Questions 1-6

A crayon manufacturer is designing a box with ten colors of crayons—M, N, O, P, R, S, T, V, W, and Y. The crayons will be arranged in two rows—front and back—and five columns, labeled 1-5 from left to right. The following conditions apply: P is in the same row as O, and there are exactly three crayons between them. S is in the same column as W.

T is directly to the left of P. If M is in the front row, W and Y are in the back row. If W is in the front row, O is in the back row. R is in the third column.

1. Which one of the following could be a possible arrangement of crayons, listed from left to right?

- Front: O , S , R , T , P (B)
	- Back: N, W, V, Y, M
- Front: Y, W, N, R, V (C)
	- Back: P, S, M, T, O
- Front: O, R, S, T, P (D)
- Back: N, V, W, Y, M
- Front: N, W, Y, V, M (E) Back: O, S, R, T, P
- 2. If O is in the first column of the front row, which one of the following must be true?
	- R is in the third column of the front row. (A)
	- M is in the third column of the front row. (B)
	- M is in the third column of the back row. (C)
	- V is in the fifth column of the back row. (D)
	- W is in the second column of the back row. (E)

- Which one of the following conditions, if true, would 6. determine the complete order for at least one of the rows?
	- T is in the fourth column in the back row. (A)
	- V is in the fourth column of the front row. (B)
	- S is in the second column in the front row. (C)
	- W is in the second column in the front row. (D)
	- (E) P is in the fifth column of the front row.
- 3. If W is in the front row, how many exact positions of crayons can be determined?
	- (A) 4
	- (B) 5
	- (C) 6
	- (D) 7
	- 8 (E)
- 4. If M is in the front row, it must be true that
	- O is in the front row (A)
	- (B) S is in the front row
	- T is in the back row (C)
	- R is in the front row (D)
	- R is in the back row (E)

If V and Y are in the front row, which one of the following could be true?

- (A) S is in the front row.
- O is in the front row. (B)
- T is in the front row. (C)
- M is in the front row. (D)
- R is in the back row. (E)

DIAGRAMS AND VALIDITY

TOP HAPPY CA

Wednesday, 30 April

An argument is FO-valid if any interpretation that makes all of the premises true also makes the conclusion true.

An argument is FO-valid if any interpretation that makes all of the premises true also makes the conclusion true.

• So it is invalid if there is at least one interpretation that makes all the premises true and makes the conclusion false.

- An argument is FO-valid if any interpretation that makes all of the premises true also makes the conclusion true.
	- So it is invalid if there is at least one interpretation that makes all the premises true and makes the conclusion false.
- An *interpretation* gives the meaning of the constants, functions, and predicates and gives a domain (so we know what 'for all x' means). - it gives enough information to know whether any particular sentence is true or false.

The Electric Anderlines, comments of David Mile

$\exists x P(x) \land \exists x Q(x) \nvdash \exists x (P(x) \land Q(x))$ - it is FO invalid

C. Lawrence Charles Comments of Charles

 $\exists x P(x) \land \exists x Q(x) \nvdash \exists x (P(x) \land Q(x))$ - it is FO invalid

Domain: Natural numbers P(x): Even numbers Q(x): Odd numbers A countermodel:

And Anderson Comments of Stand to

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A picture with one cube and one tet P(x): Cubes Q(x): Tets Another countermodel:

Constitution commit of Sports

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Domain: Natural numbers P(x): Even numbers Q(x): Odd numbers A countermodel:

A picture with one cube and one tet P(x): Cubes Q(x): Tets Another countermodel:

 $\exists x(P(x) \land Q(x))$ ⊢ $\exists x P(x) \land \exists x Q(x)$ But the other direction is correct

valid or not?

1. ∃x∃y(Square(x) ∧ Square(y) ∧ LeftOf(x,y)) 2. $\forall x \forall y (LeftOf(x, y) \rightarrow \neg Filed(x))$ 3. $\forall x \forall y((Filled(x) \land Filled(y)) \rightarrow SameRow(x,y))$

valid or not?

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Try to make the premises true and the conclusion false. To do this, first add things to make the ∃x s true (in the premises) and to make the ∀x s false (if in the conclusion).

valid or not?

1. ∃x∃y(Square(x) ∧ Square(y) ∧ LeftOf(x,y)) 2. $\forall x \forall y (LeftOf(x, y) \rightarrow \neg Filed(x))$ 3. $\forall x \forall y ((Filled(x) \land Filled(y)) \rightarrow SameRow(x,y))$

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Ans: Picture on board in class - a non-filled square left of two filled squares which aren't on the same row (for example - other pictures work).

 $1. \exists x(T(x) \land \forall y(M(y) \rightarrow A(x,y)))$ $2. \exists x(T(x) \land \forall y(M(y) \rightarrow \neg A(x,y)))$ 3. $\forall x(M(x) \rightarrow \exists y(T(y) \land A(x,y)))$

valid or not?

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valid or not?

You could try to give meanings to T, M, and A and then use Tarski's world or a shapes diagram, but it is usually easier (and safer) to draw a diagram.

 $1. \exists x(T(x) \land \forall y(M(y) \rightarrow A(x,y)))$ $2. \exists x(T(x) \land \forall y(M(y) \rightarrow \neg A(x,y)))$ $3. \forall x(M(x) \rightarrow \exists y(T(y) \land A(x,y)))$

valid or not?

You could try to give meanings to T, M, and A and then use Tarski's world or a shapes diagram, but it is usually easier (and safer) to draw a diagram.

Label the Ts and the Ms and then have $A(x,y) = x$ points to y

T_1 T_2 T_3 T_1 T_2 T_3 $T₁$ $\sqrt{11}$ M_1 M_2 M_1 M_2 M_1 M_2 M_3

CALIFORNIAL CARTER STATE

Wednesday, April 30, 2014

T_1 T_2 T_3 M_1 M_2 M_3 $T₁$ M_1 M_2

 T_1 T_2 T_3 $\sqrt{1/2}$ M_1 M_2

$\exists x(T(x) \land \forall y(M(y) \rightarrow A(x,y)))$

Wednesday, April 30, 2014

T_1 T_2 T_3 T_1 T_2 T_3 $T₁$ 1 M_1 M_2 M_1 M_2 M_1 M_2 M_3

$\exists x(T(x) \land \forall y(M(y) \rightarrow A(x,y)))$ True, False, True

T_1 T_2 T_3 M_1 M_2 M_3 T_1 M_1 M_2 T_1 T_2 T_3 $M₁$

 $\exists x(T(x) \land \forall y(M(y) \rightarrow A(x,y)))$ True, False, True $\forall x(M(x) \rightarrow \exists y(T(y) \land \neg A(y,x)))$

Wednesday, April 30, 2014

T_1 T_2 T_3 T_1 T_2 T_3 T_1 NV $M₁$ M_1 M_2 M_1 M_2 M_3

 $\exists x(T(x) \land \forall y(M(y) \rightarrow A(x,y)))$ True, False, True $\forall x(M(x) \rightarrow \exists y(T(y) \land \neg A(y,x)))$ False, False, True

Wednesday, April 30, 2014

T_1 T_2 T_3 M_1 M_2 M_3 T_1 M_1 M_2 T_1 T_2 T_3 $M₁$

 $\exists x(T(x) \land \forall y(M(y) \rightarrow A(x,y)))$ True, False, True $\forall x(M(x) \rightarrow \exists y(T(y) \land \neg A(y,x)))$ $\forall x(T(x) \rightarrow \exists y(M(y) \land A(x,y)))$ False, False, True

T_1 T_2 T_3 T_1 T_2 T_3 T_1 NV $M₁$ M_1 M_2 M_1 M_2 M_3

 $\exists x(T(x) \land \forall y(M(y) \rightarrow A(x,y)))$ True, False, True $\forall x(M(x) \rightarrow \exists y(T(y) \land \neg A(y,x)))$ $\forall x(T(x) \rightarrow \exists y(M(y) \land A(x,y)))$ False, False, True True, True, False 3. $\forall x(M(x) \rightarrow \exists y(T(y) \land A(y,x)))$ 1. $\forall x(T(x) → \exists y(M(y) ∧ A(x,y))$ 2. $\exists x(M(x) \land \forall y(T(y) \rightarrow A(y,x)))$

valid or not?

3. $\forall x(M(x) \rightarrow \exists y(T(y) \land A(y,x))$ 1. $\forall x(T(x) → \exists y(M(y) ∧ A(x,y))$ 2. $\exists x(M(x) \land \forall y(T(y) \rightarrow A(y,x))$

valid or not?

Lets try to show this is invalid using a diagram. We are trying to make both of the premises true and the conclusion false.

1. $\forall x(T(x) → \exists y(M(y) ∧ A(x,y))$ 2. $\exists x(M(x) \land \forall y(T(y) \rightarrow A(y,x)))$ 3. $\forall x(M(x) \rightarrow \exists y(T(y) \land A(y,x)))$

valid or not?

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valid or not?

For P2, we need an M that every T points to. - Lets call it M₁

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 $M₁$

 T_1

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valid or not?

For P2, we need an M that every T points to. - Lets call it M₁

The conclusion says for every M, there is a T that points to it. For this to be false, we need at least one M that nothing points to. - M₂

 $M₁$

T1

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 $M₁$ $M₂$

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For P1, we need to make sure that for every T, there is at least one M that it points to. Right now, that is true. So we are done. $M₂$

 T_1

 $M₁$

 $1. \exists x(T(x) \land \forall y(M(y) \rightarrow A(x,y)))$ $2. \exists x(T(x) \land \forall y(M(y) \rightarrow \neg A(x,y)))$ 3. $\forall x(M(x) \rightarrow \exists y(T(y) \land A(y,x)))$

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valid or not?

For PI, we need a T that points to every M. - Lets call it T₁

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valid or not?

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 $M₁$

 T_1

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For P1, we need a T that points to every M. - Lets call it T_1 For P2, we need a T that points to no M. - T₂

T1

 $1. \exists x(T(x) \land \forall y(M(y) \rightarrow A(x,y)))$ $2. \exists x(T(x) \land \forall y(M(y) \rightarrow \neg A(x,y)))$ 3. $\forall x(M(x) \rightarrow \exists y(T(y) \land A(y,x))$ valid or not?

For P1, we need a T that points to every M. - Lets call it T_1 T_1 For P2, we need a T that points to no M. - T₂ $T₂$

 $M₁$
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> But don't forget T₁ was supposed to point to every M. Every time you add to the picture, make sure you keep that true.

For P1, we need a T that points to every M. - Lets call it T_1 T1 $M₁$ For P2, we need a T that points to no M. - T₂ $T₂$ The conclusion says for every M, there is a T that points to it. For this to be false, we need $M₂$ at least one M that nothing points to. - M₂

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> ***But you can't do that here. M2 was the meeting that nothing went to. Adding an extra meeting won't help either. So it is valid.

$\forall x(M(x) \rightarrow \exists y(T(y) \land A(y,x))$ -- How to make it false?

of the later in the common way of beating

10 Let on the land and the common of the state

Wednesday, April 30, 2014

1 The process common wheels

∀xP(x) is like a big conjunction.

- ∀xP(x) is like a big conjunction.
- ¬∀xP(x) is like the negation of a big conjunction.

A Charles Companies Commandered

- ∀xP(x) is like a big conjunction.
- ¬∀xP(x) is like the negation of a big conjunction.

The the Longitude Commission to

By DeMorgan's like thinking....

- ∀xP(x) is like a big conjunction.
- ¬∀xP(x) is like the negation of a big conjunction.

A Charles Line Anderson County Principle

By DeMorgan's like thinking....

¬∀xP(x) 㱻 ∃x¬P(x) (a big disjunction of negations)

- ∀xP(x) is like a big conjunction.
- ¬∀xP(x) is like the negation of a big conjunction.

A Charles L. Corners and the County of Dentis

By DeMorgan's like thinking....

¬∀xP(x) 㱻 ∃x¬P(x) (a big disjunction of negations)

By the same thought....

- ∀xP(x) is like a big conjunction.
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A Chairman Commercial Commercial

By DeMorgan's like thinking....

¬∀xP(x) 㱻 ∃x¬P(x) (a big disjunction of negations)

By the same thought....

 $\rightarrow \exists xP(x) \Leftrightarrow \forall x\neg P(x)$

The printing and have come of the the

To make an existential true, add something to the picture.

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Committee Committee Committee Committee Committee

To make an existential true, add something to the picture. To make an existential false, don't add anything.

To make an existential true, add something to the picture. To make an existential false, don't add anything.

To make a universal true, you have to make sure that each time you add something to the picture, you go back and check the universal.

To make an existential true, add something to the picture. To make an existential false, don't add anything.

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To make a universal false, create a counterexample.

To make an existential true, add something to the picture. To make an existential false, don't add anything.

To make a universal true, you have to make sure that each time you add something to the picture, you go back and check the universal.

To make a universal false, create a counterexample.

To make ∀xP(x) false, make ∃x¬P(x) true.

$\forall x(Cube(x) \rightarrow Small(x))$ -- How to make it false?

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 $\forall x(Cube(x) \rightarrow Small(x))$ -- How to make it false?

and the late of the statement was affected that the

 $\neg \forall x(Cube(x) \rightarrow Small(x)) \Leftrightarrow$

 $\forall x(Cube(x) \rightarrow Small(x))$ -- How to make it false?

by the them and the common of the time. It

 $\neg \forall x(Cube(x) \rightarrow Small(x)) \Leftrightarrow$

 $\exists x \neg (Cube(x) \rightarrow Small(x)) \Leftrightarrow (negative the quantifier)$

 $\forall x(Cube(x) \rightarrow Small(x))$ -- How to make it false?

by a factor and the country Clear that I

 $\neg \forall x(Cube(x) \rightarrow Small(x)) \Leftrightarrow$ $\exists x \neg (Cube(x) \rightarrow Small(x)) \Leftrightarrow (negative the quantifier)$ $\exists x(Cube(x) \land \neg Small(x)) \Leftrightarrow (by taut con)$

 $\forall x(Cube(x) \rightarrow Small(x))$ -- How to make it false?

And Charles Landscher County Plantine L'

 $\neg \forall x(Cube(x) \rightarrow Small(x)) \Leftrightarrow$ $\exists x \neg (Cube(x) \rightarrow Small(x)) \Leftrightarrow (negative the quantifier)$ $\exists x(Cube(x) \land \neg Small(x)) \Leftrightarrow (by taut con)$

So there is a cube which is not small

$\forall x(M(x) \rightarrow \exists y(T(y) \land A(y,x))$ -- How to make it false?

of the later in the common way of beating

 $\forall x(M(x) \rightarrow \exists y(T(y) \land A(y,x))$ -- How to make it false?

The Lorentzian Communication the

 $\neg \forall x (M(x) \rightarrow \exists y (T(y) \wedge A(y,x)) \Leftrightarrow$

 $\forall x(M(x) \rightarrow \exists y(T(y) \land A(y,x))$ -- How to make it false?

Service Anderson and March 1990

 $\neg \forall x (M(x) \rightarrow \exists y (T(y) \land A(y,x)) \Leftrightarrow$ $\exists x \neg(M(x) \rightarrow \exists y(T(y) \land A(y,x)) \Leftrightarrow$ (negate the quantifier)

 $\forall x(M(x) \rightarrow \exists y(T(y) \land A(y,x))$ -- How to make it false?

Constitution of the Constitution

 $\neg \forall x (M(x) \rightarrow \exists y (T(y) \land A(y,x)) \Leftrightarrow$ $\exists x \neg(M(x) \rightarrow \exists y(T(y) \land A(y,x)) \Leftrightarrow$ (negate the quantifier) $\exists x(M(x) \land \neg \exists y(T(y) \land A(y,x)) \Leftrightarrow (by taut con))$

 $\forall x(M(x) \rightarrow \exists y(T(y) \land A(y,x))$ -- How to make it false?

Service Communication of Chevroleton

 $\neg \forall x (M(x) \rightarrow \exists y (T(y) \land A(y,x)) \Leftrightarrow$ $\exists x \neg(M(x) \rightarrow \exists y(T(y) \land A(y,x)) \Leftrightarrow$ (negate the quantifier) $\exists x(M(x) \land \neg \exists y(T(y) \land A(y,x)) \Leftrightarrow (by taut con))$ So there is an M where it is false that there is a T that points to it. -- So no T points to it.

 $\forall x(M(x) \rightarrow \exists y(T(y) \land A(y,x))$ -- How to make it false?

CONTRACTORS COMMUNICATIONS AND

 $\neg \forall x (M(x) \rightarrow \exists y (T(y) \land A(y,x)) \Leftrightarrow$ $\exists x \neg(M(x) \rightarrow \exists y(T(y) \land A(y,x)) \Leftrightarrow$ (negate the quantifier) $\exists x(M(x) \land \neg \exists y(T(y) \land A(y,x)) \Leftrightarrow (by taut con))$ So there is an M where it is false that there is a T that points to it. -- So no T points to it.

 $\exists x(M(x) \land \forall y(T(y) \rightarrow \neg A(y,x))$ (quantifer + taut con)

T_1 T_2 T_3 M_1 M_2 M_3 T_1 T_2 M_1 M_2 T_1 T_2 T_3 $M₃$ M_1 M_2

Wednesday, April 30, 2014

T_1 T_2 T_3 M_1 M_2 M_3 T_2 M_1 M_2 T_1 T_2 T_3 $M₃$ $M₁$

A very natural thing you might want to do is to talk not just about single teachers or single meetings, but about pairs of teachers or pairs of meetings. E.g. there is a pair of teachers who went to exactly the same meetings. Or a pair of meetings that between the two, every teacher went to.

T_1 T_2 T_3 M_1 M_2 M_3 T_1 T_2 $\begin{array}{ccc} 1 & 1 \\ \mathsf{M}_1 & \mathsf{M}_2 \end{array}$ T_1 T_2 T_3 M_1 M_2 M_3

T_1 T_2 T_3 T_1 T_2 T_1 T_2 T_3 $N_{\rm t}/$ M_1 M_2 M_1 M_2 M_3 $M₃$ M_1 M_2

$\exists x \exists y(T(x) \land T(y) \land \forall z(M(z) \rightarrow (A(x,z) \lor A(y,z))))$

T_1 T_2 T_3 $M₃$ T_2 M_1 M_2 M_1 M_2 T_1 T_2 T_3 $M₃$ M_1 M_2

 $\exists x \exists y(T(x) \land T(y) \land \forall z(M(z) \rightarrow (A(x,z) \lor A(y,z))))$ There is a pair of Ts such that for every M, at least one of those Ts went.

T_1 T_2 T_3 $M₃$ T_2 M_1 M_2 M_1 M_2 T_1 T_2 T_3 $M₃$ M_1 M_2

 $\exists x \exists y(T(x) \land T(y) \land \forall z(M(z) \rightarrow (A(x,z) \lor A(y,z))))$ There is a pair of Ts such that for every M, at least one of those Ts went.

 T, T, F
T_1 T_2 T_3 $M₃$ T_1 T_2 M_1 M_2 M_1 M_2 T_1 T_2 T_3 $M₃$ M_1 M_2

 $\exists x \exists y(T(x) \land T(y) \land \forall z(M(z) \rightarrow (A(x,z) \lor A(y,z))))$ There is a pair of Ts such that for every M, at least one of those Ts went.

T, T, F

 $\forall x \forall y((M(x) \land M(y)) \rightarrow \exists z(T(z) \land (A(z,x) \leftrightarrow A(z,y))))$

T_1 T_2 T_3 M₃ T_2 M_1 M_2 M_1 M_2 T_1 T_2 T_3 $M₃$ $M₁$

 $\exists x \exists y(T(x) \land T(y) \land \forall z(M(z) \rightarrow (A(x,z) \lor A(y,z))))$ $\forall x \forall y((M(x) \land M(y)) \rightarrow \exists z(T(z) \land (A(z,x) \leftrightarrow A(z,y))))$ There is a pair of Ts such that for every M, at least one of those Ts went. For every pair of Ms, there is a T that went to the

 T, T, F

first if and only if they went to the second.

T_1 T_2 T_3 M₃ T_2 M_1 M_2 M_1 M_2 T_1 T_2 T_3 $M₃$ $M₁$

 $\exists x \exists y(T(x) \land T(y) \land \forall z(M(z) \rightarrow (A(x,z) \lor A(y,z))))$ $\forall x \forall y((M(x) \land M(y)) \rightarrow \exists z(T(z) \land (A(z,x) \leftrightarrow A(z,y))))$ There is a pair of Ts such that for every M, at least one of those Ts went.

 T, T, F

 F, T, T

For every pair of Ms, there is a T that went to the first if and only if they went to the second.

 $\exists x \exists y(T(x) \land T(y) \land \forall z(M(z) \rightarrow (A(x,z) \lor A(y,z))))$ T, T, F T_1 T_2 T_3 M₃ T_1 T_2 M_1 M_2 M_1 M_2 T_1 T_2 T_3 $M₃$ $M₁$ $\forall x \forall y((M(x) \land M(y)) \rightarrow \exists z(T(z) \land (A(z,x) \leftrightarrow A(z,y))))$ F, T, T There is a pair of Ts such that for every M, at least one of those Ts went. For every pair of Ms, there is a T that went to the first if and only if they went to the second.

 $\forall x(M(x) \rightarrow \exists y \exists z(T(y) \wedge T(z)) \wedge A(y,x) \wedge \neg A(z,x)))$

 $\exists x \exists y(T(x) \land T(y) \land \forall z(M(z) \rightarrow (A(x,z) \lor A(y,z))))$ T, T, F T_1 T_2 T_3 M₃ T_1 T_2 M_1 M_2 $M₁$ T_1 T_2 T_3 $M₃$ $M₁$ $\forall x \forall y((M(x) \land M(y)) \rightarrow \exists z(T(z) \land (A(z,x) \leftrightarrow A(z,y))))$ $\forall x(M(x) \rightarrow \exists y \exists z(T(y) \land T(z)) \land A(y,x) \land \neg A(z,x)))$ F, T, T There is a pair of Ts such that for every M, at least one of those Ts went. For every pair of Ms, there is a T that went to the first if and only if they went to the second. For every M, there is a pair of Ts such that one went and the other didn't.

 $\exists x \exists y(T(x) \land T(y) \land \forall z(M(z) \rightarrow (A(x,z) \lor A(y,z))))$ T, T, F T_1 T_2 T_3 M₃ T_1 T_2 M_1 M_2 $M₁$ T_1 T_2 T_3 $M₃$ $M₁$ $\forall x \forall y((M(x) \land M(y)) \rightarrow \exists z(T(z) \land (A(z,x) \leftrightarrow A(z,y))))$ $\forall x(M(x) \rightarrow \exists y \exists z(T(y) \land T(z)) \land A(y,x) \land \neg A(z,x)))$ T, F, F F, T, T There is a pair of Ts such that for every M, at least one of those Ts went. For every pair of Ms, there is a T that went to the first if and only if they went to the second. For every M, there is a pair of Ts such that one went and the other didn't.

T_1 T_2 T_3 M_1 M_2 M_3 T_1 T_2 $\begin{array}{ccc} 1 & 1 \\ \mathsf{M}_1 & \mathsf{M}_2 \end{array}$ T_1 T_2 T_3 M_1 M_2 M_3

T_1 T_2 T_3 T_1 T_2 T_1 T_2 T_3 \mathcal{M} \downarrow M_1 M_2 M_1 M_2 M_3 $M₃$ M_1 M_2

 $\forall x(M(x) \rightarrow \exists y \exists z(T(y) \land T(z)) \land A(y,x) \leftrightarrow A(z,x)))$

T_1 T_2 T_3 M₃ T_2 M_1 M_2 $M₁$ T_1 T_2 T_3 $M₃$ M_1 M_2

 $\forall x(M(x) \rightarrow \exists y \exists z(T(y) \land T(z)) \land A(y,x) \leftrightarrow A(z,x)))$ For every M, there is a pair of Ts such that the first went to the M if and only if the second did.

T_1 T_2 T_3 M₃ T_2 M_1 M_2 $M₁$ T_1 T_2 T_3 $M₃$ M_1 M_2

T, T, T

 $\forall x(M(x) \rightarrow \exists y \exists z(T(y) \land T(z)) \land A(y,x) \leftrightarrow A(z,x)))$ For every M, there is a pair of Ts such that the first went to the M if and only if the second did.

T_1 T_2 T_3 M₃ T_2 M_1 M_2 M_1 M_2 T_1 T_2 T_3 $M₃$ $M₁$

T, T, T

 $\forall x(M(x) \rightarrow \exists y \exists z(T(y) \land T(z)) \land A(y,x) \leftrightarrow A(z,x)))$ For every M, there is a pair of Ts such that the first went to the M if and only if the second did.

How could this be right? What pair could work for say the first diagram? $-$ Ans, $\langle T_1, T_1 \rangle$

T_1 T_2 T_3 M₃ T_2 M_1 M_2 M_1 M_2 T_1 T_2 T_3 M₃ $M₁$

T, T, T

 $\forall x(M(x) \rightarrow \exists y \exists z(T(y) \land T(z)) \land A(y,x) \leftrightarrow A(z,x)))$ For every M, there is a pair of Ts such that the first went to the M if and only if the second did.

How could this be right? What pair could work for say the first diagram? $-$ Ans, $\langle T_1, T_1 \rangle$

In fact, the above sentence follows just from ∃z T(y)

T_1 T_2 T_3 T_1 T_2 T_1 T_2 T_3 \mathcal{M} \downarrow M_1 M_2 M_1 M_2 M_3 $M₃$ M_1 M_2

 $\forall x(M(x) \rightarrow \exists y \exists z(T(y) \land T(z)) \land A(y,x) \leftrightarrow A(z,x)))$

T_1 T_2 T_3 $M₃$ T_2 M_1 M_2 $M₁$ T_1 T_2 T_3 $M₃$ M_1 M_2

 $\forall x(M(x) \rightarrow \exists y \exists z(T(y) \land T(z)) \land A(y,x) \leftrightarrow A(z,x)))$

But sometimes you explicitly want to talk about pairs of distinct Ts - two *different* Ts. For this you need identity.

 $\forall x(M(x) \rightarrow \exists y \exists z(T(y) \land T(z)) \land A(y,x) \leftrightarrow A(z,x)))$

But sometimes you explicitly want to talk about pairs of distinct Ts - two *different* Ts. For this you need identity.

Identity is used whenever you want to *count* things

COUNTING IN DIAGRAMS

 $M₃$

T_1 T_2 T_3 M_2 M_3 T_1 T_2 M_1 M_2 $M₁$ T_1 T_2 T_3 M_1 M_2

Wednesday, April 30, 2014

COUNTING IN DIAGRAMS

A COUNTY L. Commissioner County P. Is

A very natural thing you might want to say about these diagrams essentially involves counting. For example, there is one teacher who went to three meetings and two teachers who went to none (true in diagram 3). For this, you need identity.

 $\exists x \exists y(T(x) \wedge T(y))$

$\exists x \exists y(T(x) \wedge T(y))$

Both x and y are teachers

$\exists x \exists y(T(x) \wedge T(y))$

Both x and y are teachers

- but not necessarily different!

$\exists x \exists y(T(x) \wedge T(y))$

Both x and y are teachers

- but not necessarily different!

 $\exists x\exists y(T(x) \wedge T(y) \wedge x \neq y)$

$\exists x \exists y(T(x) \wedge T(y))$

Both x and y are teachers

- but not necessarily different!

 $\exists x \exists y(T(x) \land T(y) \land x \neq y)$

There are at least two teachers

- $\exists x \exists y(T(x) \wedge T(y))$
	- Both x and y are teachers
		- but not necessarily different!
- $\exists x \exists y(T(x) \land T(y) \land x \neq y)$

There are at least two teachers

 $\exists x \exists y(T(x) \land T(y) \land x \neq y \land \forall z(M(z) \rightarrow (A(x,z) \land A(y,z)))$

- $\exists x \exists y(T(x) \wedge T(y))$
	- Both x and y are teachers
		- but not necessarily different!
- $\exists x \exists y(T(x) \land T(y) \land x \neq y)$

There are at least two teachers

 $\exists x \exists y(T(x) \land T(y) \land x \neq y \land \forall z(M(z) \rightarrow (A(x,z) \land A(y,z)))$ There are at least two teachers who attended every meeting

There are at least two painters \mathcal{L}_max there are at least two painters \mathcal{L}_max

T_1 T_2 T_1 T_2 T_3 T_1 $\overline{}$ M_1 M_2 M_1 M_2 M_3 M_1 M_2 M_3 M_1 M_2 M_3

T_1 T_2 T_3 T_1 T_2 T1 \downarrow M_1 M_2 M_1 M_2 M_3 M_1 M_2 M_3 M_1 M_2

$\exists x \exists y(T(x) \land T(y) \land x \neq y \land \forall z(M(z) \rightarrow (A(x,z) \leftrightarrow A(y,z))))$

T_1 T_2 T_3 $M₃$ T_2 M_1 M_2 M_1 M_2 M_3 M_1 M_2 M_3 T1 M_1 M_2

 $\exists x \exists y(T(x) \land T(y) \land x \neq y \land \forall z(M(z) \rightarrow (A(x,z) \leftrightarrow A(y,z))))$ There is a pair of distinct Ts that went to the same Ms

$\exists x \exists y(T(x) \land T(y) \land x \neq y \land \forall z(M(z) \rightarrow (A(x,z) \leftrightarrow A(y,z))))$ F, T, F T_1 T_2 T_3 $M₃$ T_2 M_1 M_2 M_1 M_2 M_3 M_1 M_2 M_3 T1 M_1 M_2 There is a pair of distinct Ts that went to the

same Ms

$\exists x \exists y(T(x) \land T(y) \land x \neq y \land \forall z(M(z) \rightarrow (A(x,z) \leftrightarrow A(y,z))))$ F, T, F T_1 T_2 T_3 $M₃$ T_2 M_1 M_2 M_1 M_2 M_3 M_1 M_2 M_3 T1 M_1 M_2 There is a pair of distinct Ts that went to the same Ms

∀x∀y((M(x) ∧ M(y) ∧ x≠y) → ∃z(T(z) ∧ A(z,x) ∧ A(z,y))))

 $\exists x \exists y(T(x) \land T(y) \land x \neq y \land \forall z(M(z) \rightarrow (A(x,z) \leftrightarrow A(y,z))))$ F, T, F T_1 T_2 T_3 $M₃$ T_2 M_1 M_2 M_1 M_2 M_3 M_1 M_2 M_3 T1 M_1 M_2 ∀x∀y((M(x) ∧ M(y) ∧ x≠y) → ∃z(T(z) ∧ A(z,x) ∧ A(z,y)))) There is a pair of distinct Ts that went to the same Ms For every pair of Ms, there is a T that went to both

 $\exists x \exists y(T(x) \land T(y) \land x \neq y \land \forall z(M(z) \rightarrow (A(x,z) \leftrightarrow A(y,z))))$ F, T, F T_1 T_2 T_3 $M₃$ T_2 M_1 M_2 M_1 M_2 M_3 M_1 M_2 M_3 T1 M_1 M_2 ∀x∀y((M(x) ∧ M(y) ∧ x≠y) → ∃z(T(z) ∧ A(z,x) ∧ A(z,y)))) F, F, F There is a pair of distinct Ts that went to the same Ms For every pair of Ms, there is a T that went to both

 $\exists x \exists y(T(x) \land T(y) \land x \neq y \land \forall z(M(z) \rightarrow (A(x,z) \leftrightarrow A(y,z))))$ F, T, F T_1 T_2 T_3 $M₃$ T_2 M_1 M_2 M_1 M_2 M_3 M_1 M_2 M_3 T1 M_1 M_2 ∀x∀y((M(x) ∧ M(y) ∧ x≠y) → ∃z(T(z) ∧ A(z,x) ∧ A(z,y)))) F, F, F There is a pair of distinct Ts that went to the same Ms For every pair of Ms, there is a T that went to both ∃x(T(x) ∧ ∃y∃z(y≠z ∧ M(y) ∧ M(z) ∧ A(x,y) ∧ A(x,z))

 $\exists x \exists y(T(x) \land T(y) \land x \neq y \land \forall z(M(z) \rightarrow (A(x,z) \leftrightarrow A(y,z))))$ F, T, F T_1 T_2 T_3 $M₃$ T_1 T_2 M_1 M_2 M_1 M_2 M_3 M_1 M_2 M_3 T1 M_1 M_2 ∀x∀y((M(x) ∧ M(y) ∧ x≠y) → ∃z(T(z) ∧ A(z,x) ∧ A(z,y)))) F, F, F There is a pair of distinct Ts that went to the same Ms For every pair of Ms, there is a T that went to both There is a T who went to two different Ms $\exists x(T(x) \land \exists y\exists z(y \neq z \land M(y) \land M(z) \land A(x,y) \land A(x,z))$

 $\exists x \exists y(T(x) \land T(y) \land x \neq y \land \forall z(M(z) \rightarrow (A(x,z) \leftrightarrow A(y,z))))$ F, T, F T_1 T_2 T_3 $M₃$ T_1 T_2 M_1 M_2 M_1 M_2 M_3 M_1 M_2 M_3 T1 M_1 M_2 ∀x∀y((M(x) ∧ M(y) ∧ x≠y) → ∃z(T(z) ∧ A(z,x) ∧ A(z,y)))) F, F, T F, F, F There is a pair of distinct Ts that went to the same Ms For every pair of Ms, there is a T that went to both There is a T who went to two different Ms $\exists x(T(x) \land \exists y\exists z(y \neq z \land M(y) \land M(z) \land A(x,y) \land A(x,z))$