Puzzle

On a special island populated by knights and knaves, the natives understand English perfectly, but they only answer questions in their own language. "Bal" and "Da" mean "Yes" and "No", but you don't know which is which.

You asked a native "Does Bal mean 'Yes'", and he said 'Bal'. Was the speaker a knight or a knave?

Puzzle Answer

It is not possible to tell what "Bal" means, but we can tell that the speaker must be a knight.

Suppose "Bal" means yes. Then "Bal" is the truthful answer to the question whether "Bal" means yes. So in this case, the speaker is a knight.

Suppose "Bal" means no. Then "No" is the truthful English answer to the question whether "Bal" means yes, therefore "Bal" is the truthful native answer to the question. So again, the speaker is a knight.

Multiple Quanitifiers

Wednesday, 23 April

Proofs with Multiple Quantifiers

1. $\forall x \forall y(R(x,y) \rightarrow S(x,y))$
2. $\forall x \forall y(S(x,y) \rightarrow T(x,y))$

$\forall x \forall y(R(x,y) \rightarrow T(x,y))$

1. $\forall x \forall y(R(x,y) \rightarrow S(x,y))$
2. $\forall x \forall y(S(x,y) \rightarrow T(x,y))$

$\forall x \forall y(R(x,y) \rightarrow T(x,y))$

1. $\forall x \forall y(R(x,y) \rightarrow S(x,y))$
2. $\forall x \forall y(S(x,y) \rightarrow T(x,y))$

3. a

$\forall y(R(a,y) \rightarrow T(a,y))$

$\forall x \forall y(R(x,y) \rightarrow T(x,y))$ $\forall$ Intro

1. $\forall x \forall y(R(x,y) \rightarrow S(x,y))$
2. $\forall x \forall y(S(x,y) \rightarrow T(x,y))$

3. [a]		
4. [b]		
	$R(a,b) \rightarrow T(a,b)$	
	$\forall y(R(a,y) \rightarrow T(a,y))$	$\forall$ Intro
	$\forall x \forall y(R(x,y) \rightarrow T(x,y))$	$\forall$ Intro

1. $\forall x \forall y(R(x,y) \rightarrow S(x,y))$
2. $\forall x \forall y(S(x,y) \rightarrow T(x,y))$

3. $\boxed{a}$

4. $\boxed{b}$

5. $R(a,b)$

$T(a,b))$

$R(a,b) \rightarrow T(a,b)$ $\rightarrow$ Intro

$\forall y(R(a,y) \rightarrow T(a,y))$ $\forall$ Intro

$\forall x \forall y(R(x,y) \rightarrow T(x,y))$ $\forall$ Intro

1. $\forall x \forall y(R(x,y) \rightarrow S(x,y))$
2. $\forall x \forall y(S(x,y) \rightarrow T(x,y))$

3. $\boxed{a}$

4. $\boxed{b}$

5. $R(a,b)$

6. $\forall y(R(a,y) \rightarrow S(a,y)$ $\forall$ Elim 1

$T(a,b))$

$R(a,b) \rightarrow T(a,b)$ $\rightarrow$ Intro

$\forall y(R(a,y) \rightarrow T(a,y))$ $\forall$ Intro

$\forall x \forall y(R(x,y) \rightarrow T(x,y))$ $\forall$ Intro

1. $\forall x \forall y(R(x,y) \rightarrow S(x,y))$
2. $\forall x \forall y(S(x,y) \rightarrow T(x,y))$

3. $\boxed{a}$

4. $\boxed{b}$

5. $R(a,b)$

6. $\forall y(R(a,y) \rightarrow S(a,y))$ $\forall$ Elim 1

7. $R(a,b) \rightarrow S(a,b)$ $\forall$ Elim 6

$T(a,b))$

$R(a,b) \rightarrow T(a,b)$ $\rightarrow$ Intro

$\forall y(R(a,y) \rightarrow T(a,y))$ $\forall$ Intro

$\forall x \forall y(R(x,y) \rightarrow T(x,y))$ $\forall$ Intro

1. $\forall x \forall y(R(x,y) \rightarrow S(x,y))$
2. $\forall x \forall y(S(x,y) \rightarrow T(x,y))$

3. [a]		
4. [b]		
5. R(a,b)		
6. $\forall y(R(a,y) \rightarrow S(a,y)$		$\forall$ Elim 1
7. $R(a,b) \rightarrow S(a,b)$		$\forall$ Elim 6
8. S(a,b)		$\rightarrow$ Elim 5,7
	T(a,b))	
$R(a,b) \rightarrow T(a,b)$		$\rightarrow$ Intro
$\forall y(R(a,y) \rightarrow T(a,y))$		$\forall$ Intro
$\forall x \forall y(R(x,y) \rightarrow T(x,y))$		$\forall$ Intro

1. $\forall x \forall y(R(x,y) \rightarrow S(x,y))$
2. $\forall x \forall y(S(x,y) \rightarrow T(x,y))$

3. [a]		
4. [b]		
5. R(a,b)		
6.	$\forall y(R(a,y) \rightarrow S(a,y)$	$\forall$ Elim 1
7.	$R(a,b) \rightarrow S(a,b)$	$\forall$ Elim 6
8.	$S(a,b)$	$\rightarrow$ Elim 5,7
9.	$\forall y(S(a,y) \rightarrow T(a,y))$	$\forall$ Elim 2
	$T(a,b))$	
	$R(a,b) \rightarrow T(a,b)$	$\rightarrow$ Intro
	$\forall y(R(a,y) \rightarrow T(a,y))$	$\forall$ Intro
	$\forall x \forall y(R(x,y) \rightarrow T(x,y))$	$\forall$ Intro

1. $\forall x \forall y(R(x,y) \rightarrow S(x,y))$
2. $\forall x \forall y(S(x,y) \rightarrow T(x,y))$

3. [a]		
4. [b]		
5. R(a,b)		
6.	$\forall y(R(a,y) \rightarrow S(a,y)$	$\forall$ Elim 1
7.	$R(a,b) \rightarrow S(a,b)$	$\forall$ Elim 6
8.	$S(a,b)$	$\rightarrow$ Elim 5,7
9.	$\forall y(S(a,y) \rightarrow T(a,y))$	$\forall$ Elim 2
10.	$S(a,b) \rightarrow T(a,b))$	$\forall$ Elim 9
	$T(a,b))$	
$R(a,b) \rightarrow T(a,b)$		$\rightarrow$ Intro
$\forall y(R(a,y) \rightarrow T(a,y))$		$\forall$ Intro
$\forall x \forall y(R(x,y) \rightarrow T(x,y))$		$\forall$ Intro

1. $\forall x \forall y(R(x,y) \rightarrow S(x,y))$
2. $\forall x \forall y(S(x,y) \rightarrow T(x,y))$

3. [a]	
4. [b]	
5. R(a,b)	
6. $\forall y(R(a,y) \rightarrow S(a,y)$	$\forall$ Elim 1
7. $R(a,b) \rightarrow S(a,b)$	$\forall$ Elim 6
8. S(a,b)	$\rightarrow$ Elim 5,7
9. $\forall y(S(a,y) \rightarrow T(a,y))$	$\forall$ Elim 2
10. $S(a,b) \rightarrow T(a,b))$	$\forall$ Elim 9
11.T(a,b))	$\rightarrow$ Elim 8,10
$R(a,b) \rightarrow T(a,b)$	$\rightarrow$ Intro
$\forall y(R(a,y) \rightarrow T(a,y))$	$\forall$ Intro
$\forall x \forall y(R(x,y) \rightarrow T(x,y))$	$\forall$ Intro

1. $\forall x \forall y(R(x,y) \rightarrow S(x,y))$
2. $\forall x \forall y(S(x,y) \rightarrow T(x,y))$

3. [a]	
4. [b]	
5. R(a,b)	
6. $\forall y(R(a,y) \rightarrow S(a,y)$	$\forall$ Elim 1
7. $R(a,b) \rightarrow S(a,b)$	$\forall$ Elim 6
8. S(a,b)	$\rightarrow$ Elim 5,7
9. $\forall y(S(a,y) \rightarrow T(a,y))$	$\forall$ Elim 2
10. $S(a,b) \rightarrow T(a,b))$	$\forall$ Elim 9
11. T(a,b))	$\rightarrow$ Elim 8,10
12. $R(a,b) \rightarrow T(a,b)$	$\rightarrow$ Intro 5-11
$\forall y(R(a,y) \rightarrow T(a,y))$	$\forall$ Intro
$\forall x \forall y(R(x,y) \rightarrow T(x,y))$	$\forall$ Intro

1. $\forall x \forall y(R(x,y) \rightarrow S(x,y))$
2. $\forall x \forall y(S(x,y) \rightarrow T(x,y))$

3. [a]	
4. [b]	
5. R(a,b)	
6. $\forall y(R(a,y) \rightarrow S(a,y)$	$\forall$ Elim 1
7. $R(a,b) \rightarrow S(a,b)$	$\forall$ Elim 6
8. S(a,b)	$\rightarrow$ Elim 5,7
9. $\forall y(S(a,y) \rightarrow T(a,y))$	$\forall$ Elim 2
10. $S(a,b) \rightarrow T(a,b))$	$\forall$ Elim 9
11. T(a,b))	$\rightarrow$ Elim 8,10
12. $R(a,b) \rightarrow T(a,b)$	$\rightarrow$ Intro 5-11
13. $\forall y(R(a,y) \rightarrow T(a,y))$	$\forall$ Intro 4-12
$\forall x \forall y(R(x,y) \rightarrow T(x,y))$	$\forall$ Intro

1. $\forall x \forall y(R(x,y) \rightarrow S(x,y))$
2. $\forall x \forall y(S(x,y) \rightarrow T(x,y))$

3. [a]		
4. [b]		
5. R(a,b)		
6.	$\forall y(R(a,y) \rightarrow S(a,y)$	$\forall$ Elim 1
7.	$R(a,b) \rightarrow S(a,b)$	$\forall$ Elim 6
8.	$S(a,b)$	$\rightarrow$ Elim 5,7
9.	$\forall y(S(a,y) \rightarrow T(a,y))$	$\forall$ Elim 2
10.	$S(a,b) \rightarrow T(a,b))$	$\forall$ Elim 9
11.	$T(a,b))$	$\rightarrow$ Elim 8,10
12.	$R(a,b) \rightarrow T(a,b)$	$\rightarrow$ Intro 5-11
13.	$\forall y(R(a,y) \rightarrow T(a,y))$	$\forall$ Intro 4-12
14.	$\forall x \forall y(R(x,y) \rightarrow T(x,y))$	$\forall$ Intro 3-13

1. $\forall x(A(x) \rightarrow \forall y(B(y) \rightarrow D(x,y)))$

$\forall x(B(x) \rightarrow \forall y(A(y) \rightarrow D(y,x)))$

1. $\forall x(A(x) \rightarrow \forall y(B(y) \rightarrow D(x,y)))$

Everyone on Team A defeated everyone on Team B

Everyone on Team B was defeated by everyone on Team A

$\forall x(B(x) \rightarrow \forall y(A(y) \rightarrow D(y,x)))$

1. $\forall x(A(x) \rightarrow \forall y(B(y) \rightarrow D(x,y)))$

Everyone on Team A defeated everyone on Team B

$\forall x \forall y[(A(x) \wedge B(y)) \rightarrow D(x,y)]$

$\forall x \forall y[(B(x) \wedge A(y)) \rightarrow D(y,x)]$

Everyone on Team B was defeated by everyone on Team A

$\forall x(B(x) \rightarrow \forall y(A(y) \rightarrow D(y,x)))$

1. $\forall x(A(x) \rightarrow \forall y(B(y) \rightarrow D(x,y)))$

$\forall x(B(x) \rightarrow \forall y(A(y) \rightarrow D(y,x)))$

1. $\forall x(A(x) \rightarrow \forall y(B(y) \rightarrow D(x,y)))$

2. b

$B(b) \rightarrow \forall y(A(y) \rightarrow D(y,b))$

$\forall x(B(x) \rightarrow \forall y(A(y) \rightarrow D(y,x)))$ $\forall$ Intro

1. $\forall x(A(x) \rightarrow \forall y(B(y) \rightarrow D(x,y)))$

2. $\boxed{b}$

3. $B(b)$

$\forall y(A(y) \rightarrow D(y,b))$

$B(b) \rightarrow \forall y(A(y) \rightarrow D(y,b))$ $\rightarrow$ Intro

$\forall x(B(x) \rightarrow \forall y(A(y) \rightarrow D(y,x)))$ $\forall$ Intro

1. $\forall x(A(x) \rightarrow \forall y(B(y) \rightarrow D(x,y)))$

2. $\boxed{b}$

3. $B(b)$

4. $\boxed{a}$

$A(a) \rightarrow D(a,b)$

$\forall y(A(y) \rightarrow D(y,b))$ $\forall$ Intro

$B(b) \rightarrow \forall y(A(y) \rightarrow D(y,b))$ $\rightarrow$ Intro

$\forall x(B(x) \rightarrow \forall y(A(y) \rightarrow D(y,x)))$ $\forall$ Intro

1. $\forall x(A(x) \rightarrow \forall y(B(y) \rightarrow D(x,y)))$

2. $\boxed{b}$

3. $B(b)$

4. $\boxed{a}$

5. $A(a)$

$D(a,b)$ $\quad$

$A(a) \rightarrow D(a,b)$ $\quad \rightarrow$ Intro

$\forall y(A(y) \rightarrow D(y,b))$ $\quad \forall$ Intro

$B(b) \rightarrow \forall y(A(y) \rightarrow D(y,b))$ $\quad \rightarrow$ Intro

$\forall x(B(x) \rightarrow \forall y(A(y) \rightarrow D(y,x)))$ $\quad \forall$ Intro

1. $\forall x(A(x) \rightarrow \forall y(B(y) \rightarrow D(x,y)))$

2. $\boxed{b}$

3. $B(b)$

4. $\boxed{a}$

5. $A(a)$

6. $A(a) \rightarrow \forall y(B(y) \rightarrow D(a,y))$ $\forall$ Elim 1

$D(a,b)$

$A(a) \rightarrow D(a,b)$ $\rightarrow$ Intro

$\forall y(A(y) \rightarrow D(y,b))$ $\forall$ Intro

$B(b) \rightarrow \forall y(A(y) \rightarrow D(y,b))$ $\rightarrow$ Intro

$\forall x(B(x) \rightarrow \forall y(A(y) \rightarrow D(y,x)))$ $\forall$ Intro

1. $\forall x(A(x) \rightarrow \forall y(B(y) \rightarrow D(x,y)))$

2. $\boxed{b}$

3. $B(b)$

4. $\boxed{a}$

5. $A(a)$

6. $A(a) \rightarrow \forall y(B(y) \rightarrow D(a,y))$ $\forall$ Elim 1

7. $\forall y(B(y) \rightarrow D(a,y))$ $\rightarrow$ Elim 5,6

$D(a,b)$

$A(a) \rightarrow D(a,b)$ $\rightarrow$ Intro

$\forall y(A(y) \rightarrow D(y,b))$ $\forall$ Intro

$B(b) \rightarrow \forall y(A(y) \rightarrow D(y,b))$ $\rightarrow$ Intro

$\forall x(B(x) \rightarrow \forall y(A(y) \rightarrow D(y,x)))$ $\forall$ Intro

1. $\forall x(A(x) \rightarrow \forall y(B(y) \rightarrow D(x,y)))$

2. $\boxed{b}$

3. $B(b)$

4. $\boxed{a}$

5. $A(a)$

6. $A(a) \rightarrow \forall y(B(y) \rightarrow D(a,y))$ $\forall$ Elim 1

7. $\forall y(B(y) \rightarrow D(a,y))$ $\rightarrow$ Elim 5,6

8. $B(b) \rightarrow D(a,b)$ $\forall$ Elim 7

$D(a,b)$

$A(a) \rightarrow D(a,b)$ $\rightarrow$ Intro

$\forall y(A(y) \rightarrow D(y,b))$ $\forall$ Intro

$B(b) \rightarrow \forall y(A(y) \rightarrow D(y,b))$ $\rightarrow$ Intro

$\forall x(B(x) \rightarrow \forall y(A(y) \rightarrow D(y,x)))$ $\forall$ Intro

1. $\forall x(A(x) \rightarrow \forall y(B(y) \rightarrow D(x,y)))$

2. $\boxed{b}$

3. $B(b)$

4. $\boxed{a}$

5. $A(a)$

6. $A(a) \rightarrow \forall y(B(y) \rightarrow D(a,y))$ $\forall$ Elim 1

7. $\forall y(B(y) \rightarrow D(a,y))$ $\rightarrow$ Elim 5,6

8. $B(b) \rightarrow D(a,b)$ $\forall$ Elim 7

9. $D(a,b)$ $\rightarrow$ Elim 3,8

$A(a) \rightarrow D(a,b)$ $\rightarrow$ Intro

$\forall y(A(y) \rightarrow D(y,b))$ $\forall$ Intro

$B(b) \rightarrow \forall y(A(y) \rightarrow D(y,b))$ $\rightarrow$ Intro

$\forall x(B(x) \rightarrow \forall y(A(y) \rightarrow D(y,x)))$ $\forall$ Intro

1. $\forall x(A(x) \rightarrow \forall y(B(y) \rightarrow D(x,y)))$

2. [b]

3. B(b)

4. [a]

5. A(a)

6. $A(a) \rightarrow \forall y(B(y) \rightarrow D(a,y))$ $\forall$ Elim 1

7. $\forall y(B(y) \rightarrow D(a,y))$ $\rightarrow$ Elim 5,6

8. $B(b) \rightarrow D(a,b)$ $\forall$ Elim 7

9. D(a,b) $\rightarrow$ Elim 3,8

10. $A(a) \rightarrow D(a,b)$ $\rightarrow$ Intro 5-9

$\forall y(A(y) \rightarrow D(y,b))$ $\forall$ Intro

$B(b) \rightarrow \forall y(A(y) \rightarrow D(y,b))$ $\rightarrow$ Intro

$\forall x(B(x) \rightarrow \forall y(A(y) \rightarrow D(y,x)))$ $\forall$ Intro

1. $\forall x(A(x) \rightarrow \forall y(B(y) \rightarrow D(x,y)))$

2. [b]

3. B(b)

4. [a]

5. A(a)

6. $A(a) \rightarrow \forall y(B(y) \rightarrow D(a,y))$ $\forall$ Elim 1

7. $\forall y(B(y) \rightarrow D(a,y))$ $\rightarrow$ Elim 5,6

8. $B(b) \rightarrow D(a,b)$ $\forall$ Elim 7

9. D(a,b) $\rightarrow$ Elim 3,8

10. $A(a) \rightarrow D(a,b)$ $\rightarrow$ Intro 5-9

11. $\forall y(A(y) \rightarrow D(y,b))$ $\forall$ Intro 4-10

$B(b) \rightarrow \forall y(A(y) \rightarrow D(y,b))$ $\rightarrow$ Intro

$\forall x(B(x) \rightarrow \forall y(A(y) \rightarrow D(y,x)))$ $\forall$ Intro

1. $\forall x(A(x) \rightarrow \forall y(B(y) \rightarrow D(x,y)))$

2. $\boxed{b}$

3. $B(b)$

4. $\boxed{a}$

5. $A(a)$

6. $A(a) \rightarrow \forall y(B(y) \rightarrow D(a,y))$ $\forall$ Elim 1

7. $\forall y(B(y) \rightarrow D(a,y))$ $\rightarrow$ Elim 5,6

8. $B(b) \rightarrow D(a,b)$ $\forall$ Elim 7

9. $D(a,b)$ $\rightarrow$ Elim 3,8

10. $A(a) \rightarrow D(a,b)$ $\rightarrow$ Intro 5-9

11. $\forall y(A(y) \rightarrow D(y,b))$ $\forall$ Intro 4-10

12. $B(b) \rightarrow \forall y(A(y) \rightarrow D(y,b))$ $\rightarrow$ Intro 3-11

$\forall x(B(x) \rightarrow \forall y(A(y) \rightarrow D(y,x)))$ $\forall$ Intro

1. $\forall x(A(x) \rightarrow \forall y(B(y) \rightarrow D(x,y)))$

2. [b]

3. B(b)

4. [a]

5. A(a)

6. $A(a) \rightarrow \forall y(B(y) \rightarrow D(a,y))$ $\forall$ Elim 1

7. $\forall y(B(y) \rightarrow D(a,y))$ $\rightarrow$ Elim 5,6

8. $B(b) \rightarrow D(a,b)$ $\forall$ Elim 7

9. D(a,b) $\rightarrow$ Elim 3,8

10. $A(a) \rightarrow D(a,b)$ $\rightarrow$ Intro 5-9

11. $\forall y(A(y) \rightarrow D(y,b))$ $\forall$ Intro 4-10

12. $B(b) \rightarrow \forall y(A(y) \rightarrow D(y,b))$ $\rightarrow$ Intro 3-11

13. $\forall x(B(x) \rightarrow \forall y(A(y) \rightarrow D(y,x)))$ $\forall$ Intro 2-12

1. $\forall x \forall y (R(x,y) \vee R(y,x))$

$\forall x\ R(x,x)$

1. $\forall x \forall y (R(x,y) \vee R(y,x))$

2. a

$R(a,a)$

$\forall x\, R(x,x)$ $\forall$ Intro

1. $\forall x \forall y(R(x,y) \vee R(y,x))$

2. $\boxed{a}$

3. $\forall y(R(a,y) \vee R(y,a))$ $\forall$ Elim 1

$R(a,a)$

$\forall x\, R(x,x)$ $\forall$ Intro

1. $\forall x \forall y(R(x,y) \vee R(y,x))$

2. $\boxed{a}$

3. $\forall y(R(a,y) \vee R(y,a))$ $\forall$ Elim 1

4. $R(a,a) \vee R(a,a)$ $\forall$ Elim 2

$R(a,a)$

$\forall x\, R(x,x)$ $\forall$ Intro

1. $\forall x \forall y(R(x,y) \vee R(y,x))$

2. $\boxed{a}$

3. $\forall y(R(a,y) \vee R(y,a))$ $\forall$ Elim 1

4. $R(a,a) \vee R(a,a)$ $\forall$ Elim 2

5. $R(a,a)$ Taut Con

6. $\forall x\, R(x,x)$ $\forall$ Intro 5

1. $\forall x \forall y(\text{SameRow}(x,y) \rightarrow \text{SameSize}(x,y))$
2. $\exists x \exists y(\text{Cube}(x) \wedge \text{Tet}(y) \wedge \text{SameRow}(x,y))$

$\exists x \exists y(\text{Cube}(x) \wedge \text{Tet}(y) \wedge \text{SameSize}(x,y))$

1. $\forall x \forall y(\text{SameRow}(x,y) \rightarrow \text{SameSize}(x,y))$
2. $\exists x \exists y(\text{Cube}(x) \wedge \text{Tet}(y) \wedge \text{SameRow}(x,y))$

3. $\boxed{a}$ $\exists y(\text{Cube}(a) \wedge \text{Tet}(y) \wedge \text{SameRow}(a,y))$

$\exists x \exists y(\text{Cube}(x) \wedge \text{Tet}(y) \wedge \text{SameSize}(x,y))$

$\exists x \exists y(\text{Cube}(x) \wedge \text{Tet}(y) \wedge \text{SameSize}(x,y))$ $\exists$ Elim

1. $\forall x \forall y(\text{SameRow}(x,y) \rightarrow \text{SameSize}(x,y))$
2. $\exists x \exists y(\text{Cube}(x) \wedge \text{Tet}(y) \wedge \text{SameRow}(x,y))$

3. a $\exists y(\text{Cube}(a) \wedge \text{Tet}(y) \wedge \text{SameRow}(a,y))$

4. b $\text{Cube}(a) \wedge \text{Tet}(b) \wedge \text{SameRow}(a,b)$

$\exists x \exists y(\text{Cube}(x) \wedge \text{Tet}(y) \wedge \text{SameSize}(x,y))$

$\exists x \exists y(\text{Cube}(x) \wedge \text{Tet}(y) \wedge \text{SameSize}(x,y))$ $\exists$ Elim

$\exists x \exists y(\text{Cube}(x) \wedge \text{Tet}(y) \wedge \text{SameSize}(x,y))$ $\exists$ Elim

1. ∀x∀y(SameRow(x,y) → SameSize(x,y))
2. ∃x∃y(Cube(x) ∧ Tet(y) ∧ SameRow(x,y))

3. [a] ∃y(Cube(a) ∧ Tet(y) ∧ SameRow(a,y))

4. [b] Cube(a) ∧ Tet(b) ∧ SameRow(a,b)

5. SameRow(a,b) → SameSize(a,b) ∀ Elim 1 x2 :x>a :y>b]

∃x∃y(Cube(x) ∧ Tet(y) ∧ SameSize(x,y))

∃x∃y(Cube(x) ∧ Tet(y) ∧ SameSize(x,y)) ∃ Elim

∃x∃y(Cube(x) ∧ Tet(y) ∧ SameSize(x,y)) ∃ Elim

1. $\forall x \forall y(\text{SameRow}(x,y) \rightarrow \text{SameSize}(x,y))$
2. $\exists x \exists y(\text{Cube}(x) \wedge \text{Tet}(y) \wedge \text{SameRow}(x,y))$

3. [a] $\exists y(\text{Cube}(a) \wedge \text{Tet}(y) \wedge \text{SameRow}(a,y))$

4. [b] $\text{Cube}(a) \wedge \text{Tet}(b) \wedge \text{SameRow}(a,b)$

5. $\text{SameRow}(a,b) \rightarrow \text{SameSize}(a,b)$ $\forall$ Elim 1 x2 :x>a :y>b]

6. $\text{SameRow}(a,b)$ $\wedge$ Elim 4

$\exists x \exists y(\text{Cube}(x) \wedge \text{Tet}(y) \wedge \text{SameSize}(x,y))$

$\exists x \exists y(\text{Cube}(x) \wedge \text{Tet}(y) \wedge \text{SameSize}(x,y))$ $\exists$ Elim

$\exists x \exists y(\text{Cube}(x) \wedge \text{Tet}(y) \wedge \text{SameSize}(x,y))$ $\exists$ Elim

1. $\forall x \forall y(\text{SameRow}(x,y) \rightarrow \text{SameSize}(x,y))$
2. $\exists x \exists y(\text{Cube}(x) \wedge \text{Tet}(y) \wedge \text{SameRow}(x,y))$

3. [a] $\exists y(\text{Cube}(a) \wedge \text{Tet}(y) \wedge \text{SameRow}(a,y))$

4. [b] $\text{Cube}(a) \wedge \text{Tet}(b) \wedge \text{SameRow}(a,b)$

5. $\text{SameRow}(a,b) \rightarrow \text{SameSize}(a,b)$ $\forall$ Elim 1 x2 :x>a :y>b]
6. $\text{SameRow}(a,b)$ $\wedge$ Elim 4
7. $\text{SameSize}(a,b)$ $\rightarrow$ Elim 5,6

$\exists x \exists y(\text{Cube}(x) \wedge \text{Tet}(y) \wedge \text{SameSize}(x,y))$

$\exists x \exists y(\text{Cube}(x) \wedge \text{Tet}(y) \wedge \text{SameSize}(x,y))$ $\exists$ Elim

$\exists x \exists y(\text{Cube}(x) \wedge \text{Tet}(y) \wedge \text{SameSize}(x,y))$ $\exists$ Elim

1. ∀x∀y(SameRow(x,y) → SameSize(x,y))
2. ∃x∃y(Cube(x) ∧ Tet(y) ∧ SameRow(x,y))

3. [a] ∃y(Cube(a) ∧ Tet(y) ∧ SameRow(a,y))

4. [b] Cube(a) ∧ Tet(b) ∧ SameRow(a,b)

5. SameRow(a,b) → SameSize(a,b) ∀ Elim 1 x2 :x>a :y>b]

6. SameRow(a,b) ∧ Elim 4

7. SameSize(a,b) → Elim 5,6

8. Cube(a) ∧ Tet(b) ∧ SameSize(a,b) Taut Con 4,7

∃x∃y(Cube(x) ∧ Tet(y) ∧ SameSize(x,y))

∃x∃y(Cube(x) ∧ Tet(y) ∧ SameSize(x,y)) ∃ Elim

∃x∃y(Cube(x) ∧ Tet(y) ∧ SameSize(x,y)) ∃ Elim

1. ∀x∀y(SameRow(x,y) → SameSize(x,y))
2. ∃x∃y(Cube(x) ∧ Tet(y) ∧ SameRow(x,y))

3. [a] ∃y(Cube(a) ∧ Tet(y) ∧ SameRow(a,y))

4. [b] Cube(a) ∧ Tet(b) ∧ SameRow(a,b)

5. SameRow(a,b) → SameSize(a,b) ∀ Elim 1 x2 :x>a :y>b]

6. SameRow(a,b) ∧ Elim 4

7. SameSize(a,b) → Elim 5,6

8. Cube(a) ∧ Tet(b) ∧ SameSize(a,b) Taut Con 4,7

9. ∃y(Cube(a) ∧ Tet(y) ∧ SameSize(a,y)) ∃ Intro 8

∃x∃y(Cube(x) ∧ Tet(y) ∧ SameSize(x,y))

∃x∃y(Cube(x) ∧ Tet(y) ∧ SameSize(x,y)) ∃ Elim

∃x∃y(Cube(x) ∧ Tet(y) ∧ SameSize(x,y)) ∃ Elim

1. ∀x∀y(SameRow(x,y) → SameSize(x,y))
2. ∃x∃y(Cube(x) ∧ Tet(y) ∧ SameRow(x,y))

3. [a] ∃y(Cube(a) ∧ Tet(y) ∧ SameRow(a,y))

4. [b] Cube(a) ∧ Tet(b) ∧ SameRow(a,b)

5. SameRow(a,b) → SameSize(a,b) ∀ Elim 1 x2 :x>a :y>b]

6. SameRow(a,b) ∧ Elim 4

7. SameSize(a,b) → Elim 5,6

8. Cube(a) ∧ Tet(b) ∧ SameSize(a,b) Taut Con 4,7

9. ∃y(Cube(a) ∧ Tet(y) ∧ SameSize(a,y) ∃ Intro 8

10. ∃x∃y(Cube(x) ∧ Tet(y) ∧ SameSize(x,y)) ∃ Intro 9

∃x∃y(Cube(x) ∧ Tet(y) ∧ SameSize(x,y)) ∃ Elim

∃x∃y(Cube(x) ∧ Tet(y) ∧ SameSize(x,y)) ∃ Elim

1. ∀x∀y(SameRow(x,y) → SameSize(x,y))
2. ∃x∃y(Cube(x) ∧ Tet(y) ∧ SameRow(x,y))

3. [a] ∃y(Cube(a) ∧ Tet(y) ∧ SameRow(a,y))

4. [b] Cube(a) ∧ Tet(b) ∧ SameRow(a,b)

5. SameRow(a,b) → SameSize(a,b) ∀ Elim 1 x2 :x>a :y>b]

6. SameRow(a,b) ∧ Elim 4

7. SameSize(a,b) → Elim 5,6

8. Cube(a) ∧ Tet(b) ∧ SameSize(a,b) Taut Con 4,7

9. ∃y(Cube(a) ∧ Tet(y) ∧ SameSize(a,y) ∃ Intro 8

10. ∃x∃y(Cube(x) ∧ Tet(y) ∧ SameSize(x,y)) ∃ Intro 9

11. ∃x∃y(Cube(x) ∧ Tet(y) ∧ SameSize(x,y)) ∃ Elim 3, 4-10

∃x∃y(Cube(x) ∧ Tet(y) ∧ SameSize(x,y)) ∃ Elim

1. ∀x∀y(SameRow(x,y) → SameSize(x,y))
2. ∃x∃y(Cube(x) ∧ Tet(y) ∧ SameRow(x,y))

3. [a] ∃y(Cube(a) ∧ Tet(y) ∧ SameRow(a,y))

4. [b] Cube(a) ∧ Tet(b) ∧ SameRow(a,b)

5. SameRow(a,b) → SameSize(a,b) ∀ Elim 1 x2 :x>a :y>b]

6. SameRow(a,b) ∧ Elim 4

7. SameSize(a,b) → Elim 5,6

8. Cube(a) ∧ Tet(b) ∧ SameSize(a,b) Taut Con 4,7

9. ∃y(Cube(a) ∧ Tet(y) ∧ SameSize(a,y) ∃ Intro 8

10. ∃x∃y(Cube(x) ∧ Tet(y) ∧ SameSize(x,y)) ∃ Intro 9

11. ∃x∃y(Cube(x) ∧ Tet(y) ∧ SameSize(x,y)) ∃ Elim 3, 4-10

12. ∃x∃y(Cube(x) ∧ Tet(y) ∧ SameSize(x,y)) ∃ Elim 2, 3-11

1. $\forall x \forall y(R(x,y) \rightarrow S(y,x))$
2. $\exists x \exists y(\neg S(x,y) \wedge Q(x,y))$

$\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$

1. $\forall x \forall y(R(x,y) \rightarrow S(y,x))$
2. $\exists x \exists y(\neg S(x,y) \wedge Q(x,y))$

3. a $\exists y(\neg S(a,y) \wedge Q(a,y))$

$\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$

$\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ $\exists$ Elim

1. $\forall x \forall y(R(x,y) \rightarrow S(y,x))$
2. $\exists x \exists y(\neg S(x,y) \wedge Q(x,y))$

3. a $\exists y(\neg S(a,y) \wedge Q(a,y))$

4. b $\neg S(a,b) \wedge Q(a,b)$

$\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$

$\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ $\exists$ Elim

$\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ $\exists$ Elim

1. $\forall x \forall y(R(x,y) \rightarrow S(y,x))$
2. $\exists x \exists y(\neg S(x,y) \wedge Q(x,y))$

3. [a] $\exists y(\neg S(a,y) \wedge Q(a,y))$

4. [b] $\neg S(a,b) \wedge Q(a,b)$

5. $R(b,a) \rightarrow S(a,b)$ $\forall$ Elim 1 x2 [:x>b :y>a]

$\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$

$\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ $\exists$ Elim

$\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ $\exists$ Elim

1. $\forall x \forall y(R(x,y) \rightarrow S(y,x))$
2. $\exists x \exists y(\neg S(x,y) \wedge Q(x,y))$

3. [a] $\exists y(\neg S(a,y) \wedge Q(a,y))$

4. [b] $\neg S(a,b) \wedge Q(a,b)$

5. $R(b,a) \rightarrow S(a,b)$ — $\forall$ Elim 1 x2 [:x>b :y>a]
6. $\neg R(b,a)$ — Taut Con 4,5

$\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$

$\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ — $\exists$ Elim

$\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ — $\exists$ Elim

1. $\forall x \forall y(R(x,y) \rightarrow S(y,x))$
2. $\exists x \exists y(\neg S(x,y) \wedge Q(x,y))$

3. [a] $\exists y(\neg S(a,y) \wedge Q(a,y))$

4. [b] $\neg S(a,b) \wedge Q(a,b)$

5. $R(b,a) \rightarrow S(a,b)$ — $\forall$ Elim 1 x2 [:x>b :y>a]
6. $\neg R(b,a)$ — Taut Con 4,5
7. $\neg R(b,a) \wedge Q(a,b)$ — Taut Con 4,6

$\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$

$\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ — $\exists$ Elim

$\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ — $\exists$ Elim

1. $\forall x \forall y(R(x,y) \rightarrow S(y,x))$
2. $\exists x \exists y(\neg S(x,y) \wedge Q(x,y))$

3. [a] $\exists y(\neg S(a,y) \wedge Q(a,y))$

4. [b] $\neg S(a,b) \wedge Q(a,b)$

5. $R(b,a) \rightarrow S(a,b)$ — $\forall$ Elim 1 x2 [:x>b :y>a]
6. $\neg R(b,a)$ — Taut Con 4,5
7. $\neg R(b,a) \wedge Q(a,b)$ — Taut Con 4,6
8. $\exists y(\neg R(b,y) \wedge Q(y,b))$ — $\exists$ Intro 7

$\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ — $\exists$ Elim

$\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ — $\exists$ Elim

1. $\forall x \forall y(R(x,y) \rightarrow S(y,x))$
2. $\exists x \exists y(\neg S(x,y) \wedge Q(x,y))$

3. [a] $\exists y(\neg S(a,y) \wedge Q(a,y))$

4. [b] $\neg S(a,b) \wedge Q(a,b)$

5. $R(b,a) \rightarrow S(a,b)$ — $\forall$ Elim 1 x2 [:x>b :y>a]
6. $\neg R(b,a)$ — Taut Con 4,5
7. $\neg R(b,a) \wedge Q(a,b)$ — Taut Con 4,6
8. $\exists y(\neg R(b,y) \wedge Q(y,b))$ — $\exists$ Intro 7
9. $\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ — $\exists$ Intro 8

$\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ — $\exists$ Elim

$\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ — $\exists$ Elim

1. $\forall x \forall y(R(x,y) \rightarrow S(y,x))$
2. $\exists x \exists y(\neg S(x,y) \wedge Q(x,y))$

3. [a] $\exists y(\neg S(a,y) \wedge Q(a,y))$

4. [b] $\neg S(a,b) \wedge Q(a,b)$

5. $R(b,a) \rightarrow S(a,b)$ — ∀ Elim 1 x2 [:x>b :y>a]
6. $\neg R(b,a)$ — Taut Con 4,5
7. $\neg R(b,a) \wedge Q(a,b)$ — Taut Con 4,6
8. $\exists y(\neg R(b,y) \wedge Q(y,b))$ — ∃ Intro 7
9. $\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ — ∃ Intro 8
10. $\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ — ∃ Elim 3,4-9

$\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ — ∃ Elim

1. $\forall x \forall y(R(x,y) \rightarrow S(y,x))$
2. $\exists x \exists y(\neg S(x,y) \wedge Q(x,y))$

3. [a] $\exists y(\neg S(a,y) \wedge Q(a,y))$

4. [b] $\neg S(a,b) \wedge Q(a,b)$

5. $R(b,a) \rightarrow S(a,b)$ — ∀ Elim 1 x2 [:x>b :y>a]
6. $\neg R(b,a)$ — Taut Con 4,5
7. $\neg R(b,a) \wedge Q(a,b)$ — Taut Con 4,6
8. $\exists y(\neg R(b,y) \wedge Q(y,b))$ — ∃ Intro 7
9. $\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ — ∃ Intro 8
10. $\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ — ∃ Elim 3,4-9
11. $\exists x \exists y(\neg R(x,y) \wedge Q(y,x))$ — ∃ Elim 2, 3-10