Proof?

Is there anything wrong with the following argument?

Claim: Every natural number is interesting.

Proof: If there are some uninteresting natural numbers, then there is a smallest number which is uninteresting. Call it ‘a’. But since ‘a’ is the lowest number which is uninteresting, there is something quite interesting about it. So ‘a’ can’t be the first uninteresting number. But ‘a’ was totally arbitrary. So no number can be the first uninteresting number so no number can be uninteresting so every number is interesting.

Multiple Quanitifiers

Wednesday, 16 April

Interpretations

An interpretation (world) specifies a domain that the quantifiers range over and the meaning of predicates, constants, and functions

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An interpretation (world) specifies a domain that the quantifiers range over and the meaning of predicates, constants, and functions

Example:
Domain = all people
A(x): x is on Team A
B(x): x is on Team B
D(x,y): x defeated y (the last time they played chess...)

a: Adam
b: Barbara

Translations

Everyone on Team A defeated Adam

Translations

Everyone on Team A defeated Adam

$\forall x(A(x) \rightarrow D(x,a))$

Translations

Everyone on Team A defeated Adam

$\forall x(A(x) \rightarrow D(x,a))$

Someone on Team B was defeated by Barbara

Translations

Everyone on Team A defeated Adam

$\forall x(A(x) \rightarrow D(x,a))$

Someone on Team B was defeated by Barbara

$\exists x(B(x) \wedge D(b,x))$

Translations

Everyone on Team A defeated Adam

$\forall x(A(x) \rightarrow D(x,a))$

Someone on Team B was defeated by Barbara

$\exists x(B(x) \wedge D(b,x))$

Everyone on Team A defeated someone

Translations

Everyone on Team A defeated Adam

$\forall x(A(x) \rightarrow D(x,a))$

Someone on Team B was defeated by Barbara

$\exists x(B(x) \wedge D(b,x))$

Everyone on Team A defeated someone

$\forall x(A(x) \rightarrow$

Translations

Everyone on Team A defeated Adam

$\forall x(A(x) \rightarrow D(x,a))$

Someone on Team B was defeated by Barbara

$\exists x(B(x) \wedge D(b,x))$

Everyone on Team A defeated someone

$\forall x(A(x) \rightarrow \exists y\, D(x,y))$

Translations

Everyone on Team A defeated Adam

$\forall x(A(x) \rightarrow D(x,a))$

Someone on Team B was defeated by Barbara

$\exists x(B(x) \wedge D(b,x))$

Everyone on Team A defeated someone

$\forall x(A(x) \rightarrow \exists y\, D(x,y))$

Someone on Team B defeated everyone on Team A

Translations

Everyone on Team A defeated Adam

$\forall x(A(x) \rightarrow D(x,a))$

Someone on Team B was defeated by Barbara

$\exists x(B(x) \wedge D(b,x))$

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$\exists x(B(x) \wedge D(b,x))$

Everyone on Team A defeated someone

$\forall x(A(x) \rightarrow \exists y\, D(x,y))$

Someone on Team B defeated everyone on Team A

$\exists x(B(x) \wedge \forall y(A(y) \rightarrow D(x,y)))$

Translations

If anyone on Team A defeated Adam, Barbara did

Translations

If anyone on Team A defeated Adam, Barbara did

$\exists x(A(x) \wedge D(x,a)) \rightarrow D(b,a)$

Translations

If anyone on Team A defeated Adam, Barbara did

$\exists x(A(x) \wedge D(x,a)) \rightarrow D(b,a)$

Someone on Team A other than Barbara defeated Adam

Translations

If anyone on Team A defeated Adam, Barbara did

$\exists x(A(x) \wedge D(x,a)) \rightarrow D(b,a)$

Someone on Team A other than Barbara defeated Adam

$\exists x(A(x) \wedge x \neq b \wedge D(x,a))$

Translations

If anyone on Team A defeated Adam, Barbara did

$\exists x(A(x) \wedge D(x,a)) \rightarrow D(b,a)$

Someone on Team A other than Barbara defeated Adam

$\exists x(A(x) \wedge x \neq b \wedge D(x,a)) \quad \wedge A(b)$?

Translations

If anyone on Team A defeated Adam, Barbara did

$\exists x(A(x) \wedge D(x,a)) \rightarrow D(b,a)$

Someone on Team A other than Barbara defeated Adam

$\exists x(A(x) \wedge x \neq b \wedge D(x,a)) \quad \wedge A(b)$?

Someone on Team A was not defeated by anyone on Team B

Translations

If anyone on Team A defeated Adam, Barbara did

$\exists x(A(x) \wedge D(x,a)) \rightarrow D(b,a)$

Someone on Team A other than Barbara defeated Adam

$\exists x(A(x) \wedge x \neq b \wedge D(x,a)) \quad \wedge A(b)$?

Someone on Team A was not defeated by anyone on Team B

$\exists x(A(x) \wedge \neg\exists y(B(y) \wedge D(y,x)))$

Translations

If anyone on Team A defeated Adam, Barbara did

$\exists x(A(x) \wedge D(x,a)) \rightarrow D(b,a)$

Someone on Team A other than Barbara defeated Adam

$\exists x(A(x) \wedge x \neq b \wedge D(x,a)) \quad \wedge A(b)$?

Someone on Team A was not defeated by anyone on Team B

$\exists x(A(x) \wedge \neg\exists y(B(y) \wedge D(y,x)))$

No one on Team A defeated everyone on Team B

Translations

If anyone on Team A defeated Adam, Barbara did

$\exists x(A(x) \wedge D(x,a)) \rightarrow D(b,a)$

Someone on Team A other than Barbara defeated Adam

$\exists x(A(x) \wedge x \neq b \wedge D(x,a)) \quad \wedge A(b)$?

Someone on Team A was not defeated by anyone on Team B

$\exists x(A(x) \wedge \neg\exists y(B(y) \wedge D(y,x)))$

No one on Team A defeated everyone on Team B

$\forall x(A(x) \rightarrow \neg\forall y(B(y) \rightarrow D(x,y)))$

Translations

Not everyone on Team A who defeated Adam also defeated Barbara

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Not everyone on Team A who defeated Adam also defeated Barbara

$\neg\forall x([A(x) \wedge D(x,a)] \rightarrow D(x,b))$

Translations

Not everyone on Team A who defeated Adam also defeated Barbara

$$\neg\forall x([A(x) \wedge D(x,a)] \rightarrow D(x,b))$$

Anyone on Team A who defeated anyone at all defeated Barbara

Translations

Not everyone on Team A who defeated Adam also defeated Barbara

$$\neg\forall x([A(x) \wedge D(x,a)] \rightarrow D(x,b))$$

Anyone on Team A who defeated anyone at all defeated Barbara

$$\forall x([A(x) \wedge \exists y\, D(x,y)] \rightarrow D(x,b))$$

Translations

Not everyone on Team A who defeated Adam also defeated Barbara

$$\neg\forall x([A(x) \wedge D(x,a)] \rightarrow D(x,b))$$

Anyone on Team A who defeated anyone at all defeated Barbara

$$\forall x([A(x) \wedge \exists y\, D(x,y)] \rightarrow D(x,b))$$

Only members of Team A defeated Adam

Translations

Not everyone on Team A who defeated Adam also defeated Barbara

$$\neg\forall x([A(x) \wedge D(x,a)] \rightarrow D(x,b))$$

Anyone on Team A who defeated anyone at all defeated Barbara

$$\forall x([A(x) \wedge \exists y\, D(x,y)] \rightarrow D(x,b))$$

Only members of Team A defeated Adam

$$\forall x(\neg A(x) \rightarrow \neg D(x,a))$$

Translations

Not everyone on Team A who defeated Adam also defeated Barbara

$$\neg\forall x([A(x) \wedge D(x,a)] \rightarrow D(x,b))$$

Anyone on Team A who defeated anyone at all defeated Barbara

$$\forall x([A(x) \wedge \exists y\, D(x,y)] \rightarrow D(x,b))$$

Only members of Team A defeated Adam

$$\forall x(\neg A(x) \rightarrow \neg D(x,a)) \quad \wedge \quad \exists x(A(x) \wedge D(x,a))\ ?$$

Translations

Not everyone on Team A who defeated Adam also defeated Barbara

$$\neg\forall x([A(x) \wedge D(x,a)] \rightarrow D(x,b))$$

Anyone on Team A who defeated anyone at all defeated Barbara

$$\forall x([A(x) \wedge \exists y\, D(x,y)] \rightarrow D(x,b))$$

Only members of Team A defeated Adam

$$\forall x(\neg A(x) \rightarrow \neg D(x,a)) \quad \wedge \quad \exists x(A(x) \wedge D(x,a))\ ?$$

No one on Team A defeated Barbara except those who defeated everyone

Translations

Not everyone on Team A who defeated Adam also defeated Barbara

$$\neg\forall x([A(x) \wedge D(x,a)] \rightarrow D(x,b))$$

Anyone on Team A who defeated anyone at all defeated Barbara

$$\forall x([A(x) \wedge \exists y\, D(x,y)] \rightarrow D(x,b))$$

Only members of Team A defeated Adam

$$\forall x(\neg A(x) \rightarrow \neg D(x,a)) \quad \wedge \quad \exists x(A(x) \wedge D(x,a))\ ?$$

No one on Team A defeated Barbara except those who defeated everyone

$$\forall x[A(x) \wedge D(x,b)] \rightarrow \forall y\, Dxy)$$

Step by Step...

Everyone on Team A who defeated anyone on Team B was defeated by both Barbara and someone on Team C who defeated Adam

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Everyone on Team A who defeated anyone on Team B was defeated by both Barbara and someone on Team C who defeated Adam

$\forall x([A(x) \wedge \exists y(B(y) \wedge D(x,y)] \rightarrow .....$

Step by Step...

Everyone on Team A who defeated anyone on Team B was defeated by both Barbara and someone on Team C who defeated Adam

$\forall x([A(x) \wedge \exists y(B(y) \wedge D(x,y)] \rightarrow .....$

Now say that this person (x) was defeated by both Barbara and someone on Team C who defeated Adam

Step by Step...

Everyone on Team A who defeated anyone on Team B was defeated by both Barbara and someone on Team C who defeated Adam

$\forall x([A(x) \wedge \exists y(B(y) \wedge D(x,y)] \rightarrow .....$

Now say that this person (x) was defeated by both Barbara and someone on Team C who defeated Adam

$D(b,x) \wedge \exists y([C(x) \wedge D(y,a)] \wedge D(y,x))$

Step by Step...

Everyone on Team A who defeated anyone on Team B was defeated by both Barbara and someone on Team C who defeated Adam

$$\forall x([A(x) \wedge \exists y(B(y) \wedge D(x,y)] \rightarrow$$

$$(D(b,x) \wedge \exists y([C(x) \wedge D(y,a)] \wedge D(y,x))))$$