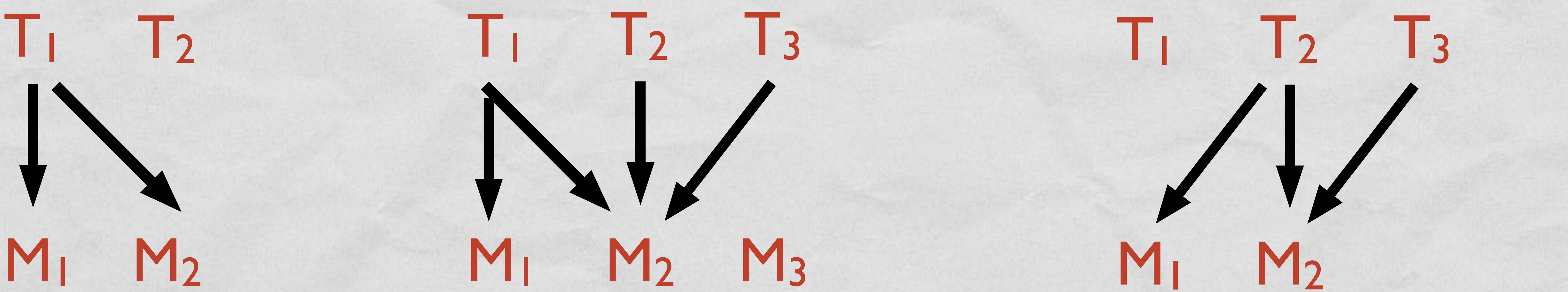
Reading Diagrams





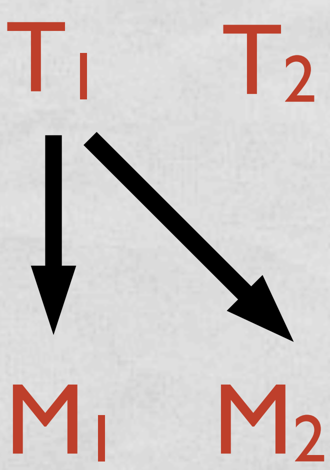
$\exists x(T(x) \wedge \forall y(M(y) \rightarrow A(x,y)))$

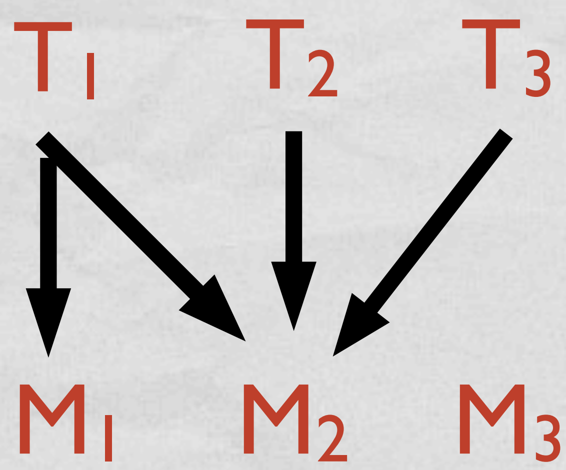
Reading Diagrams

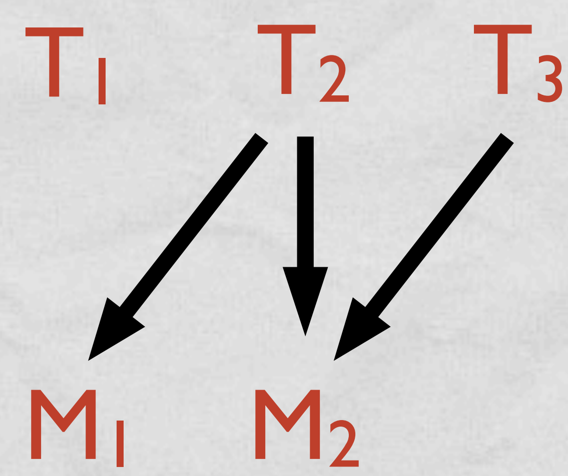


$\exists x(T(x) \wedge \forall y(M(y) \rightarrow A(x,y)))$ True, False, True

Reading Diagrams





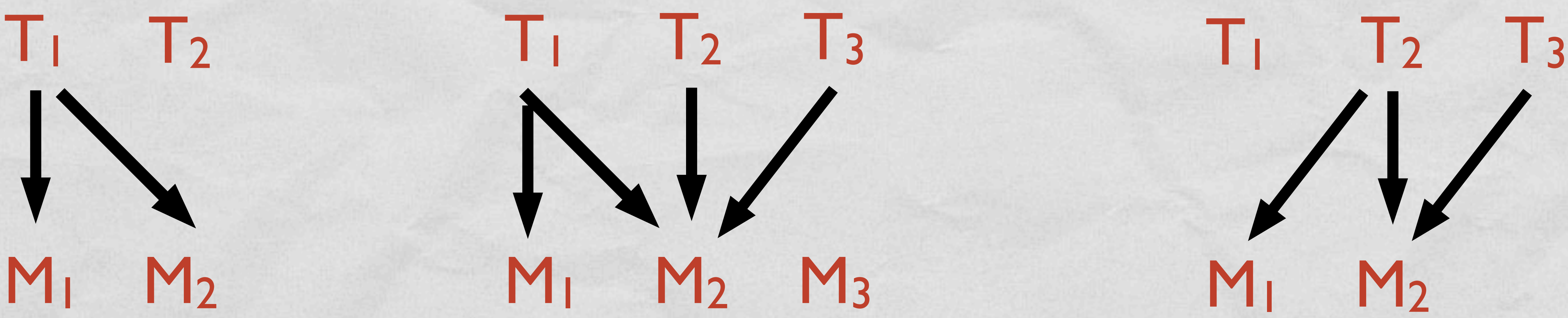


$\exists x(T(x) \wedge \forall y(M(y) \rightarrow A(x,y)))$

$\forall x(M(x) \rightarrow \exists y(T(y) \wedge A(y,x)))$

True, False, True

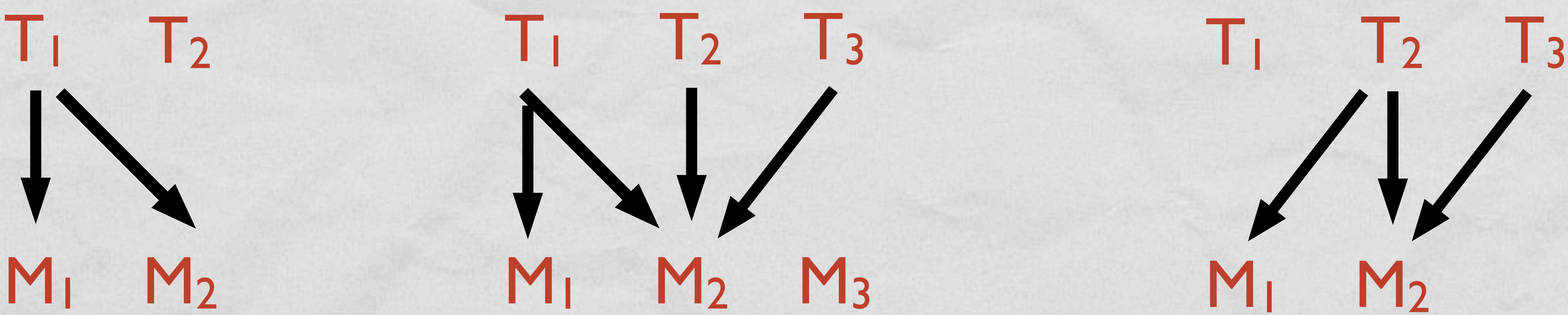
Reading Diagrams



$\exists x(T(x) \wedge \forall y(M(y) \rightarrow A(x,y)))$ True, False, True

$\forall x(M(x) \rightarrow \exists y(T(y) \wedge A(y,x)))$ True, False, True

Reading Diagrams

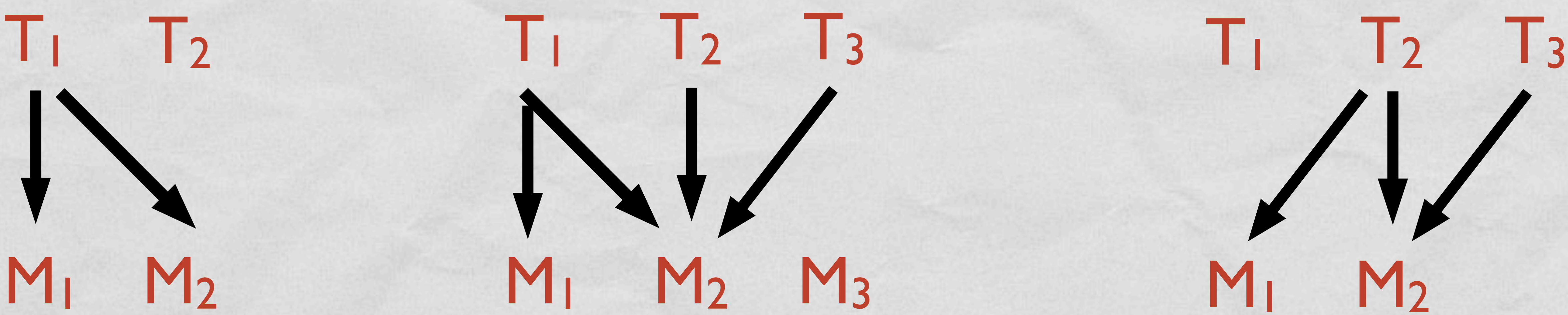


$\exists x(T(x) \wedge \forall y(M(y) \rightarrow A(x,y)))$ True, False, True

$\forall x(M(x) \rightarrow \exists y(T(y) \wedge A(y,x)))$ True, False, True

$\exists x(M(x) \wedge \forall y(T(y) \rightarrow \neg A(y,x))$

Reading Diagrams

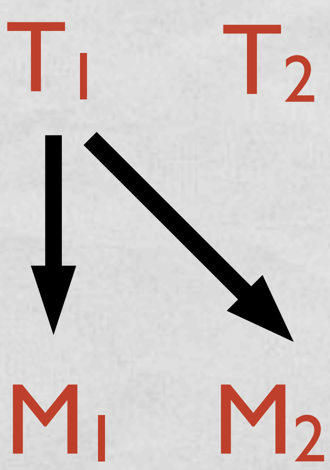


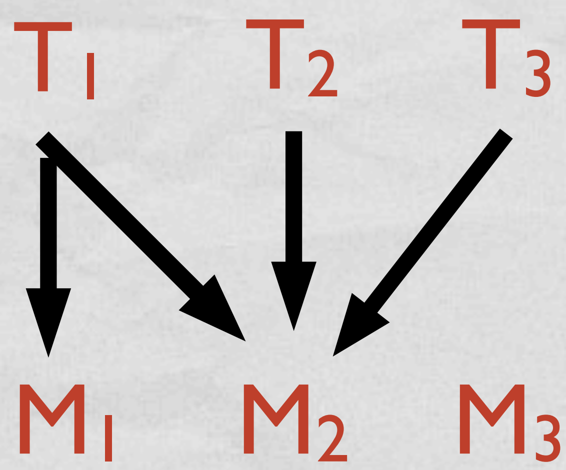
$\exists x(T(x) \wedge \forall y(M(y) \rightarrow A(x,y)))$ True, False, True

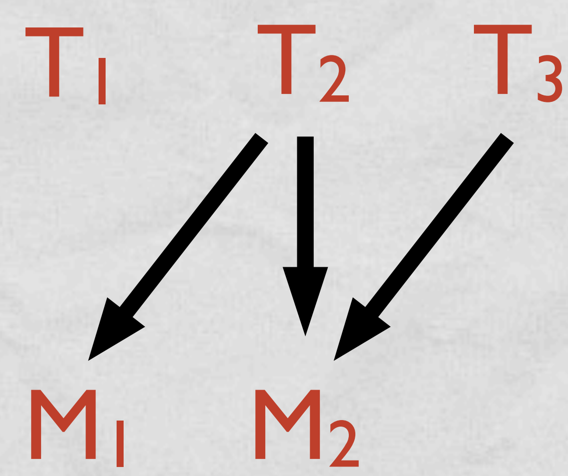
$\forall x(M(x) \rightarrow \exists y(T(y) \wedge A(y,x)))$ True, False, True

$\exists x(M(x) \wedge \forall y(T(y) \rightarrow \neg A(y,x)))$ False, True, False

Reading Diagrams





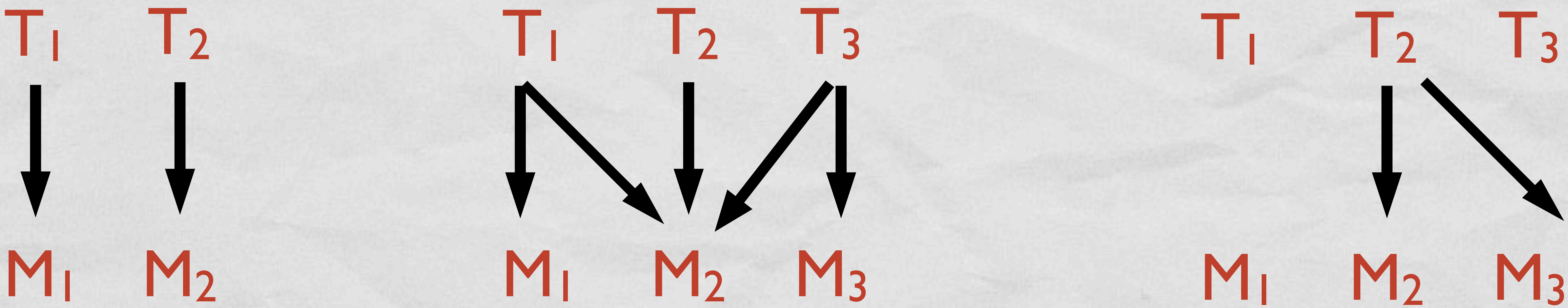


$\exists x(T(x) \wedge \forall y(M(y) \rightarrow A(x,y)))$ True, False, True

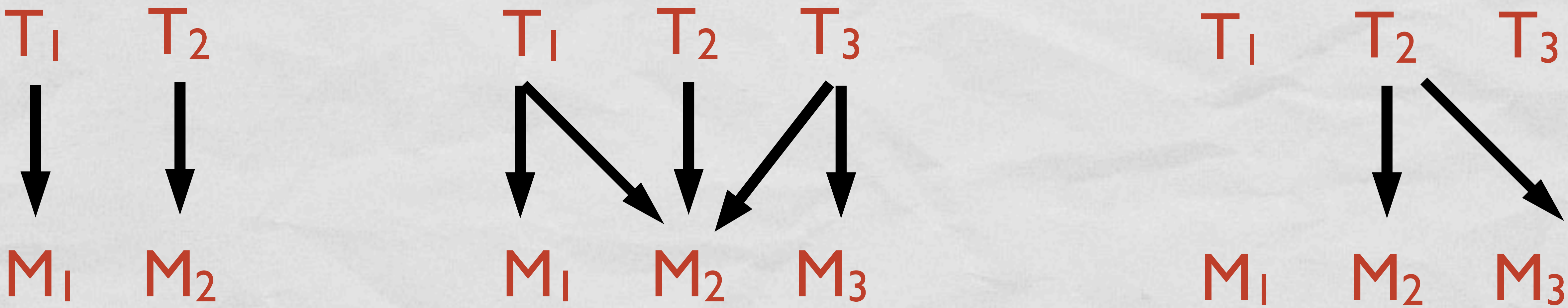
$\forall x(M(x) \rightarrow \exists y(T(y) \wedge A(y,x)))$ True, False, True

$\exists x(M(x) \wedge \forall y(T(y) \rightarrow \neg A(y,x))$ False, True, False

Notice this last sentence is the direct contradiction of the previous one

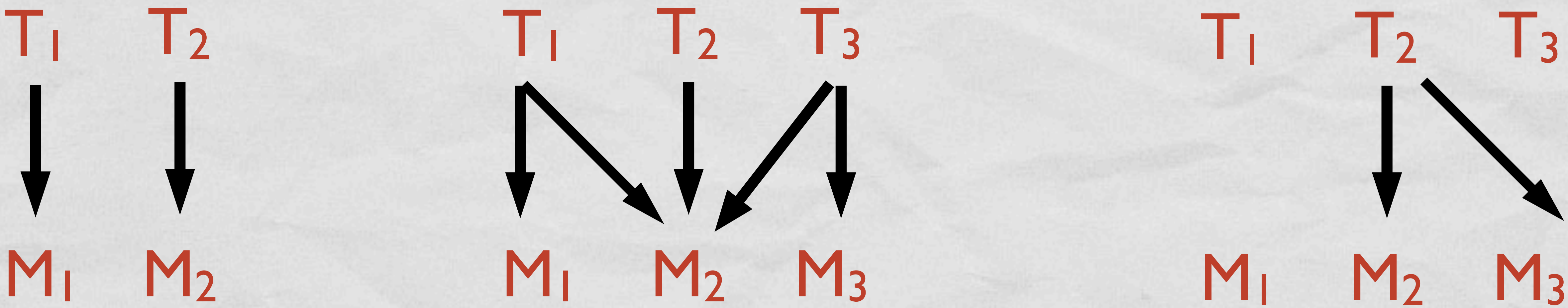


$$\exists x \exists y (T(x) \land T(y) \land \forall z (M(z) \rightarrow (A(x,z) \lor A(y,z))))$$



$$\exists x \exists y(T(x) \land T(y) \land \forall z(M(z) \rightarrow (A(x,z) \lor A(y,z))))$$

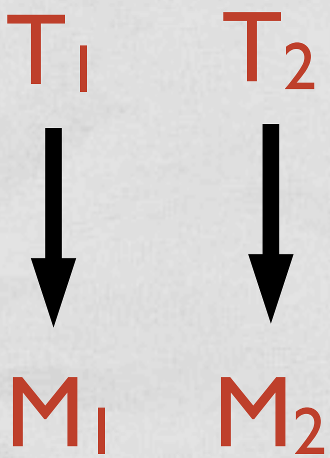
There is a pair of Ts such that for every M, either the first T went or the second one did.

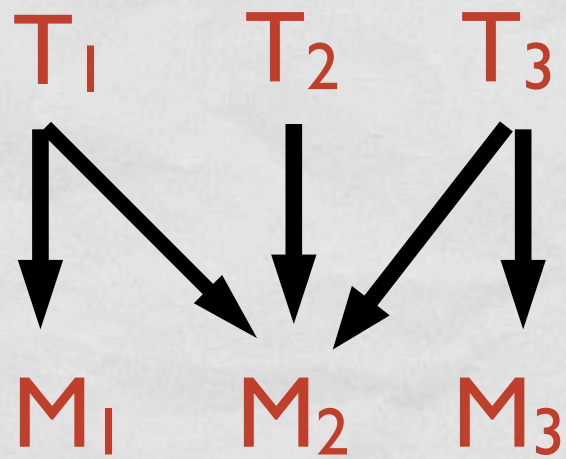


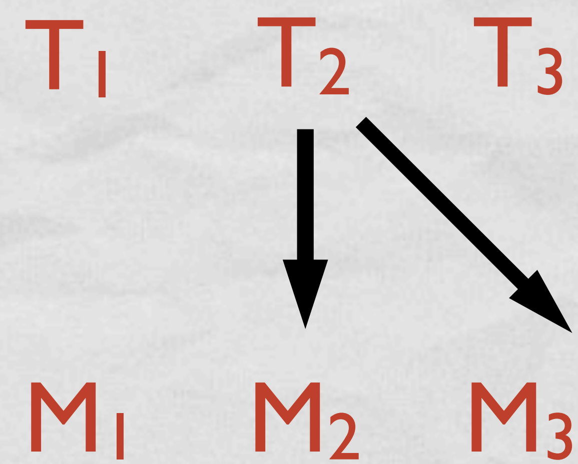
$\exists x \exists y(T(x) \wedge T(y) \wedge \forall z(M(z) \rightarrow (A(x,z) \vee A(y,z))))$

There is a pair of Ts such that for every M, either the first T went or the second one did.

So between them they cover every M.



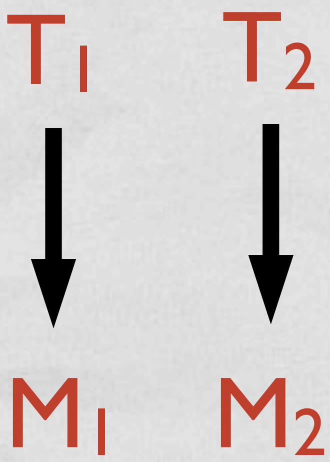


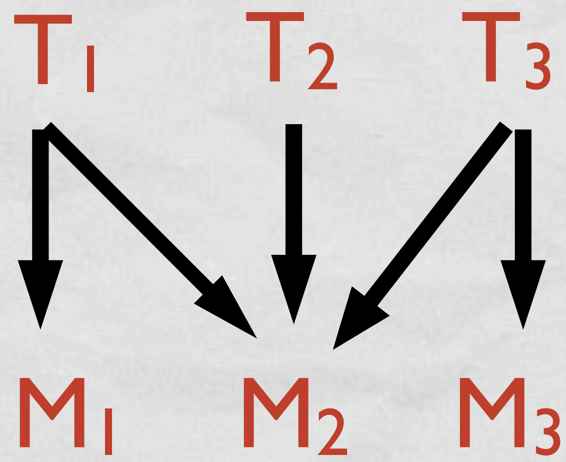


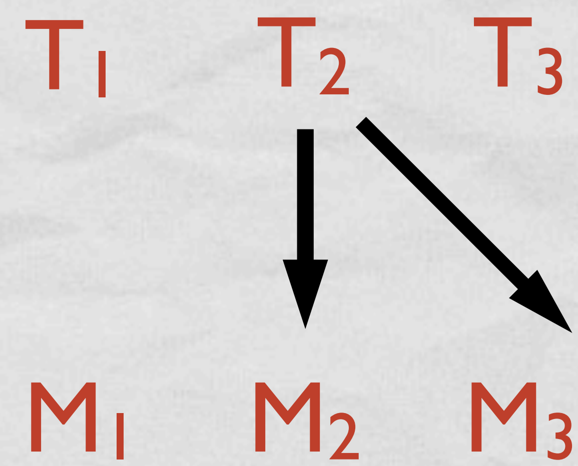
$$\exists x \exists y (T(x) \wedge T(y) \wedge \forall z (M(z) \rightarrow (A(x,z) \vee A(y,z))))$$

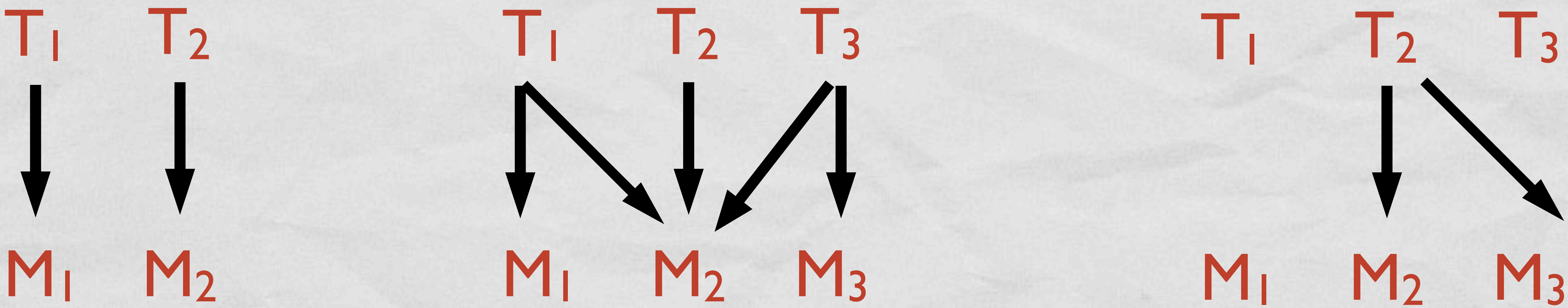
There is a pair of Ts such that for every M, either the first T went or the second one did. T, T, F

So between them they cover every M.

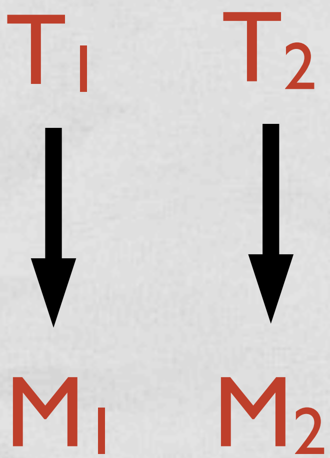

T1
T2
M1
M2

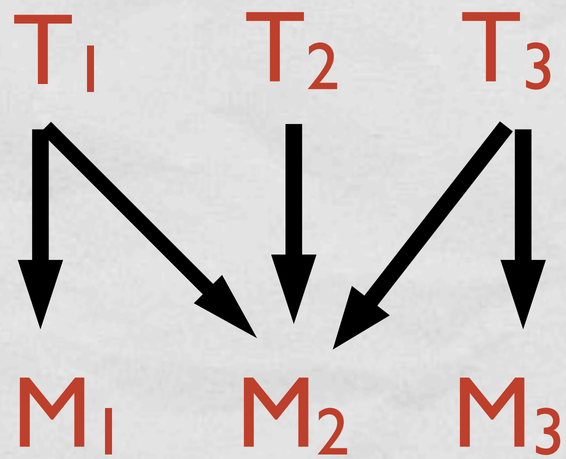

T1
T2
T3
M1
M2
M3

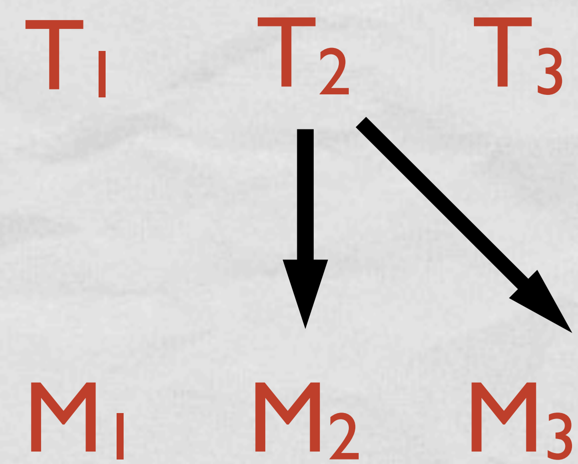

T1
T2
T3
M1
M2
M3



$$\forall x(M(x) \rightarrow \exists y \exists z(T(y) \wedge T(z) \wedge (A(y,x) \leftrightarrow A(z,x)))$$

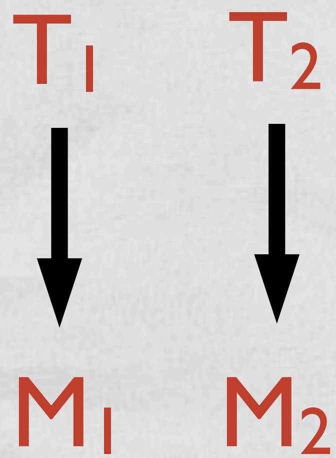


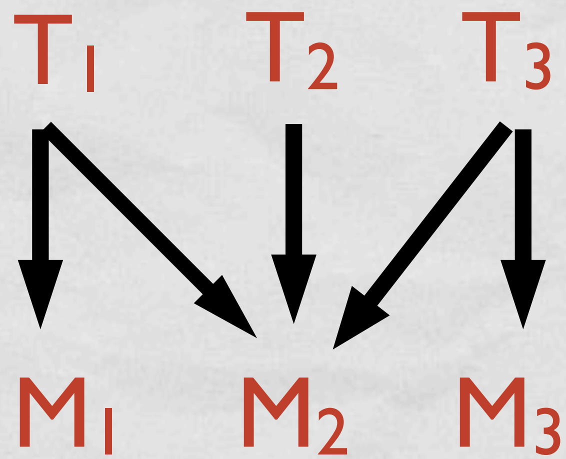


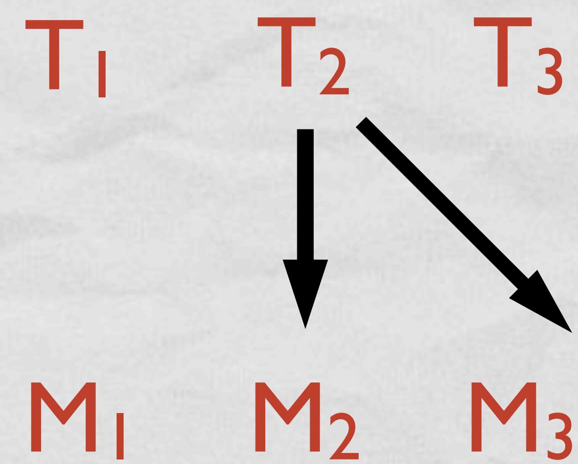


$$\forall x(M(x) \rightarrow \exists y \exists z(T(y) \wedge T(z) \wedge (A(y,x) \leftrightarrow A(z,x))))$$

For every M, there is a pair of Ts such that the first went to the M if and only if the second did.



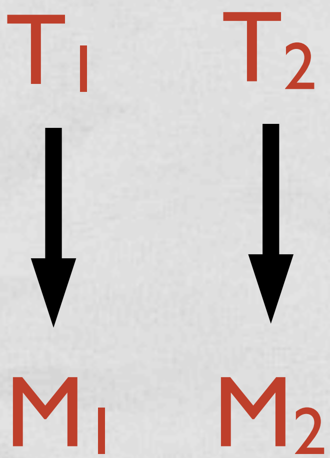


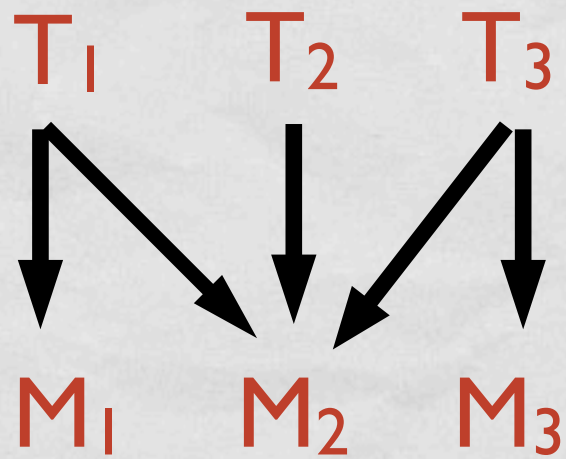


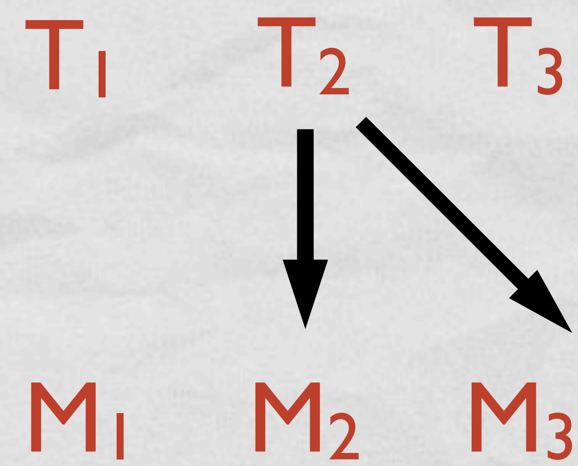
$$\forall x(M(x) \rightarrow \exists y \exists z(T(y) \wedge T(z) \wedge (A(y,x) \leftrightarrow A(z,x)))$$

For every M, there is a pair of Ts such that the first went to the M if and only if the second did.

T, T, T





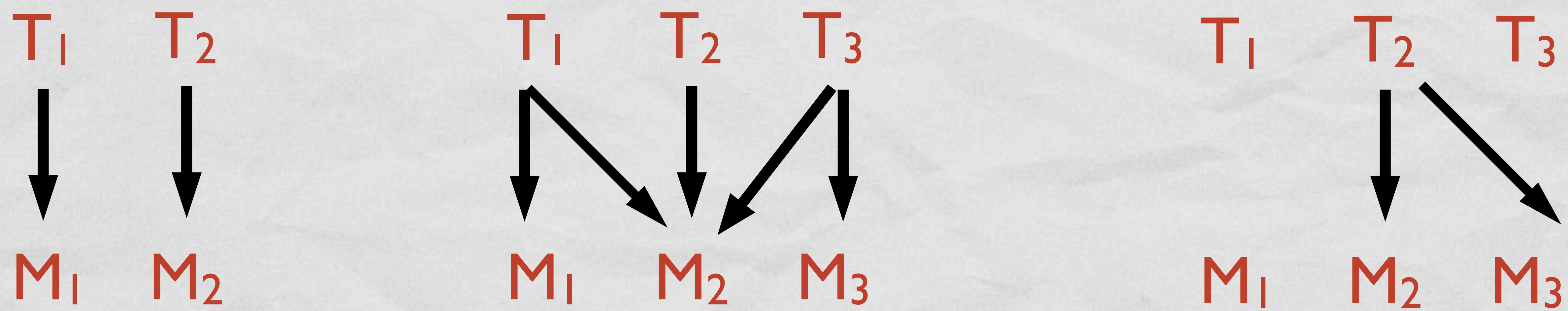


$$\forall x(M(x) \rightarrow \exists y \exists z(T(y) \wedge T(z) \wedge (A(y,x) \leftrightarrow A(z,x))))$$

For every M, there is a pair of Ts such that the first went to the M if and only if the second did.

T, T, T

How could this be right? What pair could work for say the first diagram? -- Ans, <T_1, T_1>



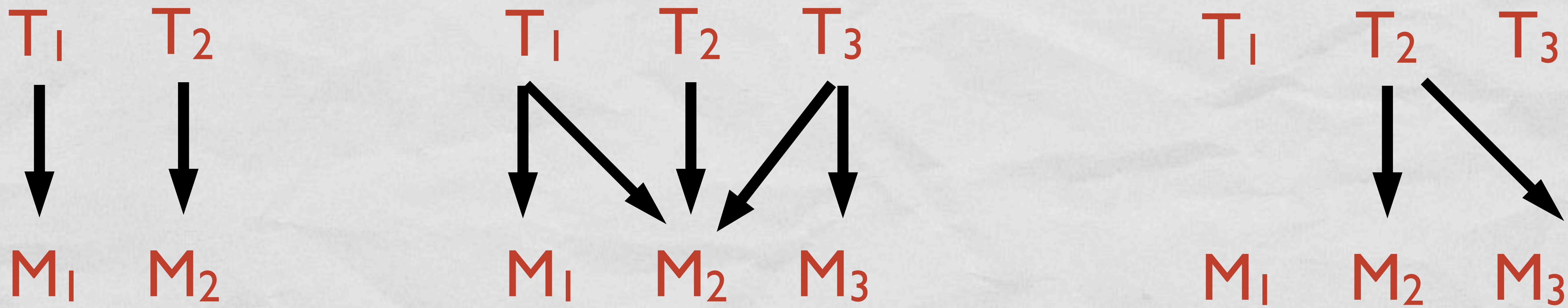
$$\forall x(M(x) \rightarrow \exists y \exists z(T(y) \wedge T(z) \wedge (A(y,x) \leftrightarrow A(z,x))))$$

For every M, there is a pair of Ts such that the first went to the M if and only if the second did.

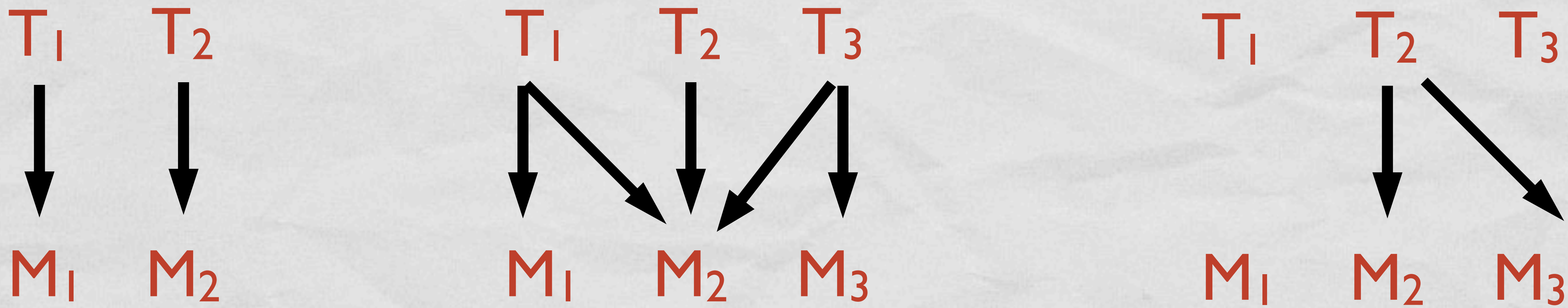
T, T, T

How could this be right? What pair could work for say the first diagram? -- Ans, <T_1, T_1>

In fact, the above sentence follows just from $\exists z\, T(y)$

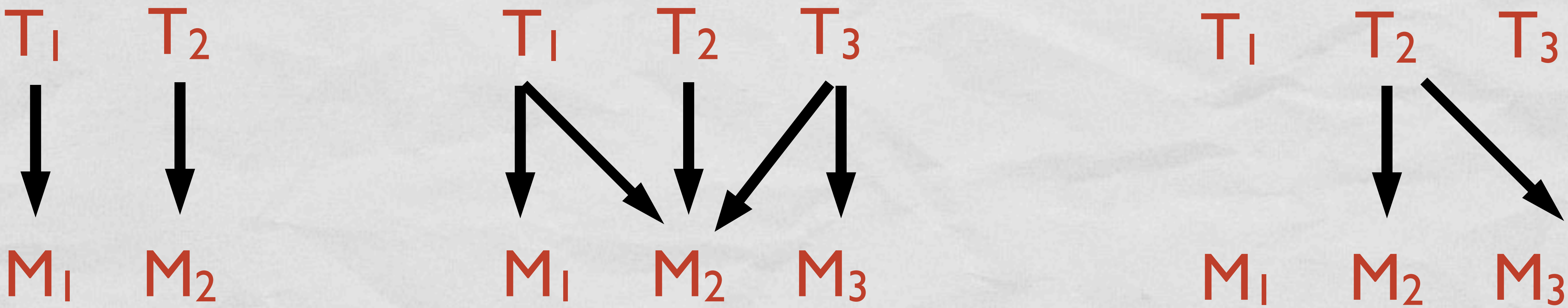


$$\forall x(M(x) \rightarrow \exists y \exists z(T(y) \wedge T(z) \wedge (A(y,x) \leftrightarrow A(z,x))))$$



$\forall x(M(x) \rightarrow \exists y \exists z(T(y) \wedge T(z) \wedge (A(y,x) \leftrightarrow A(z,x)))$

But sometimes you explicitly want to talk about pairs of distinct Ts - two *different* Ts. For this you need identity.

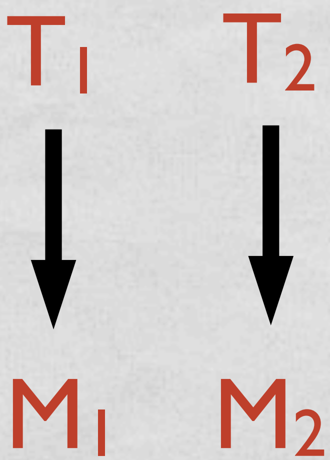


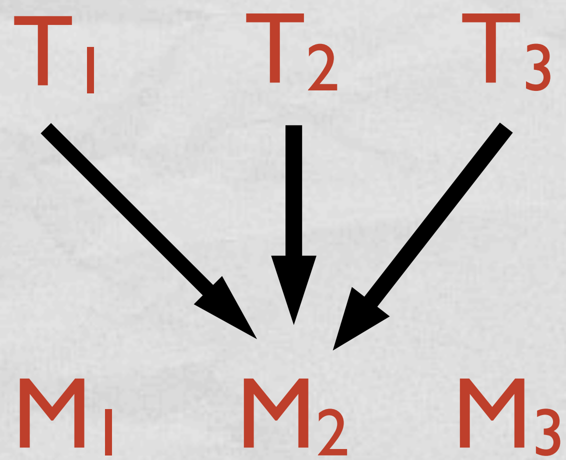
$$\forall x(M(x) \rightarrow \exists y \exists z(T(y) \wedge T(z) \wedge (A(y,x) \leftrightarrow A(z,x))))$$

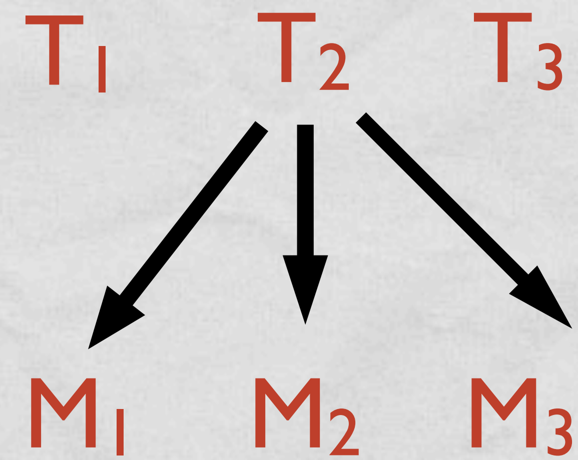
But sometimes you explicitly want to talk about pairs of distinct Ts - two *different* Ts. For this you need identity.

Identity is used whenever you want to *count* things

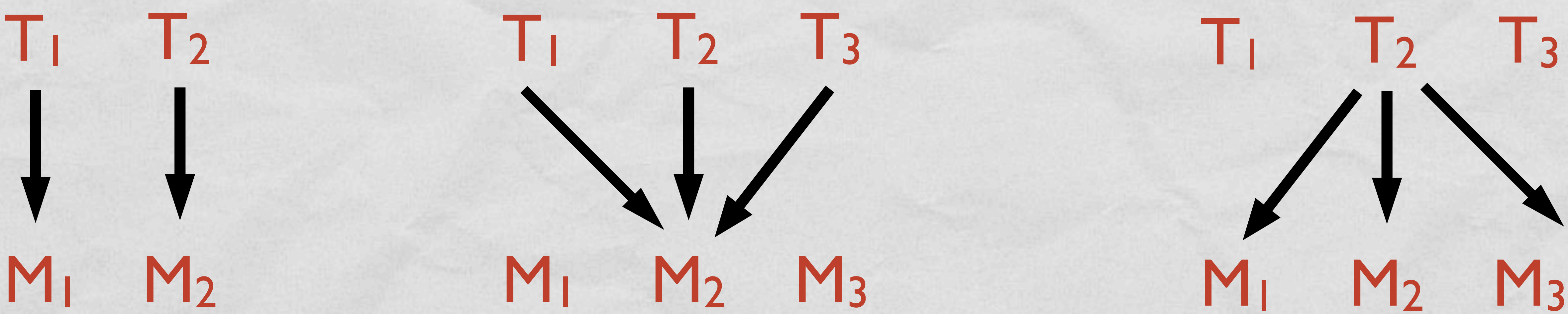
Counting in Diagrams







Counting in Diagrams



A very natural thing you might want to say about these diagrams essentially involves counting. For example, there is one teacher who went to three meetings and two teachers who went to none (true in diagram 3). For this, you need identity.

Translations with Identity

$\exists x \exists y(T(x) \land T(y))$

Translations with Identity

$\exists x \exists y(T(x) \land T(y))$

Both x and y are teachers

Translations with Identity

$\exists x \exists y(T(x) \land T(y))$

Both x and y are teachers

- but not necessarily different!

Translations with Identity

$\exists x \exists y(T(x) \wedge T(y))$

Both x and y are teachers

- but not necessarily different!

$\exists x \exists y(T(x) \wedge T(y) \wedge x \neq y)$

Translations with Identity

$\exists x \exists y(T(x) \wedge T(y))$

Both x and y are teachers

- but not necessarily different!

$\exists x \exists y(T(x) \wedge T(y) \wedge x \neq y)$

There are at least two teachers

Translations with Identity

$\exists x \exists y(T(x) \wedge T(y))$

Both x and y are teachers

- but not necessarily different!

$\exists x \exists y(T(x) \wedge T(y) \wedge x \neq y)$

There are at least two teachers

$\exists x \exists y(T(x) \wedge T(y) \wedge x \neq y \wedge \forall z(M(z) \rightarrow (A(x,z) \wedge A(y,z))))$

Translations with Identity

$\exists x \exists y(T(x) \wedge T(y))$

Both x and y are teachers

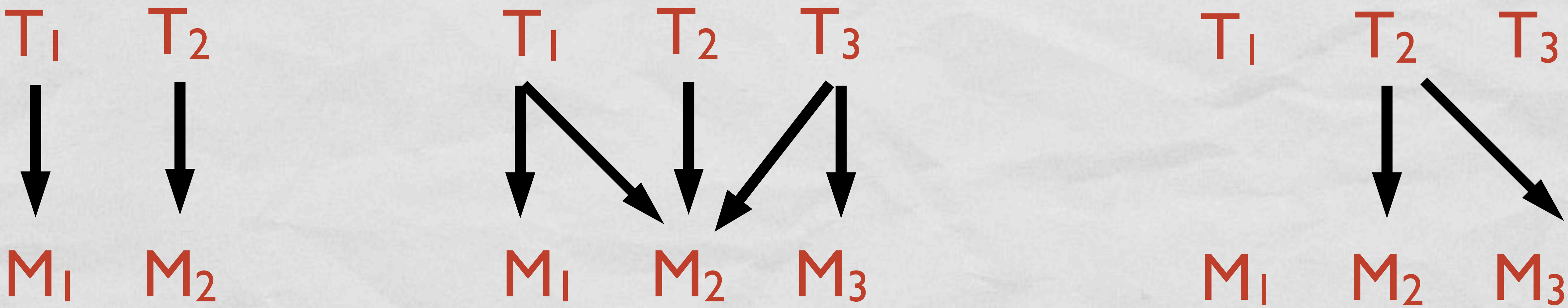
- but not necessarily different!

$\exists x \exists y(T(x) \wedge T(y) \wedge x \neq y)$

There are at least two teachers

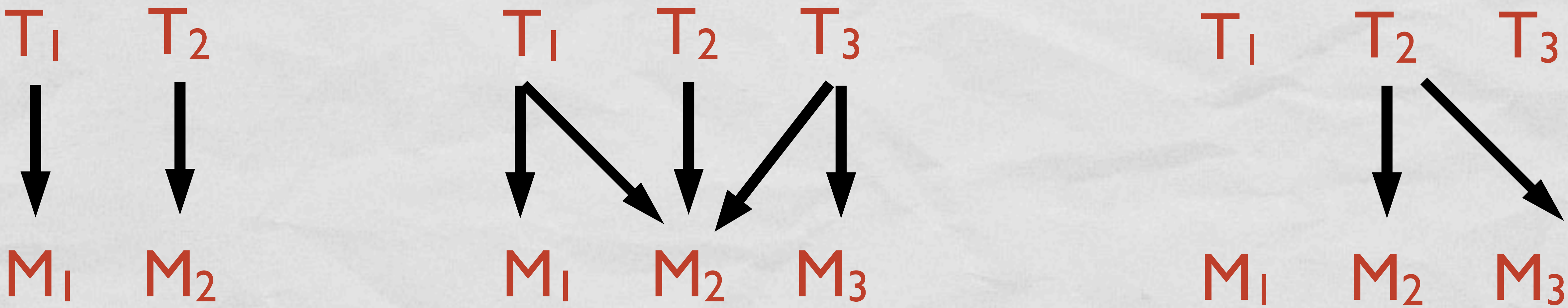
$\exists x \exists y(T(x) \wedge T(y) \wedge x \neq y \wedge \forall z(M(z) \rightarrow (A(x,z) \wedge A(y,z))))$

There are at least two teachers who attended every meeting



$\forall x(M(x) \rightarrow \exists y \exists z(T(y) \wedge T(z) \wedge (A(y,x) \leftrightarrow A(z,x)))$ T, T, T

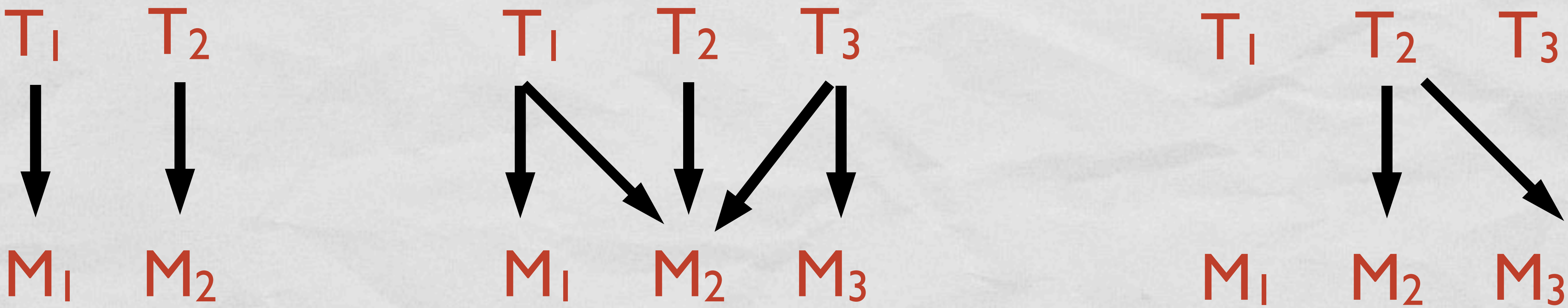
For every M, there is a pair of Ts (not necessarily different) such that the first went to the M if and only if the second did.



$\forall x(M(x) \rightarrow \exists y \exists z(T(y) \wedge T(z) \wedge (A(y,x) \leftrightarrow A(z,x)))$ T, T, T

For every M, there is a pair of Ts (not necessarily different) such that the first went to the M if and only if the second did.

$\forall x(M(x) \rightarrow \exists y \exists z(T(y) \wedge T(z) \wedge y \neq z \wedge (A(y,x) \leftrightarrow A(z,x)))$

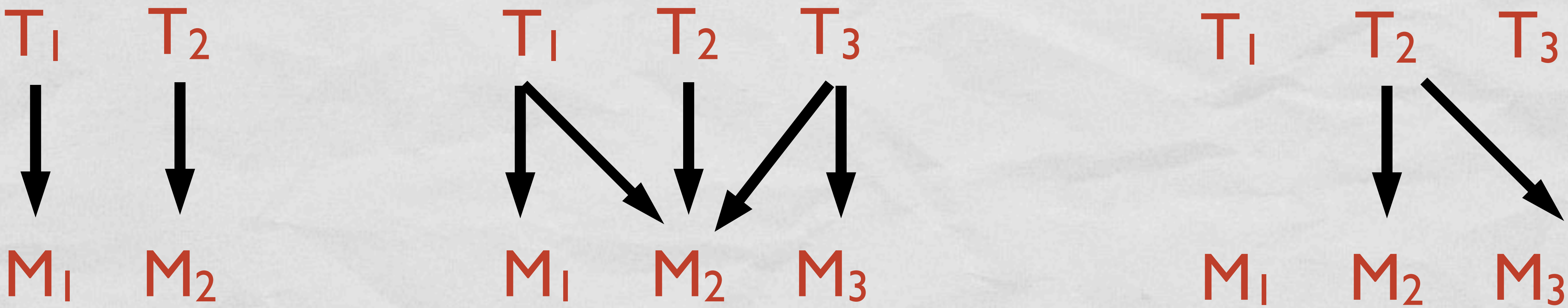


$\forall x(M(x) \rightarrow \exists y \exists z(T(y) \wedge T(z) \wedge (A(y,x) \leftrightarrow A(z,x)))$ T, T, T

For every M, there is a pair of Ts (not necessarily different) such that the first went to the M if and only if the second did.

$\forall x(M(x) \rightarrow \exists y \exists z(T(y) \wedge T(z) \wedge y \neq z \wedge (A(y,x) \leftrightarrow A(z,x)))$

For every M, there is a pair of Ts (definitely different) such that the first went to the M if and only if the second did.

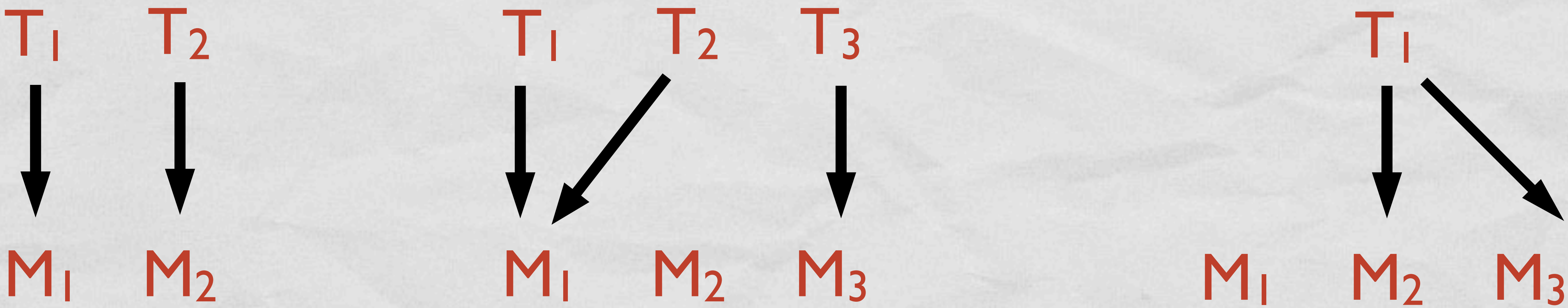


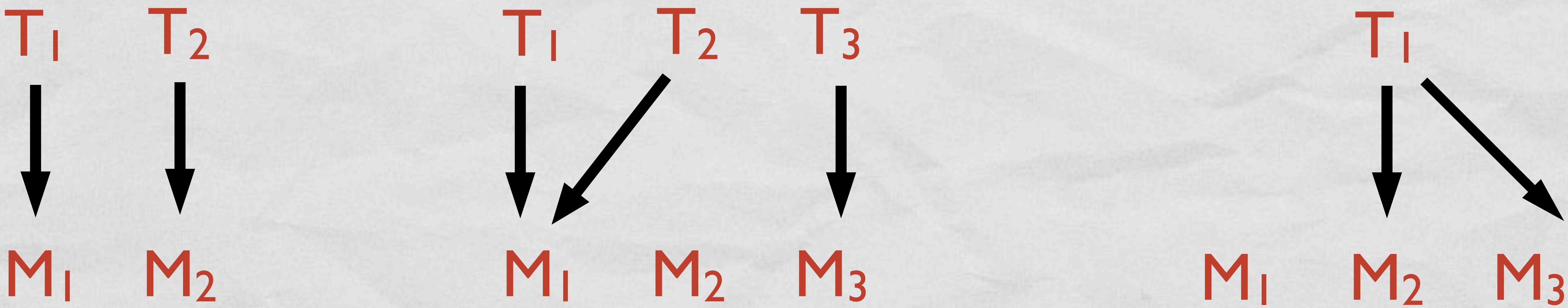
$\forall x(M(x) \rightarrow \exists y \exists z(T(y) \wedge T(z) \wedge (A(y,x) \leftrightarrow A(z,x)))$ T, T, T

For every M, there is a pair of Ts (not necessarily different) such that the first went to the M if and only if the second did.

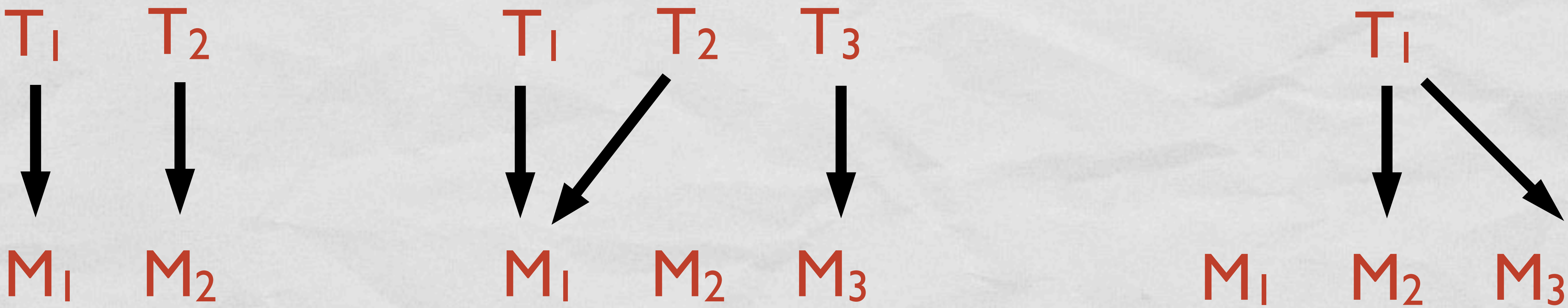
$\forall x(M(x) \rightarrow \exists y \exists z(T(y) \wedge T(z) \wedge y \neq z \wedge (A(y,x) \leftrightarrow A(z,x)))$ F, T, T

For every M, there is a pair of Ts (definitely different) such that the first went to the M if and only if the second did.


T1 T2
M1 M2
T1 T2 T3
M1 M2 M3
T1
M1 M2 M3

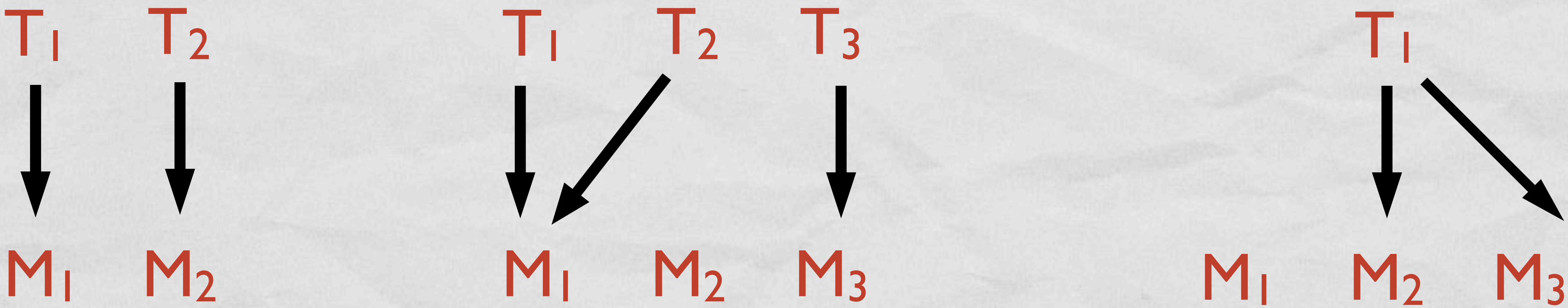


$$\exists x \exists y (T(x) \wedge T(y) \wedge x \neq y \wedge \forall z (M(z) \rightarrow (A(x,z) \leftrightarrow A(y,z))))$$



$$\exists x \exists y (T(x) \wedge T(y) \wedge x \neq y \wedge \forall z (M(z) \rightarrow (A(x,z) \leftrightarrow A(y,z))))$$

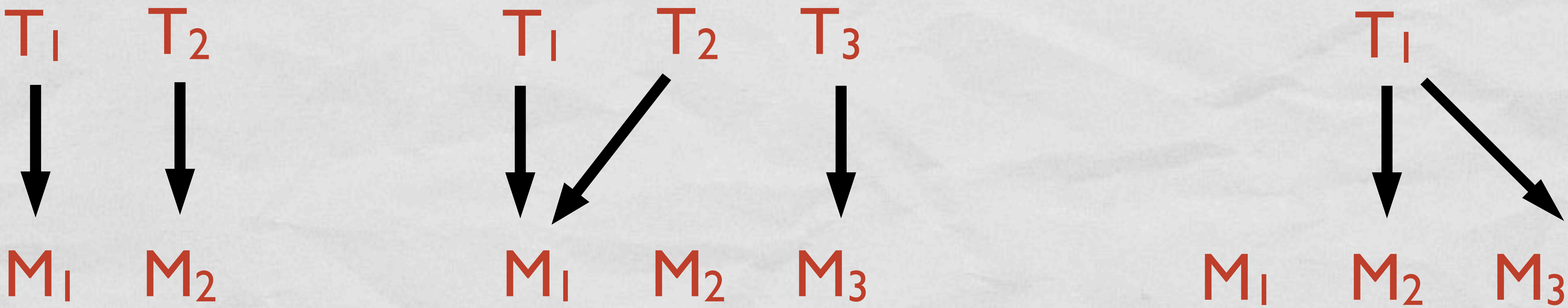
There is a pair of distinct Ts that went to the same Ms



$$\exists x \exists y (T(x) \wedge T(y) \wedge x \neq y \wedge \forall z (M(z) \rightarrow (A(x,z) \leftrightarrow A(y,z))))$$

There is a pair of distinct Ts that went to the same Ms

F, T, F

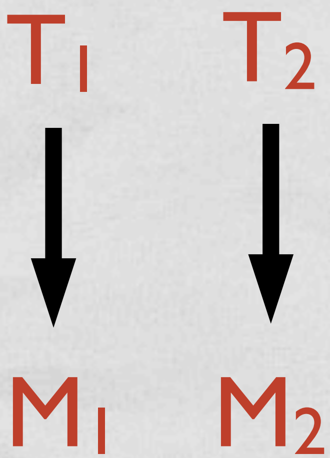


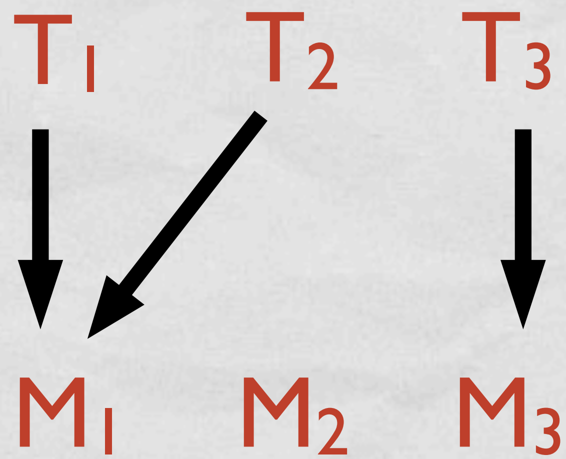
$\exists x \exists y(T(x) \wedge T(y) \wedge x \neq y \wedge \forall z(M(z) \rightarrow (A(x,z) \leftrightarrow A(y,z))))$

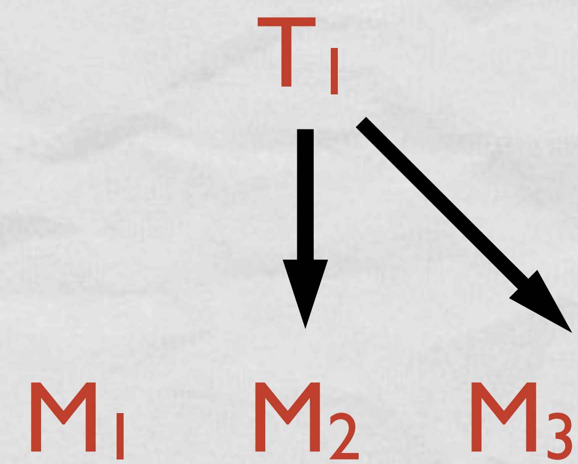
There is a pair of distinct Ts that went to the same Ms

F, T, F

$\forall x \forall y((M(x) \wedge M(y) \wedge x \neq y) \rightarrow \exists z(T(z) \wedge A(z,x) \wedge A(z,y))))$







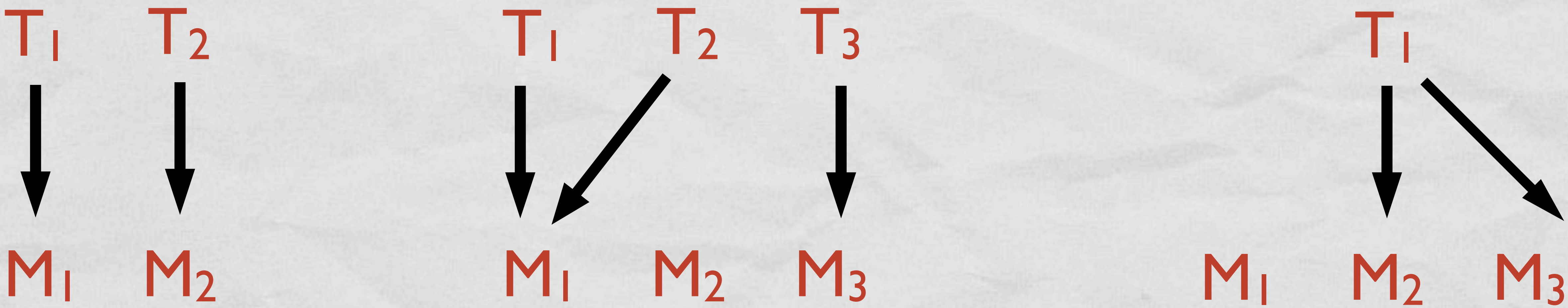
$\exists x \exists y (T(x) \wedge T(y) \wedge x \neq y \wedge \forall z (M(z) \rightarrow (A(x,z) \leftrightarrow A(y,z))))$

There is a pair of distinct Ts that went to the same Ms

F, T, F

$\forall x \forall y ((M(x) \wedge M(y) \wedge x \neq y) \rightarrow \exists z (T(z) \wedge A(z,x) \wedge A(z,y)))$

For every pair of Ms, there is a T that went to both

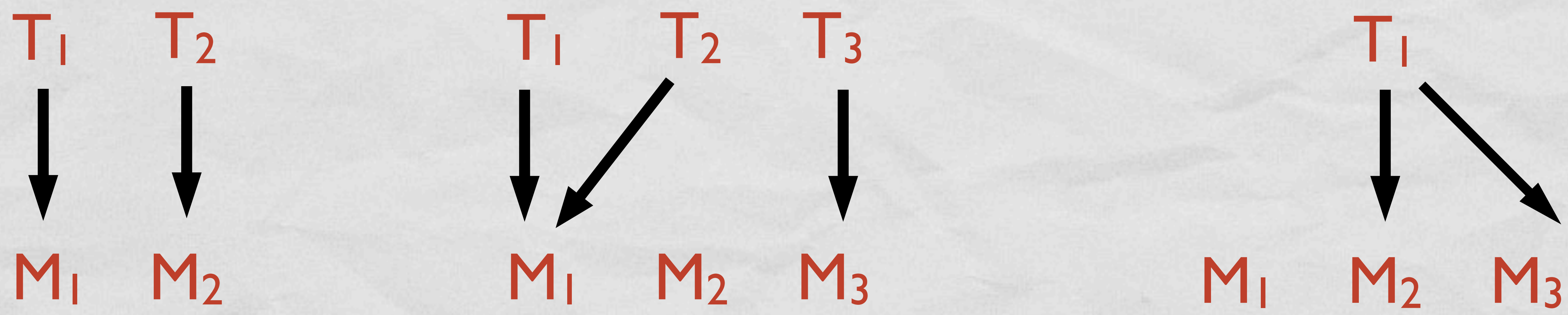


$\exists x \exists y (T(x) \wedge T(y) \wedge x \neq y \wedge \forall z (M(z) \rightarrow (A(x,z) \leftrightarrow A(y,z))))$

There is a pair of distinct Ts that went to the same Ms — F, T, F

$\forall x \forall y ((M(x) \wedge M(y) \wedge x \neq y) \rightarrow \exists z (T(z) \wedge A(z,x) \wedge A(z,y)))$

For every pair of Ms, there is a T that went to both — F, F, F



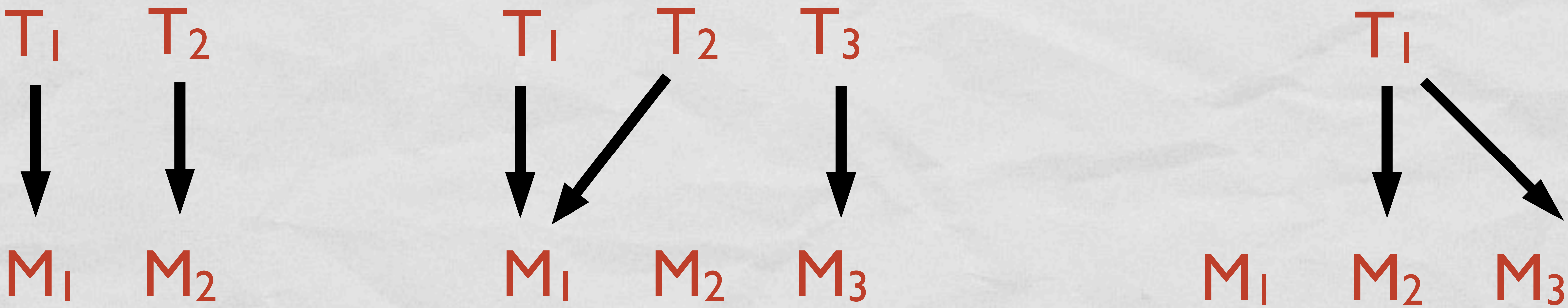
$\exists x \exists y (T(x) \wedge T(y) \wedge x \neq y \wedge \forall z (M(z) \rightarrow (A(x,z) \leftrightarrow A(y,z))))$

There is a pair of distinct Ts that went to the same Ms — F, T, F

$\forall x \forall y ((M(x) \wedge M(y) \wedge x \neq y) \rightarrow \exists z (T(z) \wedge A(z,x) \wedge A(z,y))))$

For every pair of Ms, there is a T that went to both — F, F, F

$\exists x (T(x) \wedge \exists y \exists z (y \neq z \wedge M(y) \wedge M(z) \wedge A(x,y) \wedge A(x,z))$



$$\exists x \exists y (T(x) \land T(y) \land x \neq y \land \forall z (M(z) \rightarrow (A(x,z) \leftrightarrow A(y,z))))$$

There is a pair of distinct Ts that went to the same Ms

F, T, F

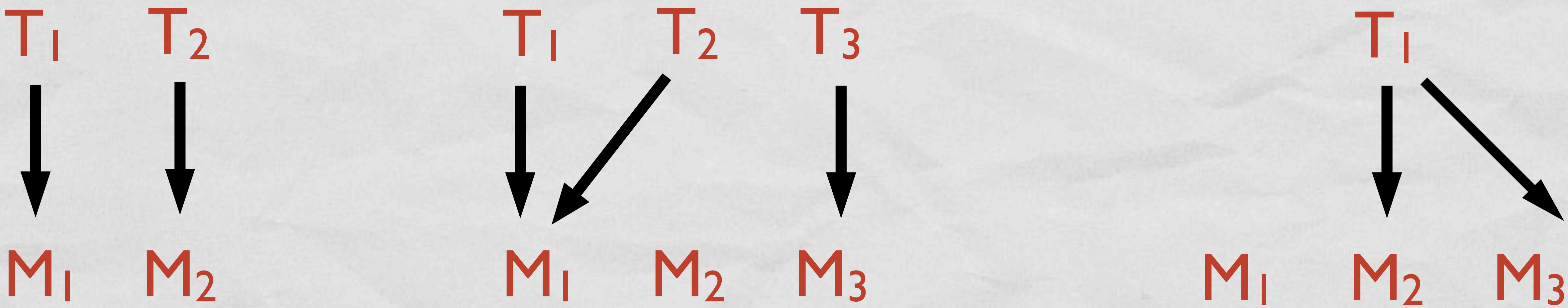
$$\forall x \forall y ((M(x) \land M(y) \land x \neq y) \rightarrow \exists z (T(z) \land A(z,x) \land A(z,y))))$$

For every pair of Ms, there is a T that went to both

F, F, F

$$\exists x (T(x) \land \exists y \exists z (y \neq z \land M(y) \land M(z) \land A(x,y) \land A(x,z))$$

There is a T who went to two different Ms



$\exists x \exists y(T(x) \wedge T(y) \wedge x \neq y \wedge \forall z(M(z) \rightarrow (A(x,z) \leftrightarrow A(y,z))))$

There is a pair of distinct Ts that went to the same Ms — F, T, F

$\forall x \forall y((M(x) \wedge M(y) \wedge x \neq y) \rightarrow \exists z(T(z) \wedge A(z,x) \wedge A(z,y))))$

For every pair of Ms, there is a T that went to both — F, F, F

$\exists x(T(x) \wedge \exists y \exists z(y \neq z \wedge M(y) \wedge M(z) \wedge A(x,y) \wedge A(x,z))$

There is a T who went to two different Ms — F, F, T

Diagrams and Validity

Diagrams and Validity

- Diagrams are often helpful ways of seeing what follows (and doesn't follow) from premises.

Diagrams and Validity

- Diagrams are often helpful ways of seeing what follows (and doesn't follow) from premises.
  - Draw a diagram that make the premises true - if you are <u>forced</u> to do something, that conclusion follows from the premises.

Diagrams and Validity

- Diagrams are often helpful ways of seeing what follows (and doesn't follow) from premises.
    - Draw a diagram that make the premises true - if you are forced to do something, that conclusion follows from the premises.
    - If you weren't forced, it is invalid.

1. $\exists x(T(x) \land \forall y(M(y) \rightarrow \neg A(x,y))$

2. $\forall x(T(x) \rightarrow \exists y(M(y) \rightarrow \neg A(x,y))$

3. $\forall x(M(x) \rightarrow \exists y(T(y) \land A(y,x))$

what follows?

1. $\exists x(T(x) \wedge \forall y(M(y) \rightarrow \neg A(x,y))$
2. $\forall x(T(x) \rightarrow \exists y(M(y) \rightarrow \neg A(x,y))$
3. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge A(y,x))$

what follows?

For P1, we need a T that points to no Ms. - Lets call it T_1

1. $\exists x(T(x) \land \forall y(M(y) \rightarrow \neg A(x,y))$
2. $\forall x(T(x) \rightarrow \exists y(M(y) \rightarrow \neg A(x,y))$
3. $\forall x(M(x) \rightarrow \exists y(T(y) \land A(y,x))$

what follows?

For P1, we need a T that points to no Ms. - Lets call it T_1

T_1

1. $\exists x(T(x) \wedge \forall y(M(y) \rightarrow \neg A(x,y))$
2. $\forall x(T(x) \rightarrow \exists y(M(y) \rightarrow \neg A(x,y))$
3. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge A(y,x))$

what follows?

For P1, we need a T that points to no Ms. - Lets call it T_1

By P2, every T skips some M. - so we need an M for T_1 to skip. Lets call it M_1.

T_1

1. $\exists x(T(x) \wedge \forall y(M(y) \rightarrow \neg A(x,y))$

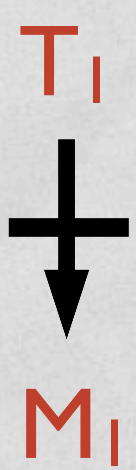
2. $\forall x(T(x) \rightarrow \exists y(M(y) \rightarrow \neg A(x,y))$

3. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge A(y,x))$

what follows?

For P1, we need a T that points to no Ms. - Lets call it T_1

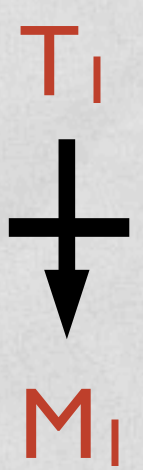
By P2, every T skips some M. - so we need an M for T_1 to skip. Lets call it M_1.



1. $\exists x(T(x) \wedge \forall y(M(y) \rightarrow \neg A(x,y))$
2. $\forall x(T(x) \rightarrow \exists y(M(y) \rightarrow \neg A(x,y))$
3. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge A(y,x))$

what follows?

For P1, we need a T that points to no Ms. - Lets call it T_1



By P2, every T skips some M. - so we need an M for T_1 to skip. Lets call it M_1.

By P3, for every M, some T had to go. - So we need a new T to go to M_1. Lets call it T_2

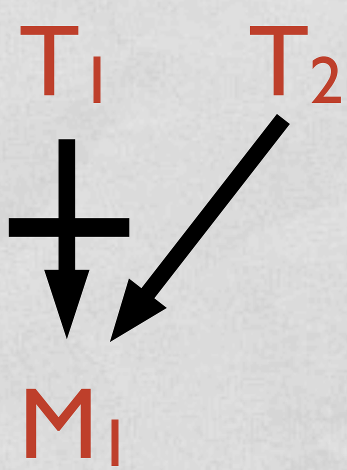
1. $\exists x(T(x) \wedge \forall y(M(y) \rightarrow \neg A(x,y))$

2. $\forall x(T(x) \rightarrow \exists y(M(y) \rightarrow \neg A(x,y))$

3. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge A(y,x))$

what follows?

For P1, we need a T that points to no Ms. - Lets call it T_1



By P2, every T skips some M. - so we need an M for T_1 to skip. Lets call it M_1.

By P3, for every M, some T had to go. - So we need a new T to go to M_1. Lets call it T_2

1. $\exists x(T(x) \wedge \forall y(M(y) \rightarrow \neg A(x,y))$
2. $\forall x(T(x) \rightarrow \exists y(M(y) \rightarrow \neg A(x,y))$
3. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge A(y,x))$

what follows?

For P1, we need a T that points to no Ms. - Lets call it T_1

By P2, every T skips some M. - so we need an M for T_1 to skip. Lets call it M_1.

By P3, for every M, some T had to go. - So we need a new T to go to M_1. Lets call it T_2

But by P2, we have to go back and have an M that T_2 skips. Lets call it M_2.

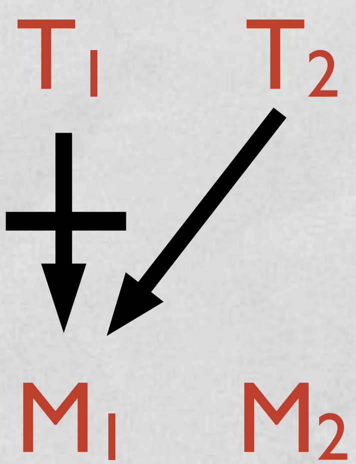
1. $\exists x(T(x) \wedge \forall y(M(y) \rightarrow \neg A(x,y))$

2. $\forall x(T(x) \rightarrow \exists y(M(y) \rightarrow \neg A(x,y))$

3. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge A(y,x))$

what follows?

For P1, we need a T that points to no Ms. - Lets call it T_1



By P2, every T skips some M. - so we need an M for T_1 to skip. Lets call it M_1.

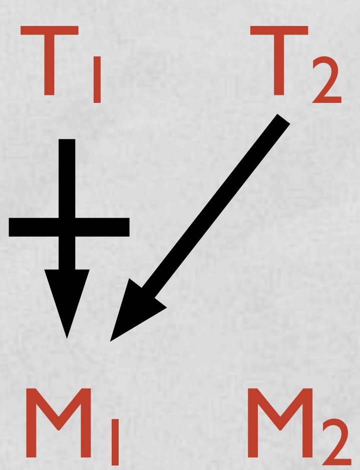
By P3, for every M, some T had to go. - So we need a new T to go to M_1. Lets call it T_2

But by P2, we have to go back and have an M that T_2 skips. Lets call it M_2.

1. $\exists x(T(x) \wedge \forall y(M(y) \rightarrow \neg A(x,y))$
2. $\forall x(T(x) \rightarrow \exists y(M(y) \rightarrow \neg A(x,y))$
3. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge A(y,x))$

what follows?

For P1, we need a T that points to no Ms. - Lets call it T_1



By P2, every T skips some M. - so we need an M for T_1 to skip. Lets call it M_1.

By P3, for every M, some T had to go. - So we need a new T to go to M_1. Lets call it T_2

But by P2, we have to go back and have an M that T_2 skips. Lets call it M_2.

Now by P3, we have to go back and have a T that goes to M_2. Lets call it T_3.

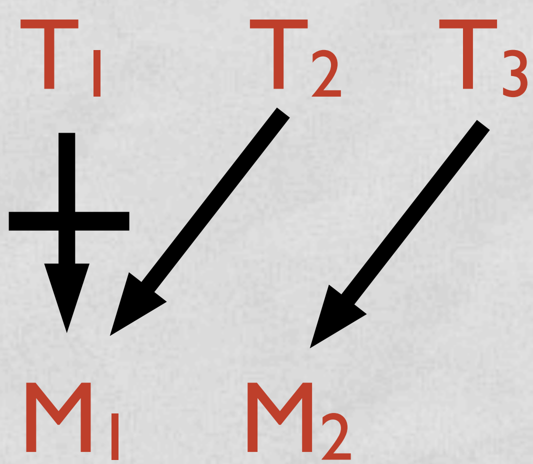
1. $\exists x(T(x) \wedge \forall y(M(y) \rightarrow \neg A(x,y))$

2. $\forall x(T(x) \rightarrow \exists y(M(y) \rightarrow \neg A(x,y))$

3. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge A(y,x))$

what follows?

For P1, we need a T that points to no Ms. - Lets call it T_1



By P2, every T skips some M. - so we need an M for T_1 to skip. Lets call it M_1.

By P3, for every M, some T had to go. - So we need a new T to go to M_1. Lets call it T_2

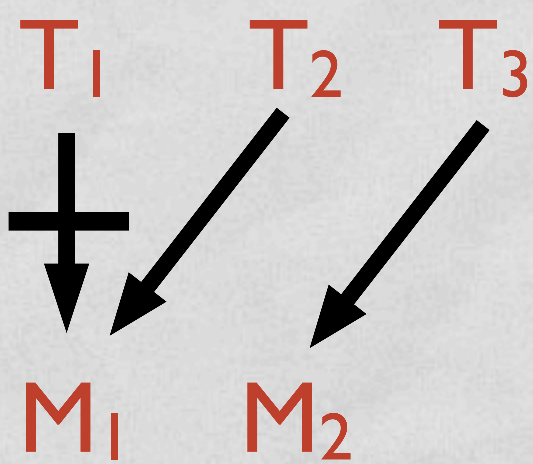
But by P2, we have to go back and have an M that T_2 skips. Lets call it M_2.

Now by P3, we have to go back and have a T that goes to M_2. Lets call it T_3.

1. $\exists x(T(x) \wedge \forall y(M(y) \rightarrow \neg A(x,y))$
2. $\forall x(T(x) \rightarrow \exists y(M(y) \rightarrow \neg A(x,y))$
3. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge A(y,x))$

what follows?

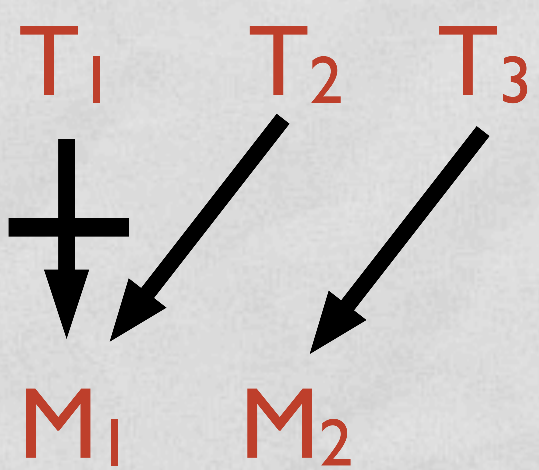
By thinking about what we just did, it it is pretty clear that the following sentences are entailed:



1. $\exists x(T(x) \land \forall y(M(y) \rightarrow \neg A(x,y))$
2. $\forall x(T(x) \rightarrow \exists y(M(y) \rightarrow \neg A(x,y))$
3. $\forall x(M(x) \rightarrow \exists y(T(y) \land A(y,x))$

what follows?

By thinking about what we just did, it it is pretty clear that the following sentences are entailed:



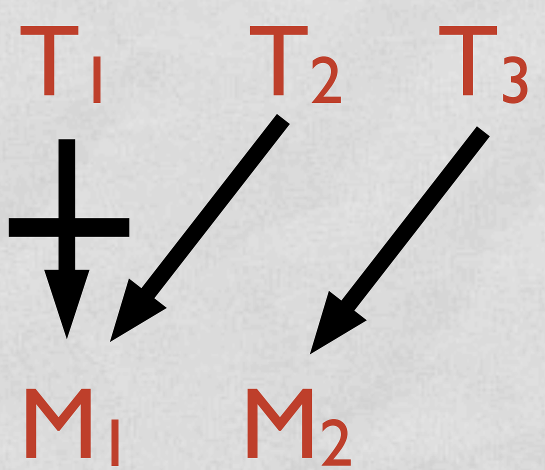
$\exists x \exists y(M(x) \land M(y) \land x \neq y)$

-- there are at least two meetings

1. $\exists x(T(x) \wedge \forall y(M(y) \rightarrow \neg A(x,y))$
2. $\forall x(T(x) \rightarrow \exists y(M(y) \rightarrow \neg A(x,y))$
3. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge A(y,x))$

what follows?

By thinking about what we just did, it it is pretty clear that the following sentences are entailed:



$\exists x \exists y(M(x) \wedge M(y) \wedge x \neq y)$

-- there are at least two meetings

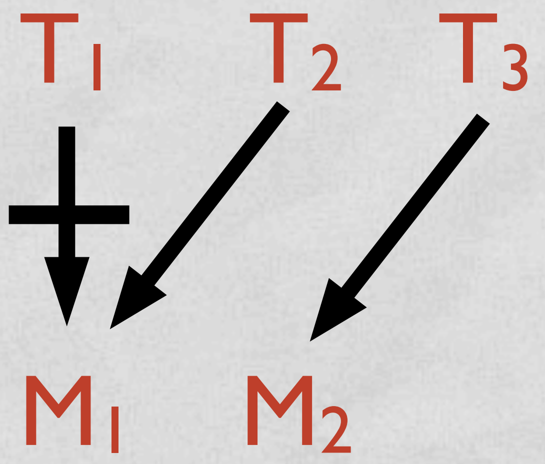
$\exists x \exists y \exists z(T(x) \wedge T(y) \wedge T(z) \wedge x \neq y \wedge y \neq z \wedge x \neq z)$

-- There are at least three teachers

1. $\exists x(T(x) \wedge \forall y(M(y) \rightarrow \neg A(x,y))$
2. $\forall x(T(x) \rightarrow \exists y(M(y) \rightarrow \neg A(x,y))$
3. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge A(y,x))$

what follows?

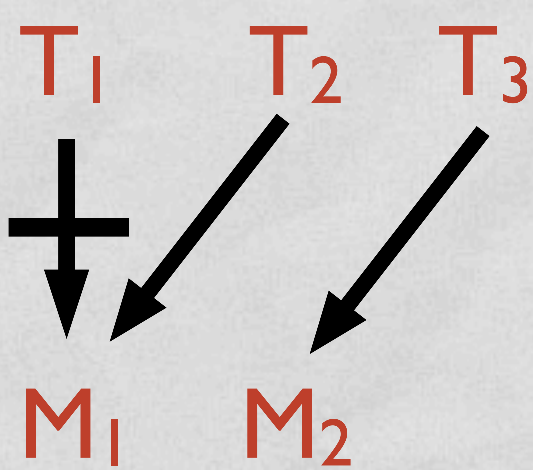
But also, since this diagram makes all the premises true, anything not true on the diagram doesn't follow



1. $\exists x(T(x) \wedge \forall y(M(y) \rightarrow \neg A(x,y))$
2. $\forall x(T(x) \rightarrow \exists y(M(y) \rightarrow \neg A(x,y))$
3. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge A(y,x))$

what follows?

But also, since this diagram makes all the premises true, anything not true on the diagram doesn't follow



$\exists x \exists y \exists z(M(x) \wedge M(y) \wedge M(z) \wedge x \neq y \wedge y \neq z \wedge x \neq z)$

-- There are at least three meetings -- doesn't follow

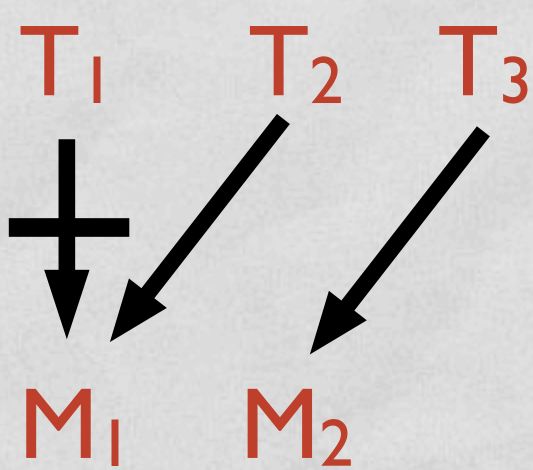
1. $\exists x(T(x) \wedge \forall y(M(y) \rightarrow \neg A(x,y))$

2. $\forall x(T(x) \rightarrow \exists y(M(y) \rightarrow \neg A(x,y))$

3. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge A(y,x))$

what follows?

But also, since this diagram makes all the premises true, anything not true on the diagram doesn't follow



$\exists x(M(x) \wedge \exists y \exists z(T(y) \wedge T(z) \wedge y \neq z \wedge A(y,x) \wedge A(z,x)))$

-- There is a meeting that at least two teachers went to

-- doesn't follow

Diagrams and Mechanical Verification

Diagrams and Mechanical Verification

- We could mechanically produce diagrams to check all the problems we have looked at before. But this kind of structure $\forall x\ldots\ \exists y\ldots$ can lead to disaster. To make it true, plug in something for x, create a new object for y, go back and plug it in for y, create a new object... We might never finish checking.

Diagrams and Mechanical Verification

- We could mechanically produce diagrams to check all the problems we have looked at before. But this kind of structure $\forall x\ldots\ \exists y\ldots$ can lead to disaster. To make it true, plug in something for x, create a new object for y, go back and plug it in for y, create a new object... We might never finish checking.
- There are real life problems with exactly this form:

Diagrams and Mechanical Verification

- We could mechanically produce diagrams to check all the problems we have looked at before. But this kind of structure $\forall x\ldots \exists y\ldots$ can lead to disaster. To make it true, plug in something for x, create a new object for y, go back and plug it in for y, create a new object... We might never finish checking.
- There are real life problems with exactly this form:
  - Goldbach's Conjecture: Every even number is the sum of two prime numbers. True or False? *We don't know*

Diagrams and Mechanical Verification

Diagrams and Mechanical Verification

- We can just write problems like this in FOL and ask if they follow from some axioms (like the Peano axioms for arithmetic).

Diagrams and Mechanical Verification

- We can just write problems like this in FOL and ask if they follow from some axioms (like the Peano axioms for arithmetic).
- We know that if it does follow, we could do a proof (say in Fitch). (By the completeness theorem).

Diagrams and Mechanical Verification

- We can just write problems like this in FOL and ask if they follow from some axioms (like the Peano axioms for arithmetic).
- We know that if it does follow, we could do a proof (say in Fitch). (By the completeness theorem).
- We know that if it doesn't follow, we could give an interpretation to show this (give a counterexample).

Diagrams and Mechanical Verification

- We can just write problems like this in FOL and ask if they follow from some axioms (like the Peano axioms for arithmetic).
- We know that if it does follow, we could do a proof (say in Fitch). (By the completeness theorem).
- We know that if it doesn't follow, we could give an interpretation to show this (give a counterexample).
- But there is no general algorithm for solving these problems. No computer program could possibly solve every logic problem. This fact is called Church's Undecidability Theorem.