PUZZLE

It is clearly impossible for a human being to count to 10^20 in their lifetime, but what is wrong with the following argument?

1. It is possible for a human to count to 100.
2. If it is possible to count to x, then it is possible to count to x+1.

Therefore 3. It is possible to count to 10^20.

Note that $\forall x(\mathrm{CanCount}(x) \rightarrow \mathrm{CanCount}(x+1)) \Leftrightarrow$
$\neg\exists x(\mathrm{CanCount}(x) \wedge \neg\mathrm{CanCount}(x+1))$

More Quantifier Translations

Monday, 24 March

Aristotelian Forms

Forms:

QL sentence:

Aristotelian Forms

Forms:	QL sentence:
All Ps are Qs.	$\forall x(P(x) \rightarrow Q(x))$

Aristotelian Forms

Forms:	QL sentence:
All Ps are Qs.	$\forall x(P(x) \rightarrow Q(x))$
All cubes are large	$\forall x(Cube(x) \rightarrow Large(x))$

Aristotelian Forms

Forms:	QL sentence:
All Ps are Qs.	$\forall x(P(x) \rightarrow Q(x))$
All cubes are large	$\forall x(Cube(x) \rightarrow Large(x))$
Some Ps are Qs.	$\exists x(P(x) \wedge Q(x))$

Aristotelian Forms

Forms:	QL sentence:
• All Ps are Qs.	$\forall x(P(x)\rightarrow Q(x))$
• All cubes are large	$\forall x(Cube(x)\rightarrow Large(x))$
• Some Ps are Qs.	$\exists x(P(x)\wedge Q(x))$
• Some dodecs are small	$\exists x(Dodec(x)\wedge Small(x))$

Aristotelian Forms

Forms:

QL sentence:

Aristotelian Forms

Forms:	QL sentence:
• No Ps are Qs.	$\forall x(P(x) \rightarrow \neg Q(x))$

Aristotelian Forms

Forms:	QL sentence:
No Ps are Qs.	$\forall x(P(x) \rightarrow \neg Q(x))$
No cubes are small	$\forall x(Cube(x) \rightarrow \neg Small(x))$

Aristotelian Forms

Forms:	QL sentence:
No Ps are Qs.	$\forall x(P(x) \rightarrow \neg Q(x))$
No cubes are small	$\forall x(Cube(x) \rightarrow \neg Small(x))$
Some Ps are not Qs.	$\exists x(P(x) \wedge \neg Q(x))$

Aristotelian Forms

Forms:	QL sentence:
No Ps are Qs.	$\forall x(P(x) \rightarrow \neg Q(x))$
No cubes are small	$\forall x(Cube(x) \rightarrow \neg Small(x))$
Some Ps are not Qs.	$\exists x(P(x) \wedge \neg Q(x))$
Some large things are not dodecs	$\exists x(Large(x) \wedge \neg Dodec(x))$

Two Place Predicates

Everything to the right of *a* is a cube

Two Place Predicates

Everything to the right of *a* is a cube

$$\forall x(RightOf(x,a) \rightarrow Cube(x))$$

Two Place Predicates

Everything to the right of *a* is a cube

$\forall x(\text{RightOf}(x,a) \rightarrow \text{Cube}(x))$

There is a dodec in the same row as *b*

Two Place Predicates

Everything to the right of *a* is a cube

$$\forall x(\mathrm{RightOf}(x,a) \rightarrow \mathrm{Cube}(x))$$

There is a dodec in the same row as *b*

$$\exists x(\mathrm{Dodec}(x) \land \mathrm{SameRow}(x,b))$$

Two Place Predicates

Everything to the right of *a* is a cube

$\forall x(\text{RightOf}(x,a) \rightarrow \text{Cube}(x))$

There is a dodec in the same row as *b*

$\exists x(\text{Dodec}(x) \wedge \text{SameRow}(x,b))$

a and *b* have a cube between them

Two Place Predicates

Everything to the right of *a* is a cube

$\forall x(\text{RightOf}(x,a) \rightarrow \text{Cube}(x))$

There is a dodec in the same row as *b*

$\exists x(\text{Dodec}(x) \wedge \text{SameRow}(x,b))$

a and *b* have a cube between them

$\exists x(\text{Cube}(x) \wedge \text{Between}(x,a,b))$

Two Place Predicates

There aren't any dodecs in the same row as *a*

Two Place Predicates

There aren't any dodecs in the same row as *a*

$$\neg\exists x(\mathrm{Dodec}(x) \wedge \mathrm{SameRow}(x,a))$$

Two Place Predicates

There aren't any dodecs in the same row as *a*

$$\neg\exists x(\text{Dodec}(x) \land \text{SameRow}(x,a))$$

$$\forall x(\text{Dodec}(x) \rightarrow \neg\, \text{SameRow}(x,a))$$

Two Place Predicates

There aren't any dodecs in the same row as *a*

$$\neg\exists x(\mathrm{Dodec}(x) \wedge \mathrm{SameRow}(x,a))$$

$$\forall x(\mathrm{Dodec}(x) \rightarrow \neg\,\mathrm{SameRow}(x,a))$$

$$\forall x(\mathrm{SameRow}(x,a) \rightarrow \neg\,\mathrm{Dodec}(x))$$

Two Place Predicates

There aren't any dodecs in the same row as *a*

$$\neg\exists x(\text{Dodec}(x) \land \text{SameRow}(x,a))$$

$$\forall x(\text{Dodec}(x) \rightarrow \neg\,\text{SameRow}(x,a))$$

$$\forall x(\text{SameRow}(x,a) \rightarrow \neg\,\text{Dodec}(x))$$

Not every dodec is in the same row as *a*

Two Place Predicates

There aren't any dodecs in the same row as *a*

$$\neg\exists x(\mathrm{Dodec}(x) \wedge \mathrm{SameRow}(x,a))$$

$$\forall x(\mathrm{Dodec}(x) \rightarrow \neg\, \mathrm{SameRow}(x,a))$$

$$\forall x(\mathrm{SameRow}(x,a) \rightarrow \neg\, \mathrm{Dodec}(x))$$

Not every dodec is in the same row as *a*

$$\neg\, \forall x(\mathrm{Dodec}(x) \rightarrow \mathrm{SameRow}(x,a))$$

Complex Predicates

Some Ps are Qs $\exists x(P(x) \wedge Q(x))$

Complex Predicates

Some Ps are Qs

$\exists x(P(x) \wedge Q(x))$

Some Ps that are also Rs are Qs

$\exists x([P(x) \wedge R(x)] \wedge Q(x))$

Complex Predicates

Some Ps are Qs $\exists x(P(x) \wedge Q(x))$

Some Ps that are also Rs are Qs $\exists x([P(x) \wedge R(x)] \wedge Q(x))$

Some Ps are Rs and Qs $\exists x(P(x) \wedge [R(x) \wedge Q(x)])$

Complex Predicates

Some Ps are Qs

$\exists x(P(x) \wedge Q(x))$

Some Ps that are also Rs are Qs

$\exists x([P(x) \wedge R(x)] \wedge Q(x))$

Some Ps are Rs and Qs

$\exists x(P(x) \wedge [R(x) \wedge Q(x)])$

These are obviously equivalent

Complex Predicates

Some Ps are Qs	$\exists x(P(x) \wedge Q(x))$
Some Ps that are also Rs are Qs	$\exists x([P(x) \wedge R(x)] \wedge Q(x))$
Some Ps are Rs and Qs	$\exists x(P(x) \wedge [R(x) \wedge Q(x)])$

Complex Predicates

Some Ps are Qs

$\exists x(P(x) \wedge Q(x))$

Some Ps that are also Rs are Qs

$\exists x([P(x) \wedge R(x)] \wedge Q(x))$

Some Ps are Rs and Qs

$\exists x(P(x) \wedge [R(x) \wedge Q(x)])$

Some small cubes are to the right of *a*

$\exists x(\mathrm{Small}(x) \wedge \mathrm{Cubes}(x) \wedge \mathrm{RightOf}(x,a))$

Complex Predicates

There is a large cube to the left of *b*

Complex Predicates

There is a large cube to the left of *b*

$$\exists x(Large(x) \wedge Cube(x) \wedge LeftOf(x,b))$$

Complex Predicates

There is a large cube to the left of *b*

$$\exists x(\text{Large}(x) \land \text{Cube}(x) \land \text{LeftOf}(x,b))$$

There is a cube to the left of *b*
which is in the same row as *c*

Complex Predicates

There is a large cube to the left of *b*

$$\exists x(\mathrm{Large}(x) \wedge \mathrm{Cube}(x) \wedge \mathrm{LeftOf}(x,b))$$

There is a cube to the left of *b*
which is in the same row as *c*

$$\exists x(\mathrm{Cube}(x) \wedge \mathrm{LeftOf}(x,b) \wedge \mathrm{SameRow}(x,c))$$

Complex Predicates

Complex Predicates

b is in the same row as a large cube

Complex Predicates

b is in the same row as a large cube

$$\exists x(\mathrm{Large}(x) \wedge \mathrm{Cube}(x) \wedge \mathrm{SameRow}(b,x))$$

Complex Predicates

b is in the same row as a large cube

$$\exists x(\mathrm{Large}(x) \wedge \mathrm{Cube}(x) \wedge \mathrm{SameRow}(b,x))$$

There is a small cube that isn't in the same row as *a* or *b*

Complex Predicates

b is in the same row as a large cube

$$\exists x(Large(x) \wedge Cube(x) \wedge SameRow(b,x))$$

There is a small cube that isn't in the same row as *a* or *b*

$$\exists x(Small(x) \wedge Cube(x) \wedge \neg SameRow(x,a) \wedge \neg SameRow(x,b))$$

Complex Predicates

All Ps are Qs $\forall x(P(x) \rightarrow Q(x))$

Complex Predicates

All Ps are Qs

$\forall x(P(x) \rightarrow Q(x))$

All Ps that are also Rs are Qs

$\forall x([P(x) \wedge R(x)] \rightarrow Q(x))$

Complex Predicates

All Ps are Qs

$\forall x(P(x) \rightarrow Q(x))$

All Ps that are also Rs are Qs

$\forall x([P(x) \wedge R(x)] \rightarrow Q(x))$

All Ps are Rs and Qs

$\forall x(P(x) \rightarrow (R(x) \wedge Q(x)))$

Complex Predicates

All Ps are Qs

$$\forall x(P(x) \rightarrow Q(x))$$

All Ps that are also Rs are Qs

$$\forall x([P(x) \wedge R(x)] \rightarrow Q(x))$$

All Ps are Rs and Qs

$$\forall x(P(x) \rightarrow (R(x) \wedge Q(x)))$$

These are NOT equivalent

Complex Predicates

Every cube is either large or small

Complex Predicates

Every cube is either large or small

$$\forall x(\text{Cube}(x) \rightarrow (\text{Large}(x) \vee \text{Small}(x))$$

Complex Predicates

Every cube is either large or small

$$\forall x(\mathrm{Cube}(x) \rightarrow (\mathrm{Large}(x) \vee \mathrm{Small}(x))$$

All small cubes are to the right of *a*

Complex Predicates

Every cube is either large or small

$$\forall x(\text{Cube}(x) \rightarrow (\text{Large}(x) \vee \text{Small}(x))$$

All small cubes are to the right of *a*

$$\forall x([\text{Small}(x) \wedge \text{Cubes}(x)] \rightarrow \text{RightOf}(x,a))$$

Complex Predicates

Every cube is either large or small

$$\forall x(\mathrm{Cube}(x) \rightarrow (\mathrm{Large}(x) \vee \mathrm{Small}(x))$$

All small cubes are to the right of *a*

$$\forall x([\mathrm{Small}(x) \wedge \mathrm{Cubes}(x)] \rightarrow \mathrm{RightOf}(x,a))$$

Every cube in the same row as *a* is small

Complex Predicates

Every cube is either large or small

$$\forall x(\text{Cube}(x) \rightarrow (\text{Large}(x) \vee \text{Small}(x))$$

All small cubes are to the right of *a*

$$\forall x([\text{Small}(x) \wedge \text{Cubes}(x)] \rightarrow \text{RightOf}(x,a))$$

Every cube in the same row as *a* is small

$$\forall x([\text{Cube}(x) \wedge \text{SameRow}(x,a)] \rightarrow \text{Small}(x))$$

Complex Predicates

The only things in back of *a* are large cubes

Complex Predicates

The only things in back of *a* are large cubes

$$\forall x(\mathrm{BackOf}(x) \rightarrow (\mathrm{Large}(x) \land \mathrm{Cube}(x))$$

Complex Predicates

The only things in back of *a* are large cubes

$$\forall x(\text{BackOf}(x) \rightarrow (\text{Large}(x) \land \text{Cube}(x))$$

No small cubes are in the same row as *a*

Complex Predicates

The only things in back of *a* are large cubes

$$\forall x(\text{BackOf}(x) \rightarrow (\text{Large}(x) \wedge \text{Cube}(x))$$

No small cubes are in the same row as *a*

$$\forall x([\text{Small}(x) \wedge \text{Cubes}(x)] \rightarrow \neg \text{SameRow}(x,a))$$

Complex Predicates

The only things in back of *a* are large cubes

$$\forall x(\mathrm{BackOf}(x) \rightarrow (\mathrm{Large}(x) \wedge \mathrm{Cube}(x))$$

No small cubes are in the same row as *a*

$$\forall x([\mathrm{Small}(x) \wedge \mathrm{Cubes}(x)] \rightarrow \neg \mathrm{SameRow}(x,a))$$

Nothing in the same row as *a* is a small cube

Complex Predicates

The only things in back of *a* are large cubes

$$\forall x(\text{BackOf}(x) \rightarrow (\text{Large}(x) \wedge \text{Cube}(x))$$

No small cubes are in the same row as *a*

$$\forall x([\text{Small}(x) \wedge \text{Cubes}(x)] \rightarrow \neg\text{SameRow}(x,a))$$

Nothing in the same row as *a* is a small cube

$$\forall x(\text{SameRow}(x,a) \rightarrow \neg(\text{Small}(x) \wedge \text{Cube}(x))$$