Can we trust the results of Science? J&B Coffee 6:00 pm tonight (with me)

Abstract:

Scientific theories often make claims that don't appear to be directly testable--for example, about the mass of some particular subatomic particle or about what happened one second after the big bang (not to mention that there was actually a big bang in the first place!). The obvious argument for believing these theories is that science just works so well--what else could explain its predictive success? But there are apparently powerful arguments that pull in the opposite direction--we know that there are competing theories that would also work very well and most of these competitors have never even been thought of. And reflection on the history of science should make us worry that we now know that so many apparently successful scientific theories of the past were just wrong. Should we really think we are any better off than they were?

Formal Proofs with Quantifiers II

Wednesday, 9 April

Universal Elimination

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- From any sentence of the form $\forall x\ P(x)$, we can infer $P(c)$ (for any constant c).

Universal Elimination

- From any sentence of the form ∀x P(x), we can infer P(c) (for any constant c).

1. ∀x P(x)	
2. P(c)	∀ Elim: 1

Universal Elimination

- From any sentence of the form ∀x P(x), we can infer P(c) (for any constant c).

1. ∀x P(x)	
2. P(c)	∀ Elim: 1

Note: If there are multiple xs in the sentence, replace them all with c

Universal Introduction

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- For a constant c naming an arbitrary object, if we show in a subproof that P(c), we can conclude that ∀x P(x).

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```
| | 1. [c]
| |----
| | ...
| | j. P(c)
| k. ∀x P(x)        ∀ Intro: 1-j
```

Universal Quantifier Proofs

1. $\forall x(Cube(x) \vee Tet(x))$
2. $\forall x(Cube(x) \leftrightarrow Large(x))$

$\forall x(\neg Large(x) \rightarrow Tet(x))$

Universal Quantifier Proofs

1. ∀x(Cube(x) ∨ Tet(x))
2. ∀x(Cube(x) ↔ Large(x))

∀x(¬Large(x) → Tet(x)) ∀ Intro

Universal Quantifier Proofs

1. ∀x(Cube(x) ∨ Tet(x))
2. ∀x(Cube(x) ↔ Large(x))

3. a

¬Large(a) → Tet(a)

∀x(¬Large(x) → Tet(x))   ∀ Intro

Universal Quantifier Proofs

1. $\forall x(\text{Cube}(x) \vee \text{Tet}(x))$
2. $\forall x(\text{Cube}(x) \leftrightarrow \text{Large}(x))$

  3. $\boxed{a}$

    4. $\neg\text{Large}(a)$

    $\text{Tet}(a)$

  $\neg\text{Large}(a) \rightarrow \text{Tet}(a)$ — → Intro

$\forall x(\neg\text{Large}(x) \rightarrow \text{Tet}(x))$ — ∀ Intro

Universal Quantifier Proofs

1. $\forall x(\text{Cube}(x) \lor \text{Tet}(x))$
2. $\forall x(\text{Cube}(x) \leftrightarrow \text{Large}(x))$

3. [a]

4. $\neg\text{Large}(a)$

5. $\text{Cube}(a) \leftrightarrow \text{Large}(a)$ ∀ Elim 2

$\text{Tet}(a)$

$\neg\text{Large}(a) \rightarrow \text{Tet}(a)$ → Intro

$\forall x(\neg\text{Large}(x) \rightarrow \text{Tet}(x))$ ∀ Intro

Universal Quantifier Proofs

1. $\forall x(\text{Cube}(x) \vee \text{Tet}(x))$
2. $\forall x(\text{Cube}(x) \leftrightarrow \text{Large}(x))$

3. a

4. $\neg\text{Large}(a)$

5. $\text{Cube}(a) \leftrightarrow \text{Large}(a)$ — ∀ Elim 2
6. $\neg\text{Cube}(a)$ — Taut Con 4,5

$\text{Tet}(a)$

$\neg\text{Large}(a) \rightarrow \text{Tet}(a)$ — → Intro

$\forall x(\neg\text{Large}(x) \rightarrow \text{Tet}(x))$ — ∀ Intro

Universal Quantifier Proofs

1. $\forall x(Cube(x) \vee Tet(x))$
2. $\forall x(Cube(x) \leftrightarrow Large(x))$

---

3. [a]

---

4. $\neg Large(a)$

---

5. $Cube(a) \leftrightarrow Large(a)$ — ∀ Elim 2
6. $\neg Cube(a)$ — Taut Con 4,5
7. $Cube(a) \vee Tet(a)$ — ∀ Elim 1

$Tet(a)$

$\neg Large(a) \rightarrow Tet(a)$ — → Intro

$\forall x(\neg Large(x) \rightarrow Tet(x))$ — ∀ Intro

Universal Quantifier Proofs

1.	$\forall x(Cube(x) \lor Tet(x))$	
2.	$\forall x(Cube(x) \leftrightarrow Large(x))$	
3.	[a]	
4.	$\neg Large(a)$	
5.	$Cube(a) \leftrightarrow Large(a)$	∀ Elim 2
6.	$\neg Cube(a)$	Taut Con 4,5
7.	$Cube(a) \lor Tet(a)$	∀ Elim 1
8.	$Tet(a)$	Taut Con 6,7
	$\neg Large(a) \rightarrow Tet(a)$	→ Intro
	$\forall x(\neg Large(x) \rightarrow Tet(x))$	∀ Intro

UNIVERSAL QUANTIFIER PROOFS

1. ∀x(Cube(x) ∨ Tet(x))	
2. ∀x(Cube(x) ↔ Large(x))	
3. [a]	
4. ¬Large(a)	
5. Cube(a) ↔ Large(a)	∀ Elim 2
6. ¬Cube(a)	Taut Con 4,5
7. Cube(a) ∨ Tet(a)	∀ Elim 1
8. Tet(a)	Taut Con 6,7
9. ¬Large(a) → Tet(a)	→ Intro 4-8
∀x(¬Large(x) → Tet(x))	∀ Intro

Universal Quantifier Proofs

1.	$\forall x(\text{Cube}(x) \vee \text{Tet}(x))$	
2.	$\forall x(\text{Cube}(x) \leftrightarrow \text{Large}(x))$	
3.	[a]	
4.	$\neg\text{Large}(a)$	
5.	$\text{Cube}(a) \leftrightarrow \text{Large}(a)$	∀ Elim 2
6.	$\neg\text{Cube}(a)$	Taut Con 4,5
7.	$\text{Cube}(a) \vee \text{Tet}(a)$	∀ Elim 1
8.	$\text{Tet}(a)$	Taut Con 6,7
9.	$\neg\text{Large}(a) \rightarrow \text{Tet}(a)$	→ Intro 4-8
10.	$\forall x(\neg\text{Large}(x) \rightarrow \text{Tet}(x))$	∀ Intro 3-9

Universal Quantifier Proofs

$\forall x(\text{Cube}(x) \vee \text{Tet}(x))$

$\forall x(\text{Cube}(x) \leftrightarrow \text{Large}(x))$

$\forall x(\neg\text{Large}(x) \rightarrow \text{Tet}(x))$

is a valid argument

Universal Quantifier Proofs

$\forall x(Cube(x) \lor Tet(x))$

$\forall x(Cube(x) \leftrightarrow Large(x))$

$\forall x(\neg Large(x) \rightarrow Tet(x))$

is a valid argument

but notice that this is valid too

$Cube(a) \lor Tet(a)$

$Cube(a) \leftrightarrow Large(a)$

$\neg Large(a) \rightarrow Tet(a)$

Universal Quantifier Proofs

$\forall x(\text{Cube}(x) \vee \text{Tet}(x))$

$\forall x(\text{Cube}(x) \leftrightarrow \text{Large}(x))$

$\forall x(\neg\text{Large}(x) \rightarrow \text{Tet}(x))$

is a valid argument

but notice that this is valid too

$\text{Cube}(a) \vee \text{Tet}(a)$

$\text{Cube}(a) \leftrightarrow \text{Large}(a)$

$\neg\text{Large}(a) \rightarrow \text{Tet}(a)$

and obviously, the 'a' could be anything

From Exam 2

$$\begin{array}{|l} (P \to S) \to T \\ \hline \neg T \to P \end{array}$$

is a valid argument

From Exam 2

| (P → S) → T
|---
| ¬T → P

is a valid argument

| (P → S) → T	
|---	
| | ¬T	
| |---	
| | ¬(P → S)	Taut Con
| | P ∧ ¬ S	Taut Con
| | P	∧ Elim
| ¬T → P	→ Intro

From Exam 2

$(P \rightarrow S) \rightarrow T$

$\neg T \rightarrow P$

is a valid argument

True of *any* P, S, T

$(P(a) \rightarrow S(a)) \rightarrow T(a)$	
$\quad \neg T(a)$	
$\quad \neg(P(a) \rightarrow S(a))$	Taut Con
$\quad P(a) \land \neg S(a)$	Taut Con
$\quad P(a)$	$\land$ Elim
$\neg T(a) \rightarrow P(a)$	$\rightarrow$ Intro

From Exam 2

$\forall x[(P(x) \rightarrow S(x)) \rightarrow T(x)]$

[a]	
$(P(a) \rightarrow S(a)) \rightarrow T(a)$	$\forall$ Elim
$\neg T(a)$	
$\neg(P(a) \rightarrow S(a))$	Taut Con
$P(a) \wedge \neg S(a)$	Taut Con
$P(a)$	$\wedge$ Elim
$\neg T(a) \rightarrow P(a)$	$\rightarrow$ Intro
$\forall x[\neg T(x) \rightarrow P(x)]$	$\forall$ Intro

True of *any* constant *a*

1. $\forall x(\text{SameRow}(x,a) \lor \text{SameRow}(x,b))$
2. $\forall x(\text{Cube}(x) \leftrightarrow \text{Large}(x))$
3. $\forall x(\text{SameSize}(x,c) \rightarrow \neg\text{SameRow}(x,a))$

$\forall x((\text{Cube}(x) \land \text{SameSize}(x,c)) \rightarrow (\text{SameRow}(x,b) \land \text{Large}(x)))$

1. $\forall x(\text{SameRow}(x,a) \lor \text{SameRow}(x,b))$
2. $\forall x(\text{Cube}(x) \leftrightarrow \text{Large}(x))$
3. $\forall x(\text{SameSize}(x,c) \rightarrow \neg\text{SameRow}(x,a))$

4. $\boxed{f}$

$(\text{Cube}(f) \land \text{SameSize}(f,c)) \rightarrow (\text{SameRow}(f,b) \land \text{Large}(f))$

$\forall x((\text{Cube}(x) \land \text{SameSize}(x,c)) \rightarrow (\text{SameRow}(x,b) \land \text{Large}(x)))$

1. $\forall x(\text{SameRow}(x,a) \lor \text{SameRow}(x,b))$
2. $\forall x(\text{Cube}(x) \leftrightarrow \text{Large}(x))$
3. $\forall x(\text{SameSize}(x,c) \rightarrow \neg\text{SameRow}(x,a))$

4. $\boxed{f}$

5. $\text{SameRow}(f,a) \lor \text{SameRow}(f,b)$ $\quad$ $\forall$ Elim 1

$(\text{Cube}(f) \land \text{SameSize}(f,c)) \rightarrow (\text{SameRow}(f,b) \land \text{Large}(f))$

$\forall x((\text{Cube}(x) \land \text{SameSize}(x,c)) \rightarrow (\text{SameRow}(x,b) \land \text{Large}(x)))$

1. $\forall x(\text{SameRow}(x,a) \vee \text{SameRow}(x,b))$

2. $\forall x(\text{Cube}(x) \leftrightarrow \text{Large}(x))$

3. $\forall x(\text{SameSize}(x,c) \rightarrow \neg\text{SameRow}(x,a))$

4. f

5. $\text{SameRow}(f,a) \vee \text{SameRow}(f,b)$ ∀ Elim 1

6. $\text{Cube}(f) \leftrightarrow \text{Large}(f)$ ∀ Elim 2

$(\text{Cube}(f) \wedge \text{SameSize}(f,c)) \rightarrow (\text{SameRow}(f,b) \wedge \text{Large}(f))$

$\forall x((\text{Cube}(x) \wedge \text{SameSize}(x,c)) \rightarrow (\text{SameRow}(x,b) \wedge \text{Large}(x)))$

1. $\forall x(\text{SameRow}(x,a) \lor \text{SameRow}(x,b))$

2. $\forall x(\text{Cube}(x) \leftrightarrow \text{Large}(x))$

3. $\forall x(\text{SameSize}(x,c) \to \neg\text{SameRow}(x,a))$

4. $\boxed{f}$

5. $\text{SameRow}(f,a) \lor \text{SameRow}(f,b)$ ∀ Elim 1

6. $\text{Cube}(f) \leftrightarrow \text{Large}(f)$ ∀ Elim 2

7. $\text{SameSize}(f,c) \to \neg\text{SameRow}(f,a)$ ∀ Elim 3

$(\text{Cube}(f) \land \text{SameSize}(f,c)) \to (\text{SameRow}(f,b) \land \text{Large}(f))$

$\forall x((\text{Cube}(x) \land \text{SameSize}(x,c)) \to (\text{SameRow}(x,b) \land \text{Large}(x)))$

1. $\forall x(\text{SameRow}(x,a) \lor \text{SameRow}(x,b))$

2. $\forall x(\text{Cube}(x) \leftrightarrow \text{Large}(x))$

3. $\forall x(\text{SameSize}(x,c) \rightarrow \neg\text{SameRow}(x,a))$

4. f

5. $\text{SameRow}(f,a) \lor \text{SameRow}(f,b)$ — $\forall$ Elim 1

6. $\text{Cube}(f) \leftrightarrow \text{Large}(f)$ — $\forall$ Elim 2

7. $\text{SameSize}(f,c) \rightarrow \neg\text{SameRow}(f,a)$ — $\forall$ Elim 3

8. $(\text{Cube}(f) \land \text{SameSize}(f,c)) \rightarrow (\text{SameRow}(f,b) \land \text{Large}(f))$

Taut Con 5,6,7

$\forall x((\text{Cube}(x) \land \text{SameSize}(x,c)) \rightarrow (\text{SameRow}(x,b) \land \text{Large}(x)))$

1. $\forall x(\text{SameRow}(x,a) \lor \text{SameRow}(x,b))$

2. $\forall x(\text{Cube}(x) \leftrightarrow \text{Large}(x))$

3. $\forall x(\text{SameSize}(x,c) \rightarrow \neg\text{SameRow}(x,a))$

4. [f]

5. $\text{SameRow}(f,a) \lor \text{SameRow}(f,b)$ ∀ Elim 1

6. $\text{Cube}(f) \leftrightarrow \text{Large}(f)$ ∀ Elim 2

7. $\text{SameSize}(f,c) \rightarrow \neg\text{SameRow}(f,a)$ ∀ Elim 3

8. $(\text{Cube}(f) \land \text{SameSize}(f,c)) \rightarrow (\text{SameRow}(f,b) \land \text{Large}(f))$
Taut Con 5,6,7

9. $\forall x((\text{Cube}(x) \land \text{SameSize}(x,c)) \rightarrow (\text{SameRow}(x,b) \land \text{Large}(x)))$
∀ Intro 4-8