Puzzle

In a certain place, all the inhabitants are either Knights or Knaves. Knights always tell the truth and Knaves never tell the truth.

You meet two inhabitants, A and B. A says "Both of us are knights." B says "A is a knave." What, if anything, can you infer from this?

THE LOGIC OF ATOMIC SENTENCES: PROOFS OF (IN)VALIDITY

Friday, 24 January

Validity in FOL

- A sentence S is a <u>logical consequence</u> of sentences $P_1 \ldots P_n$ iff the argument with $P_1 \ldots P_n$ as the premises and S as the conclusion is valid.

  - A formal deduction in $\mathcal{F}$ proves validity.

- A sentence S is a <u>nonconsequence</u> of sentences $P_1 \ldots P_n$ iff the argument with $P_1 \ldots P_n$ as the premises and S as the conclusion is invalid.

  - A counterexample (such as a world in Tarski's World) proves invalidity.

PROVING NONCONSEQUENCE

Example:

1. SameSize(a, b)
2. Small(c)

3. Small(a)
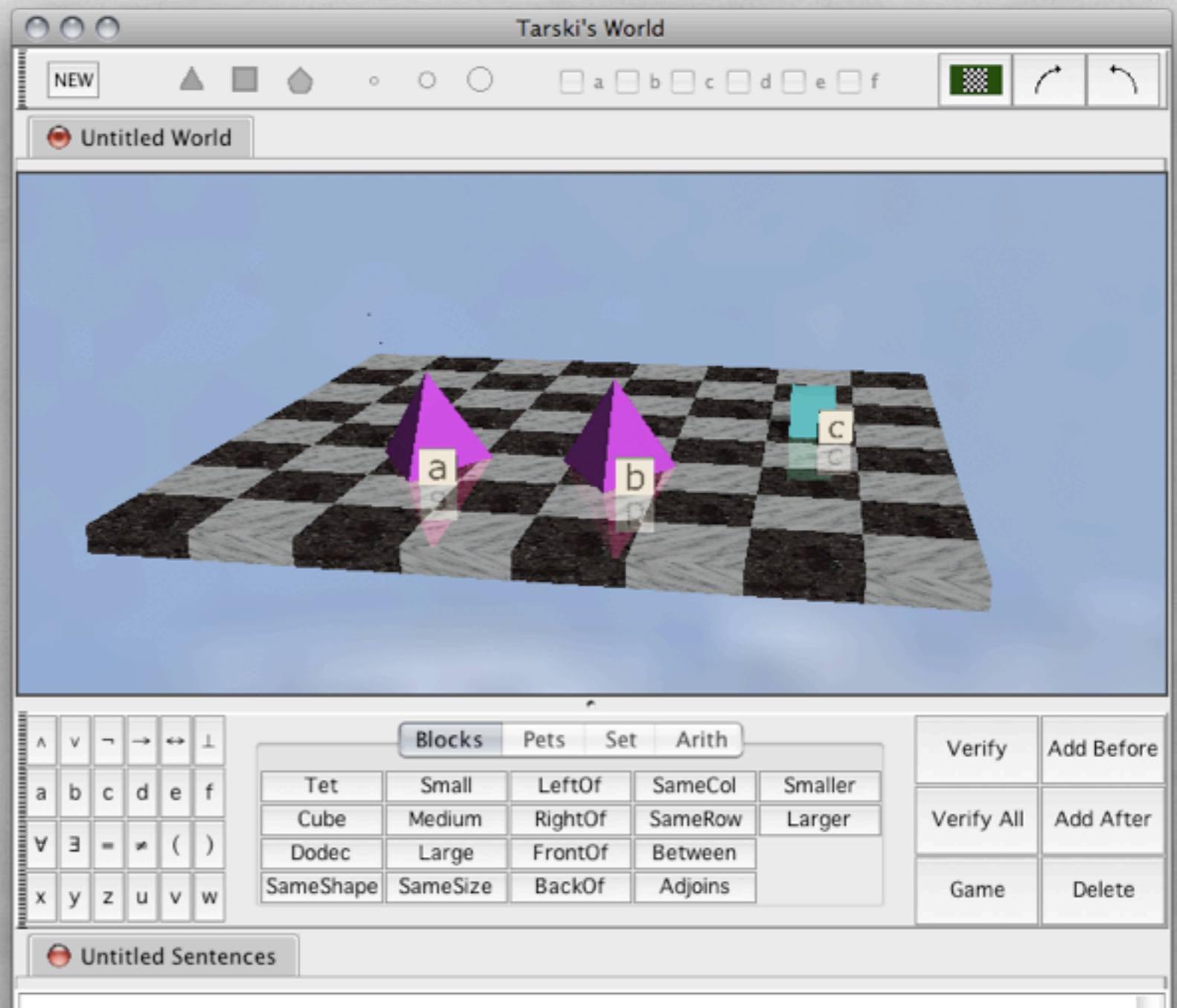
EXAMPLES

Example:

1. LeftOf(a, b)
2. LeftOf(a, c)
3. b=d

___

4. LeftOf(d,c)

Invalid

Example:

1. SameSize(a, b)
2. SameSize(a, c)
3. Medium(b)

___

4. Medium(c)

Valid

HOW DO YOU KNOW?

- If you can find a counterexample, then you know the argument is **invalid**

- If you can't find a counterexample, you might need to keep looking

    - Or you could try to prove that the conclusion follows

    - If you do find a proof, you know it is **valid**

FITCH-STYLE DEDUCTIVE SYSTEM

1. P
2. Q

3. S_1 Justification 1
4. S_2 Justification 2
…
n. S_n Justification n

FITCH-STYLE DEDUCTIVE SYSTEM

Rules of the system $\mathcal{F}$:

- = Intro

- = Elim

- Reit (Reiteration): "we have already shown that P"

The Computer Program Fitch

Premises →

Steps →

Justifications

Conclusion →

PROOFS IN FITCH

Example:

1. SameSize(a, b)	
2. a=c	
3. b=d	
4. SameSize(c,d)	

Valid

1. SameSize(a, b)	Premise	
2. a=c	Premise	
3. b=d	Premise	
4. SameSize(c,b)	= Elim 1,2
5. SameSize(c,d)	= Elim 3,4

ANALYTICAL CONSEQUENCE

- There are no rules in $\mathcal{F}$ that take advantage of the meaning of the blocks world predicates

  - LeftOf(a,b) is a logical consequence of RightOf(b,a) but you can't prove this in $\mathcal{F}$

- The rule 'Analytical Consequence' (Ana Con) in Fitch makes use of these meanings

ANALYTICAL CONSEQUENCE

Example:

1. SameSize(a, b)
2. SameSize(a, c)
3. Medium(b)

4. Medium(c)

Valid

Proof:

1. SameSize(a, b)
2. SameSize(a, c)
3. Medium(b)

4. SameSize(b,c)   Ana Con 1,2

5. Medium(c)   Ana Con 3,4