

PUZZLE

In a certain place, all the inhabitants are either Knights or Knaves. Knights always tell the truth and Knaves never tell the truth.

You meet two inhabitants, A and B. A says “Both of us are knights.” B says “A is a knave.” What, if anything, can you infer from this?

THE LOGIC OF ATOMIC SENTENCES: PROOFS OF (IN)VALIDITY

Friday, 24 January

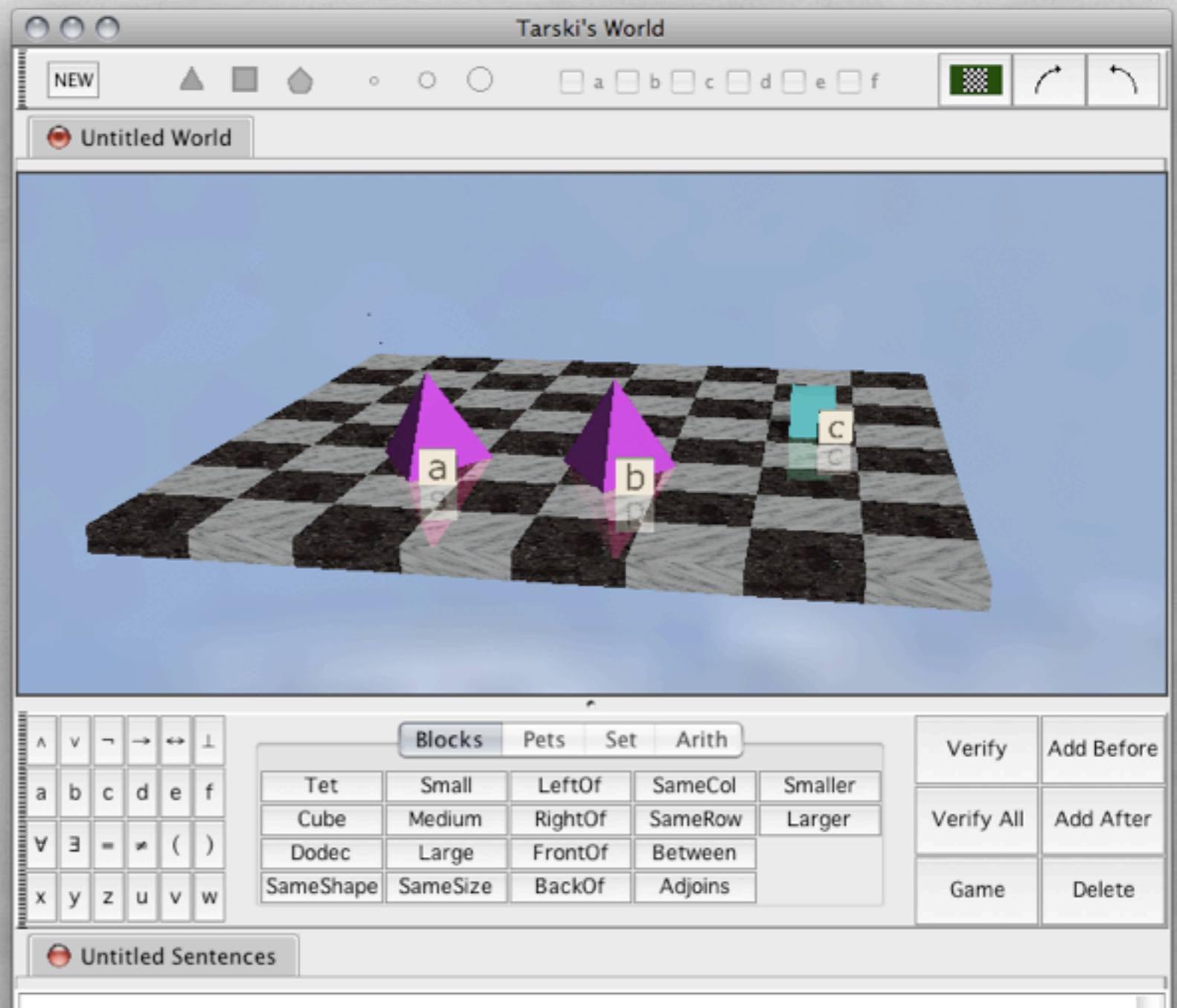
VALIDITY IN FOL

- A sentence S is a logical consequence of sentences $P_1 \dots P_n$ iff the argument with $P_1 \dots P_n$ as the premises and S as the conclusion is valid.
 - A formal deduction in \mathcal{F} proves validity.
- A sentence S is a nonconsequence of sentences $P_1 \dots P_n$ iff the argument with $P_1 \dots P_n$ as the premises and S as the conclusion is invalid.
 - A counterexample (such as a world in Tarski's World) proves invalidity.

PROVING NONCONSEQUENCE

Example:

1. SameSize(a, b)
2. Small(c)
3. Small(a)



EXAMPLES

Example:

1. LeftOf(a, b)
2. LeftOf(a, c)
3. $b=d$
4. LeftOf(d,c)

Invalid

Example:

1. SameSize(a, b)
2. SameSize(a, c)
3. Medium(b)
4. Medium(c)

Valid

HOW DO YOU KNOW?

- If you can find a counterexample, then you know the argument is **invalid**
- If you can't find a counterexample, you might need to keep looking
 - Or you could try to prove that the conclusion follows
 - If you do find a proof, you know it is **valid**

FITCH-STYLE DEDUCTIVE SYSTEM

1. P

2. Q

3. S_1

4. S_2

...

n. S_n

Justification 1

Justification 2

Justification n

FITCH-STYLE DEDUCTIVE SYSTEM

Rules of the system \mathcal{F} :

- = Intro
- = Elim
- Reit (Reiteration): “we have already shown that P”

THE COMPUTER PROGRAM FITCH

Premises →

Steps →

Conclusion →

The screenshot shows the Fitch software interface with a window titled "Untitled 3". The interface includes a toolbar with navigation and editing icons, a menu bar with options like Tet, Small, LeftOf, SameCol, and Adjoin, and buttons for "Check Step", "Verify Proof", and "Goal Constraints". The main workspace displays a list of logical statements: SameSize(b,b), a = b, a = a, b = a, and SameSize(a,b). The last statement, SameSize(a,b), is highlighted in purple. To the right of this statement, there are three justification entries: a blue checkmark and a dropdown arrow followed by "= Intro", another blue checkmark and dropdown arrow followed by "= Elim", and a third blue checkmark and dropdown arrow followed by "= Elim". A red arrow points from the text "Justifications" to the third justification entry. At the bottom of the window, a light blue bar labeled "Goals" contains the statement SameSize(a,b) with a blue checkmark next to it.

Justifications

PROOFS IN FITCH

Example:

1. SameSize(a, b)

2. a=c

3. b=d

4. SameSize(c,d)

1. SameSize(a, b) Premise

2. a=c Premise

3. b=d Premise

4. SameSize(c,b) = Elim 1,2

5. SameSize(c,d) = Elim 3,4

Valid

ANALYTICAL CONSEQUENCE

- There are no rules in \mathcal{F} that take advantage of the meaning of the blocks world predicates
- $\text{LeftOf}(a,b)$ is a logical consequence of $\text{RightOf}(b,a)$ but you can't prove this in \mathcal{F}
- The rule 'Analytical Consequence' (Ana Con) in Fitch makes use of these meanings

ANALYTICAL CONSEQUENCE

Example:

1. SameSize(a, b)
2. SameSize(a, c)
3. Medium(b)
4. Medium(c)

Proof:

1. SameSize(a, b)
2. SameSize(a, c)
3. Medium(b)
4. SameSize(b,c)
5. Medium(c)

Ana Con 1,2

Ana Con 3,4

Valid