

MULTIPLE QUANTIFIERS

Friday, 25 April

$$I. \forall x \forall y (R(x,y) \vee R(y,x))$$

$$\forall x R(x,x)$$

1. $\forall x \forall y (R(x,y) \vee R(y,x))$

2. a

$R(a,a)$

$\forall x R(x,x)$

\forall Intro

1. $\forall x \forall y (R(x,y) \vee R(y,x))$

2. a

3. $\forall y (R(a,y) \vee R(y,a))$ \forall Elim 1

$R(a,a)$

$\forall x R(x,x)$

\forall Intro

1. $\forall x \forall y (R(x,y) \vee R(y,x))$

2. a

3. $\forall y (R(a,y) \vee R(y,a))$ \forall Elim 1

4. $R(a,a) \vee R(a,a)$ \forall Elim 2

$R(a,a)$

$\forall x R(x,x)$

\forall Intro

1. $\forall x \forall y (R(x,y) \vee R(y,x))$

2. a

3. $\forall y (R(a,y) \vee R(y,a))$ \forall Elim 1

4. $R(a,a) \vee R(a,a)$ \forall Elim 2

5. $R(a,a)$

Taut Con 4

$\forall x R(x,x)$

\forall Intro

1. $\forall x \forall y (R(x,y) \vee R(y,x))$

2. a

3. $\forall y (R(a,y) \vee R(y,a))$ \forall Elim 1

4. $R(a,a) \vee R(a,a)$ \forall Elim 2

5. $R(a,a)$ Taut Con 4

6. $\forall x R(x,x)$ \forall Intro 2-5

1. $\forall x \forall y (R(x,y) \vee R(y,x))$

2. a

3. $\forall y (R(a,y) \vee R(y,a))$ \forall Elim 1

4. $R(a,a) \vee R(a,a)$ \forall Elim 2

5. $R(a,a)$ Taut Con 4

6. $\forall x R(x,x)$ \forall Intro 2-5

1. $\forall x \forall y (R(x,y) \rightarrow S(y,x))$
2. $\exists x \exists y (\neg S(x,y) \wedge Q(x,y))$

$$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$$

$$1. \forall x \forall y (R(x,y) \rightarrow S(y,x))$$

$$2. \exists x \exists y (\neg S(x,y) \wedge Q(x,y))$$

$$3. \boxed{a} \exists y (\neg S(a,y) \wedge Q(a,y))$$

$$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$$

$$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$$

\exists Elim

$$1. \forall x \forall y (R(x,y) \rightarrow S(y,x))$$

$$2. \exists x \exists y (\neg S(x,y) \wedge Q(x,y))$$

$$3. \boxed{a} \exists y (\neg S(a,y) \wedge Q(a,y))$$

$$4. \boxed{b} \neg S(a,b) \wedge Q(a,b)$$

$$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$$

$$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$$

\exists Elim

$$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$$

\exists Elim

$$1. \forall x \forall y (R(x,y) \rightarrow S(y,x))$$

$$2. \exists x \exists y (\neg S(x,y) \wedge Q(x,y))$$

$$3. \boxed{a} \exists y (\neg S(a,y) \wedge Q(a,y))$$

$$4. \boxed{b} \neg S(a,b) \wedge Q(a,b)$$

$$5. R(_,_) \rightarrow S(_,_) \quad \forall \text{ Elim } 1 \ x2$$

$$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$$

$$\exists x \exists y (\neg R(x,y) \wedge Q(y,x)) \quad \exists \text{ Elim}$$

$$\exists x \exists y (\neg R(x,y) \wedge Q(y,x)) \quad \exists \text{ Elim}$$

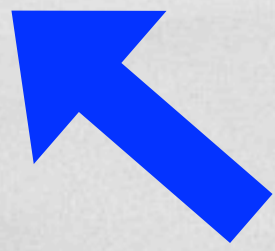
$$1. \forall x \forall y (R(x,y) \rightarrow S(y,x))$$

$$2. \exists x \exists y (\neg S(x,y) \wedge Q(x,y))$$

$$3. \boxed{a} \exists y (\neg S(a,y) \wedge Q(a,y))$$

$$4. \boxed{b} \neg S(a,b) \wedge Q(a,b)$$

$$5. R(_,_) \rightarrow S(_,_) \quad \forall \text{ Elim } 1 \ x2$$


Want S(a,b) here

$$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$$

$$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$$

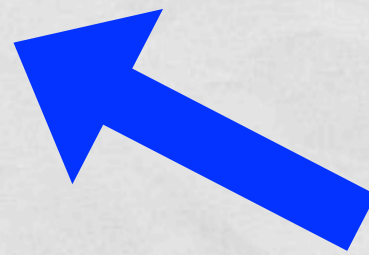
\exists Elim

$$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$$

\exists Elim

$$1. \forall x \forall y (R(x,y) \rightarrow S(y,x))$$

$$2. \exists x \exists y (\neg S(x,y) \wedge Q(x,y))$$



So make $x=b, y=a$

$$3. \boxed{a} \exists y (\neg S(a,y) \wedge Q(a,y))$$

$$4. \boxed{b} \neg S(a,b) \wedge Q(a,b)$$

$$5. R(_,_) \rightarrow S(_,_)$$

\forall Elim 1 x2



Want $S(a,b)$ here

$$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$$

$$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$$

\exists Elim

$$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$$

\exists Elim

$$1. \forall x \forall y (R(x,y) \rightarrow S(y,x))$$

$$2. \exists x \exists y (\neg S(x,y) \wedge Q(x,y))$$

$$3. \boxed{a} \exists y (\neg S(a,y) \wedge Q(a,y))$$

$$4. \boxed{b} \neg S(a,b) \wedge Q(a,b)$$

$$5. R(b,a) \rightarrow S(a,b)$$

\forall Elim 1 x2 [:x>b :y>a]

$$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$$

$$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$$

\exists Elim

$$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$$

\exists Elim

1. $\forall x \forall y (R(x,y) \rightarrow S(y,x))$

2. $\exists x \exists y (\neg S(x,y) \wedge Q(x,y))$

3. a $\exists y (\neg S(a,y) \wedge Q(a,y))$

4. b $\neg S(a,b) \wedge Q(a,b)$

5. $R(b,a) \rightarrow S(a,b)$

\forall Elim 1 $x2$ [:x>b :y>a]

6. $\neg R(b,a)$

Taut Con 4,5

$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$

$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$

\exists Elim

$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$

\exists Elim

$$1. \forall x \forall y (R(x,y) \rightarrow S(y,x))$$

$$2. \exists x \exists y (\neg S(x,y) \wedge Q(x,y))$$

$$3. \boxed{a} \exists y (\neg S(a,y) \wedge Q(a,y))$$

$$4. \boxed{b} \neg S(a,b) \wedge Q(a,b)$$

$$5. R(b,a) \rightarrow S(a,b)$$

\forall Elim 1 $x2$ $[:x>b :y>a]$

$$6. \neg R(b,a)$$

Taut Con 4,5

$$7. \neg R(b,a) \wedge Q(a,b)$$

Taut Con 4,6

$$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$$

$$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$$

\exists Elim

$$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$$

\exists Elim

1. $\forall x \forall y (R(x,y) \rightarrow S(y,x))$

2. $\exists x \exists y (\neg S(x,y) \wedge Q(x,y))$

3. a $\exists y (\neg S(a,y) \wedge Q(a,y))$

4. b $\neg S(a,b) \wedge Q(a,b)$

5. $R(b,a) \rightarrow S(a,b)$

\forall Elim 1 x2 [:x>b :y>a]

6. $\neg R(b,a)$

Taut Con 4,5

7. $\neg R(b,a) \wedge Q(a,b)$

Taut Con 4,6

8. $\exists y (\neg R(b,y) \wedge Q(y,b))$

\exists Intro 7

$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$

$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$

\exists Elim

$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$

\exists Elim

1. $\forall x \forall y (R(x,y) \rightarrow S(y,x))$

2. $\exists x \exists y (\neg S(x,y) \wedge Q(x,y))$

3. a $\exists y (\neg S(a,y) \wedge Q(a,y))$

4. b $\neg S(a,b) \wedge Q(a,b)$

5. $R(b,a) \rightarrow S(a,b)$

\forall Elim 1 x2 [:x>b :y>a]

6. $\neg R(b,a)$

Taut Con 4,5

7. $\neg R(b,a) \wedge Q(a,b)$

Taut Con 4,6

8. $\exists y (\neg R(b,y) \wedge Q(y,b))$

\exists Intro 7

9. $\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$

\exists Intro 8

$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$

\exists Elim

$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$

\exists Elim

1. $\forall x \forall y (R(x,y) \rightarrow S(y,x))$

2. $\exists x \exists y (\neg S(x,y) \wedge Q(x,y))$

3. **a** $\exists y (\neg S(a,y) \wedge Q(a,y))$

4. **b** $\neg S(a,b) \wedge Q(a,b)$

5. $R(b,a) \rightarrow S(a,b)$

\forall Elim 1 x2 **$[:x>b :y>a]$**

6. $\neg R(b,a)$

Taut Con 4,5

7. $\neg R(b,a) \wedge Q(a,b)$

Taut Con 4,6

8. $\exists y (\neg R(b,y) \wedge Q(y,b))$

\exists Intro 7

9. $\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$

\exists Intro 8

10. $\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$

\exists Elim 3,4-9

$\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$

\exists Elim

1. $\forall x \forall y (R(x,y) \rightarrow S(y,x))$

2. $\exists x \exists y (\neg S(x,y) \wedge Q(x,y))$

3. a $\exists y (\neg S(a,y) \wedge Q(a,y))$

4. b $\neg S(a,b) \wedge Q(a,b)$

5. $R(b,a) \rightarrow S(a,b)$

\forall Elim 1 x2 [:x>b :y>a]

6. $\neg R(b,a)$

Taut Con 4,5

7. $\neg R(b,a) \wedge Q(a,b)$

Taut Con 4,6

8. $\exists y (\neg R(b,y) \wedge Q(y,b))$

\exists Intro 7

9. $\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$

\exists Intro 8

10. $\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$

\exists Elim 3,4-9

11. $\exists x \exists y (\neg R(x,y) \wedge Q(y,x))$

\exists Elim 2, 3-10

$$1. \forall x \forall y \forall z ([R(x,y) \wedge R(x,z)] \rightarrow R(y,z))$$

$$2. \forall x R(x,x)$$

$$\forall x \forall y (R(x,y) \rightarrow R(y,x))$$

$$1. \forall x \forall y \forall z ([R(x,y) \wedge R(x,z)] \rightarrow R(y,z))$$

$$2. \forall x R(x,x)$$

3. a

$$\forall y (R(a,y) \rightarrow R(y,a))$$

$$\forall x \forall y (R(x,y) \rightarrow R(y,x))$$

\forall Intro

1. $\forall x \forall y \forall z ([R(x,y) \wedge R(x,z)] \rightarrow R(y,z))$

2. $\forall x R(x,x)$

3. a

4. b

$R(a,b) \rightarrow R(b,a)$

$\forall y (R(a,y) \rightarrow R(y,a))$

$\forall x \forall y (R(x,y) \rightarrow R(y,x))$

\forall Intro

\forall Intro

1. $\forall x \forall y \forall z ([R(x,y) \wedge R(x,z)] \rightarrow R(y,z))$

2. $\forall x R(x,x)$

3. a

4. b

5. $R(a,b)$

$R(b,a)$

$R(a,b) \rightarrow R(b,a)$

$\forall y (R(a,y) \rightarrow R(y,a))$

$\forall x \forall y (R(x,y) \rightarrow R(y,x))$

\rightarrow Intro

\forall Intro

\forall Intro

$$1. \forall x \forall y \forall z ([R(x,y) \wedge R(x,z)] \rightarrow R(y,z))$$

$$2. \forall x R(x,x)$$

$$3. \boxed{a}$$

$$4. \boxed{b}$$

$$5. R(a,b)$$

$$R(b,a)$$

$$R(a,b) \rightarrow R(b,a)$$

$$\forall y (R(a,y) \rightarrow R(y,a))$$

$$\forall x \forall y (R(x,y) \rightarrow R(y,x))$$



Want $R(b,a)$ here

\rightarrow Intro

\forall Intro

\forall Intro

1. $\forall x \forall y \forall z ([R(x,y) \wedge R(x,z)] \rightarrow R(y,z))$

2. $\forall x R(x,x)$

3. a

4. b

5. $R(a,b)$

6. $R(x,b) \wedge R(x,a) \rightarrow R(b,a)$ \forall Elim 1 x3

$R(b,a)$

$R(a,b) \rightarrow R(b,a)$


$\forall y (R(a,y) \rightarrow R(y,a))$

$\forall x \forall y (R(x,y) \rightarrow R(y,x))$

\rightarrow Intro

\forall Intro

\forall Intro


Want $R(b,a)$ here

1. $\forall x \forall y \forall z ([R(x,y) \wedge R(x,z)] \rightarrow R(y,z))$


2. $\forall x R(x,x)$

3. a

4. b

5. $R(a,b)$

6. $R(x,b) \wedge R(x,a) \rightarrow R(b,a)$ \forall Elim 1 x3


Want $R(b,a)$ here


We can pick any x we want here

$R(b,a)$

$R(a,b) \rightarrow R(b,a)$

\rightarrow Intro

$\forall y (R(a,y) \rightarrow R(y,a))$

\forall Intro

$\forall x \forall y (R(x,y) \rightarrow R(y,x))$

\forall Intro

$$1. \forall x \forall y \forall z ([R(x,y) \wedge R(x,z)] \rightarrow R(y,z))$$

$$2. \forall x R(x,x)$$

$$3. \boxed{a}$$

$$4. \boxed{b}$$

$$5. R(a,b)$$

$$6. R(x,b) \wedge R(x,a) \rightarrow R(b,a) \quad \forall \text{ Elim } 1 \ x3$$

$$R(b,a)$$

$$R(a,b) \rightarrow R(b,a)$$

$$\forall y (R(a,y) \rightarrow R(y,a))$$

$$\forall x \forall y (R(x,y) \rightarrow R(y,x))$$



Want $R(b,a)$ here

\rightarrow Intro

\forall Intro

\forall Intro

$$1. \forall x \forall y \forall z ([R(x,y) \wedge R(x,z)] \rightarrow R(y,z))$$

$$2. \forall x R(x,x)$$

$$3. \boxed{a}$$

$$4. \boxed{b}$$

$$5. R(a,b)$$

$$6. R(x,b) \wedge R(x,a) \rightarrow R(b,a) \quad \forall \text{ Elim } 1 \ x3$$

So make $x = a$

$$R(b,a)$$

$$R(a,b) \rightarrow R(b,a)$$


$$\forall y (R(a,y) \rightarrow R(y,a))$$

$$\forall x \forall y (R(x,y) \rightarrow R(y,x))$$

\rightarrow Intro

\forall Intro

\forall Intro


Want $R(b,a)$ here

1. $\forall x \forall y \forall z ([R(x,y) \wedge R(x,z)] \rightarrow R(y,z))$

2. $\forall x R(x,x)$

3. a

4. b

5. $R(a,b)$

6. $[R(a,b) \wedge R(a,a)] \rightarrow R(b,a)$ \forall Elim 1 x3

$R(a,b) \rightarrow R(b,a)$

\rightarrow Intro

$\forall y (R(a,y) \rightarrow R(y,a))$

\forall Intro

$\forall x \forall y (R(x,y) \rightarrow R(y,x))$

\forall Intro

1. $\forall x \forall y \forall z ([R(x,y) \wedge R(x,z)] \rightarrow R(y,z))$

2. $\forall x R(x,x)$

3. a

4. b

5. $R(a,b)$

6. $[R(a,b) \wedge R(a,a)] \rightarrow R(b,a)$ \forall Elim 1 x3

7. $R(a,a)$ \forall Elim 2

$R(a,b) \rightarrow R(b,a)$

\rightarrow Intro

$\forall y (R(a,y) \rightarrow R(y,a))$

\forall Intro

$\forall x \forall y (R(x,y) \rightarrow R(y,x))$

\forall Intro

1. $\forall x \forall y \forall z ([R(x,y) \wedge R(x,z)] \rightarrow R(y,z))$

2. $\forall x R(x,x)$

3. a

4. b

5. $R(a,b)$

6. $[R(a,b) \wedge R(a,a)] \rightarrow R(b,a)$ \forall Elim 1 x3

7. $R(a,a)$ \forall Elim 2

8. $R(b,a)$

Taut Con 5,6,7

$R(a,b) \rightarrow R(b,a)$

\rightarrow Intro

$\forall y (R(a,y) \rightarrow R(y,a))$

\forall Intro

$\forall x \forall y (R(x,y) \rightarrow R(y,x))$

\forall Intro

1. $\forall x \forall y \forall z ([R(x,y) \wedge R(x,z)] \rightarrow R(y,z))$

2. $\forall x R(x,x)$

3. a

4. b

5. $R(a,b)$

6. $[R(a,b) \wedge R(a,a)] \rightarrow R(b,a)$ \forall Elim 1 x3

7. $R(a,a)$ \forall Elim 2

8. $R(b,a)$

Taut Con 5,6,7

9. $R(a,b) \rightarrow R(b,a)$

\rightarrow Intro 5-8

$\forall y (R(a,y) \rightarrow R(y,a))$

\forall Intro

$\forall x \forall y (R(x,y) \rightarrow R(y,x))$

\forall Intro

1. $\forall x \forall y \forall z ([R(x,y) \wedge R(x,z)] \rightarrow R(y,z))$

2. $\forall x R(x,x)$

3. a

4. b

5. $R(a,b)$

6. $[R(a,b) \wedge R(a,a)] \rightarrow R(b,a)$ \forall Elim 1 x3

7. $R(a,a)$ \forall Elim 2

8. $R(b,a)$

Taut Con 5,6,7

9. $R(a,b) \rightarrow R(b,a)$

\rightarrow Intro 5-8

10. $\forall y (R(a,y) \rightarrow R(y,a))$

\forall Intro 4-9

$\forall x \forall y (R(x,y) \rightarrow R(y,x))$

\forall Intro

1. $\forall x \forall y \forall z ([R(x,y) \wedge R(x,z)] \rightarrow R(y,z))$

2. $\forall x R(x,x)$

3. a

4. b

5. $R(a,b)$

6. $[R(a,b) \wedge R(a,a)] \rightarrow R(b,a)$ \forall Elim 1 x3

7. $R(a,a)$ \forall Elim 2

8. $R(b,a)$

Taut Con 5,6,7

9. $R(a,b) \rightarrow R(b,a)$

\rightarrow Intro 5-8

10. $\forall y (R(a,y) \rightarrow R(y,a))$

\forall Intro 4-9

11. $\forall x \forall y (R(x,y) \rightarrow R(y,x))$

\forall Intro 5-10