Surprised?

At the beginning of the class, the professor told her students "I will do something that you don't expect today and you will be surprised." The students waited until the end of class and nothing surprising seemed to happen. At the end of class, Bob said, "Hey, you said you would do something we didn't expect. But I wasn't surprised at all." The professor said "You expected that you would be surprised in class today right?" Bob: "Yes" Prof: "But you weren't. Therefore something happened that you didn't expect. So I surprised you after all."

QUANTIFIERS

Friday, 14 March

Limits of Truth-Functions

All men are mortal
Socrates is a man
———
Socrates is mortal

All men are tall
Not every man is bald
———
Some tall people aren't bald

No apples are rotten
Some fruits are rotten
———
Some fruits aren't apples

For any number, there is a larger prime number
———
There is no largest prime number

None are truth-functionally valid

- We need a stronger logical system

QUANTIFIERS

Quantifiers

Two <u>quantifier</u> symbols:

QUANTIFIERS

Two quantifier symbols:

- $\forall$ means "everything" or "for all".

Quantifiers

Two quantifier symbols:

- $\forall$ means "everything" or "for all".
- $\exists$ means "something" or "there exists at least one".

Quantifiers

Two quantifier symbols:

- ∀ means "everything" or "for all".
- ∃ means "something" or "there exists at least one".
- Just these two quantifiers can be used to capture many of the quantifications we want to talk about. For example, all, every, any, none, not all of, some, some are not, at least one, at least two, exactly two, etc.

Sentences in FOL

Cube(a)

True in a world if *a* is a cube in that world

Sentences in FOL

Cube(a)

True in a world if *a* is a cube in that world

$\forall x Cube(x)$

True in a world if every object in that world is a cube

Sentences in FOL

$Cube(a)$

True in a world if *a* is a cube in that world

$\forall x Cube(x)$

True in a world if every object in that world is a cube

For every object x, x is a cube

SENTENCES IN FOL

Cube(a)

True in a world if *a* is a cube in that world

Sentences in FOL

Cube(a)

True in a world if *a* is a cube in that world

∃xCube(x)

True in a world if at least one object in that world is a cube

SENTENCES IN FOL

Cube(a)

True in a world if *a* is a cube in that world

∃xCube(x)

True in a world if at least one object in that world is a cube

For some object x, x is a cube

Sentences in FOL

Cube(a)

True in a world if *a* is a cube in that world

∃xCube(x)

True in a world if at least one object in that world is a cube

For some object x, x is a cube

Cube(x) - Not true or false - not even a sentence

The Existential Quantifier

The Existential Quantifier

- Something is a cube (There is at least one cube)

The Existential Quantifier

- Something is a cube (There is at least one cube)
  - $\exists x\, Cube(x)$

The Existential Quantifier

- Something is a cube (There is at least one cube)
  - $\exists x$ Cube(x)
- Something is a small cube

The Existential Quantifier

- Something is a cube (There is at least one cube)
  - $\exists x\, Cube(x)$
- Something is a small cube
  - $\exists x(Cube(x) \wedge Small(x))$

The Existential Quantifier

- Something is a cube (There is at least one cube)
  - $\exists x \, Cube(x)$
- Something is a small cube
  - $\exists x (Cube(x) \wedge Small(x))$
- Some cubes are small

The Existential Quantifier

- Something is a cube (There is at least one cube)
  - $\exists x\, Cube(x)$
- Something is a small cube
  - $\exists x(Cube(x) \wedge Small(x))$
- Some cubes are small
  - $\exists x(Cube(x) \wedge Small(x))$

The Existential Quantifier

- Something is a cube (There is at least one cube)
  - $\exists x\, Cube(x)$
- Something is a small cube
  - $\exists x(Cube(x) \land Small(x))$
- Some cubes are small
  - $\exists x(Cube(x) \land Small(x))$
- Some small things are cubes

THE EXISTENTIAL QUANTIFIER

- Something is a cube (There is at least one cube)
  - $\exists x\, Cube(x)$
- Something is a small cube
  - $\exists x(Cube(x) \wedge Small(x))$
- Some cubes are small
  - $\exists x(Cube(x) \wedge Small(x))$
- Some small things are cubes
  - $\exists x(Small(x) \wedge Cube(x))$ (Obviously equivalent)

The Existential Quantifier

The Existential Quantifier

- Something is a cube (There is at least one cube)

The Existential Quantifier

- Something is a cube (There is at least one cube)
  - $\exists x$ Cube(x)

The Existential Quantifier

- Something is a cube (There is at least one cube)
  - $\exists x$ Cube(x)
- Nothing is a cube (There are no cubes)

The Existential Quantifier

- Something is a cube (There is at least one cube)
  - $\exists x\ Cube(x)$
- Nothing is a cube (There are no cubes)
  - $\neg \exists x\ Cube(x)$

The Existential Quantifier

- Something is a cube (There is at least one cube)
  - ∃x Cube(x)
- Nothing is a cube (There are no cubes)
  - ¬∃x Cube(x)
- Some cubes are not small

The Existential Quantifier

- Something is a cube (There is at least one cube)
  - $\exists x\, Cube(x)$
- Nothing is a cube (There are no cubes)
  - $\neg\exists x\, Cube(x)$
- Some cubes are not small
  - $\exists x(Cube(x) \wedge \neg Small(x))$

The Existential Quantifier

- Something is a cube (There is at least one cube)
  - $\exists x\, Cube(x)$
- Nothing is a cube (There are no cubes)
  - $\neg\exists x\, Cube(x)$
- Some cubes are not small
  - $\exists x(Cube(x) \wedge \neg Small(x))$
- Some small things are not cubes

The Existential Quantifier

- Something is a cube (There is at least one cube)
  - $\exists x\, Cube(x)$
- Nothing is a cube (There are no cubes)
  - $\neg\exists x\, Cube(x)$
- Some cubes are not small
  - $\exists x(Cube(x) \wedge \neg Small(x))$
- Some small things are not cubes
  - $\exists x(Small(x) \wedge \neg Cube(x))$ (not equivalent)

The Universal Quantifier

The Universal Quantifier

- Everything is a cube

The Universal Quantifier

- Everything is a cube
  - $\forall x\ Cube(x)$

The Universal Quantifier

- Everything is a cube
  - $\forall x\ Cube(x)$
- Everything is a small cube

The Universal Quantifier

- Everything is a cube
  - $\forall x\ Cube(x)$
- Everything is a small cube
  - $\forall x (Cube(x) \land Small(x))$

The Universal Quantifier

- Everything is a cube
  - $\forall x\ Cube(x)$
- Everything is a small cube
  - $\forall x(Cube(x) \land Small(x))$
- Everything is either small or is cube

The Universal Quantifier

- Everything is a cube
  - $\forall x\ Cube(x)$
- Everything is a small cube
  - $\forall x(Cube(x) \land Small(x))$
- Everything is either small or is cube
  - $\forall x(Cube(x) \lor Small(x))$

The Universal Quantifier

- Everything is a cube
  - $\forall x\ Cube(x)$
- Everything is a small cube
  - $\forall x(Cube(x) \land Small(x))$
- Everything is either small or is cube
  - $\forall x(Cube(x) \lor Small(x))$
- Every small thing is a cube

The Universal Quantifier

- Everything is a cube
  - $\forall x\ Cube(x)$
- Everything is a small cube
  - $\forall x(Cube(x) \land Small(x))$
- Everything is either small or is cube
  - $\forall x(Cube(x) \lor Small(x))$
- Every small thing is a cube
  - $\forall x(Small(x) \rightarrow Cube(x))$

The Universal Quantifier

The Universal Quantifier

- Every cube is small (All the cubes are small)

The Universal Quantifier

- Every cube is small (All the cubes are small)
  - $\forall x(Cube(x) \rightarrow Small(x))$

The Universal Quantifier

- Every cube is small (All the cubes are small)
  - $\forall x(Cube(x) \rightarrow Small(x))$
- Not every cube is small (Not all the cubes are small)

The Universal Quantifier

- Every cube is small (All the cubes are small)
  - $\forall x(Cube(x) \rightarrow Small(x))$
- Not every cube is small (Not all the cubes are small)
  - $\neg\forall x(Cube(x) \rightarrow Small(x))$

The Universal Quantifier

- Every cube is small (All the cubes are small)
  - $\forall x(Cube(x) \rightarrow Small(x))$
- Not every cube is small (Not all the cubes are small)
  - $\neg\forall x(Cube(x) \rightarrow Small(x))$
- Every cube is not small (None of the cubes are small)

The Universal Quantifier

- Every cube is small (All the cubes are small)
    - $\forall x(Cube(x) \rightarrow Small(x))$
- Not every cube is small (Not all the cubes are small)
    - $\neg\forall x(Cube(x) \rightarrow Small(x))$
- Every cube is not small (None of the cubes are small)
    - $\forall x(Cube(x) \rightarrow \neg Small(x))$

The Universal Quantifier

- Every cube is small (All the cubes are small)
  - $\forall x(Cube(x) \rightarrow Small(x))$
- Not every cube is small (Not all the cubes are small)
  - $\neg\forall x(Cube(x) \rightarrow Small(x))$
- Every cube is not small (None of the cubes are small)
  - $\forall x(Cube(x) \rightarrow \neg Small(x))$
- $\forall x(\neg Cube(x) \rightarrow Small(x))$??

The Universal Quantifier

- Every cube is small (All the cubes are small)
  - $\forall x(Cube(x) \rightarrow Small(x))$
- Not every cube is small (Not all the cubes are small)
  - $\neg\forall x(Cube(x) \rightarrow Small(x))$
- Every cube is not small (None of the cubes are small)
  - $\forall x(Cube(x) \rightarrow \neg Small(x))$
- $\forall x(\neg Cube(x) \rightarrow Small(x))$??
  - All of the non-cubes are small

Aristotelian Forms

Forms:

Examples:

Aristotelian Forms

Forms:

- All Ps are Qs.

Examples:

All mammals are animals.

Aristotelian Forms

Forms:	Examples:
All Ps are Qs.	All mammals are animals.
Some Ps are Qs.	Some mammals live in water.

Aristotelian Forms

Forms:	Examples:
All Ps are Qs.	All mammals are animals.
Some Ps are Qs.	Some mammals live in water.
No Ps are Qs.	No humans have wings.

Aristotelian Forms

Forms:	Examples:
All Ps are Qs.	All mammals are animals.
Some Ps are Qs.	Some mammals live in water.
No Ps are Qs.	No humans have wings.
Some Ps are not Qs.	Some birds cannot fly.

Aristotelian Forms

All Ps are Qs

All mammals are animals

Aristotelian Forms

All Ps are Qs

All mammals are animals

For any x, if x is a P,
then x is a Q

Aristotelian Forms

All Ps are Qs

All mammals are animals

For any x, if x is a P,
then x is a Q

For any x, P(x) $\rightarrow$ Q(x)

Aristotelian Forms

All Ps are Qs

All mammals are animals

For any x, if x is a P,
then x is a Q

For any x, P(x) → Q(x)

$\forall x(P(x) \rightarrow Q(x))$

Aristotelian Forms

All Ps are Qs

All mammals are animals

For any x, if x is a P, then x is a Q

For any x, P(x) $\rightarrow$ Q(x)

$\forall x(P(x) \rightarrow Q(x))$

$\forall x(Mammal(x) \rightarrow Animal(x))$

Aristotelian Forms

Some Ps are Qs

Some mammals live in water

Aristotelian Forms

Some Ps are Qs

Some mammals live in water

There is at least one P that is also a Q

Aristotelian Forms

Some Ps are Qs

Some mammals live in water

There is at least one P that is also a Q

There is at least one thing x
such that x is both P and Q

Aristotelian Forms

Some Ps are Qs

Some mammals live in water

There is at least one P that is also a Q

There is at least one thing x such that x is both P and Q

There is at least one thing x such that $P(x) \wedge Q(x)$

ARISTOTELIAN FORMS

Some Ps are Qs

Some mammals live in water

There is at least one P that is also a Q

There is at least one thing x such that x is both P and Q

There is at least one thing x such that $P(x) \wedge Q(x)$

$$\exists x(P(x) \wedge Q(x))$$

Aristotelian Forms

Some Ps are Qs

Some mammals live in water

There is at least one P that is also a Q

There is at least one thing x
such that x is both P and Q

There is at least one thing x
such that $P(x) \wedge Q(x)$

$\exists x(P(x) \wedge Q(x))$

$\exists x(Mammal(x) \wedge LiWa(x))$

Aristotelian Forms

No Ps are Qs

No humans have wings

Aristotelian Forms

No Ps are Qs

No humans have wings

For any x, if x is a P, then x is not a Q

Aristotelian Forms

No Ps are Qs

No humans have wings

For any x, if x is a P,
then x is not a Q

For any x, $P(x) \rightarrow \neg Q(x)$

Aristotelian Forms

No Ps are Qs

No humans have wings

For any x, if x is a P, then x is not a Q

For any x, $P(x) \rightarrow \neg Q(x)$

$\forall x(P(x) \rightarrow \neg Q(x))$

Aristotelian Forms

No Ps are Qs

No humans have wings

For any x, if x is a P, then x is not a Q

For any x, $P(x) \rightarrow \neg Q(x)$

$\forall x(P(x) \rightarrow \neg Q(x))$

$\forall x(Human(x) \rightarrow \neg Wings(x))$

Aristotelian Forms

No Ps are Qs

No humans have wings

For any x, if x is a P, then x is not a Q

For any x, $P(x) \rightarrow \neg Q(x)$

$\forall x(P(x) \rightarrow \neg Q(x))$

$\forall x(Human(x) \rightarrow \neg Wings(x))$

$\neg\exists x(P(x) \land Q(x))$

$\neg\exists x(Human(x) \land Wings(x))$

Aristotelian Forms

Some Ps are not Qs

Some birds can't fly

Aristotelian Forms

Some Ps are not Qs

Some birds can't fly

There is at least one P that is not a Q

Aristotelian Forms

Some Ps are not Qs

Some birds can't fly

There is at least one P that is not a Q

There is at least one thing x such that x is P but not Q

Aristotelian Forms

Some Ps are not Qs

Some birds can't fly

There is at least one P that is not a Q

There is at least one thing x such that x is P but not Q

There is at least one thing x such that $P(x) \wedge \neg Q(x)$

Aristotelian Forms

Some Ps are not Qs

Some birds can't fly

There is at least one P that is not a Q

There is at least one thing x such that x is P but not Q

There is at least one thing x such that $P(x) \wedge \neg Q(x)$

$\exists x(P(x) \wedge \neg Q(x))$

Aristotelian Forms

Some Ps are not Qs

Some birds can't fly

There is at least one P that is not a Q

There is at least one thing x such that x is P but not Q

There is at least one thing x such that $P(x) \land \neg Q(x)$

$\exists x(P(x) \land \neg Q(x))$

$\exists x(Bird(x) \land \neg Fly(x))$

Aristotelian Forms

Some Ps are not Qs

Some birds can't fly

There is at least one P that is not a Q

There is at least one thing x such that x is P but not Q

There is at least one thing x such that $P(x) \wedge \neg Q(x)$

$\exists x(P(x) \wedge \neg Q(x))$

$\exists x(Bird(x) \wedge \neg Fly(x))$

$\neg \forall x(P(x) \rightarrow Q(x))$

$\neg \forall x(Bird(x) \rightarrow Fly(x))$

Aristotelian Forms

Forms:

QL sentence:

ARISTOTELIAN FORMS

Forms:	QL sentence:
All Ps are Qs.	$\forall x(P(x) \rightarrow Q(x))$

Aristotelian Forms

Forms:	QL sentence:
All Ps are Qs.	$\forall x(P(x) \rightarrow Q(x))$
Some Ps are Qs.	$\exists x(P(x) \wedge Q(x))$

Aristotelian Forms

Forms:	QL sentence:
All Ps are Qs.	$\forall x(P(x) \rightarrow Q(x))$
Some Ps are Qs.	$\exists x(P(x) \wedge Q(x))$
No Ps are Qs.	$\forall x(P(x) \rightarrow \neg Q(x))$

Aristotelian Forms

Forms:	QL sentence:
All Ps are Qs.	$\forall x(P(x) \rightarrow Q(x))$
Some Ps are Qs.	$\exists x(P(x) \wedge Q(x))$
No Ps are Qs.	$\forall x(P(x) \rightarrow \neg Q(x))$
Some Ps are not Qs.	$\exists x(P(x) \wedge \neg Q(x))$

Complex Predicates

Some Ps are Qs $\exists x(P(x) \wedge Q(x))$

Complex Predicates

Some Ps are Qs

$\exists x(P(x) \wedge Q(x))$

Some Ps that are also Rs are Qs

$\exists x([P(x) \wedge R(x)] \wedge Q(x))$

Complex Predicates

Some Ps are Qs

$\exists x(P(x) \wedge Q(x))$

Some Ps that are also Rs are Qs

$\exists x([P(x) \wedge R(x)] \wedge Q(x))$

Some cubes are to the right of *a*

$\exists x(Cubes(x) \wedge RightOf(x,a))$

Complex Predicates

Some Ps are Qs

$\exists x(P(x) \wedge Q(x))$

Some Ps that are also Rs are Qs

$\exists x([P(x) \wedge R(x)] \wedge Q(x))$

Some cubes are to the right of *a*

$\exists x(Cubes(x) \wedge RightOf(x,a))$

Some small cubes are to the right of *a*

$\exists x([Small(x) \wedge Cubes(x)] \wedge RightOf(x,a))$

Complex Predicates

There is a large cube
to the left of *b*

Complex Predicates

There is a large cube to the left of *b*

$\exists x(L(x) \wedge C(x) \wedge LO(x,b))$

Complex Predicates

There is a large cube to the left of *b*

$\exists x(L(x) \wedge C(x) \wedge LO(x,b))$

There is a cube to the left of *b* which is in the same row as *c*

Complex Predicates

There is a large cube to the left of *b*

$\exists x(L(x) \wedge C(x) \wedge LO(x,b))$

There is a cube to the left of *b* which is in the same row as *c*

$\exists x(C(x) \wedge LO(x,b) \wedge SR(x,c))$

Complex Predicates

There is a large cube to the left of *b*

$\exists x(L(x) \land C(x) \land LO(x,b))$

There is a cube to the left of *b* which is in the same row as *c*

$\exists x(C(x) \land LO(x,b) \land SR(x,c))$

b is in the same row as a large cube

Complex Predicates

There is a large cube to the left of *b*

$\exists x(L(x) \land C(x) \land LO(x,b))$

There is a cube to the left of *b* which is in the same row as *c*

$\exists x(C(x) \land LO(x,b) \land SR(x,c))$

b is in the same row as a large cube

$\exists x(L(x) \land C(x) \land SR(b,x))$

Complex Predicates

All Ps are Qs $\forall x(P(x) \rightarrow Q(x))$

Complex Predicates

All Ps are Qs

$\forall x(P(x) \rightarrow Q(x))$

All Ps that are also Rs are Qs

$\forall x([P(x) \wedge R(x)] \rightarrow Q(x))$

Complex Predicates

All Ps are Qs

$\forall x(P(x) \rightarrow Q(x))$

All Ps that are also Rs are Qs

$\forall x([P(x) \wedge R(x)] \rightarrow Q(x))$

All cubes are to the right of *a*

$\forall x(\text{Cubes}(x) \rightarrow \text{RightOf}(x,a))$

Complex Predicates

All Ps are Qs

$\forall x(P(x) \rightarrow Q(x))$

All Ps that are also Rs are Qs

$\forall x([P(x) \land R(x)] \rightarrow Q(x))$

All cubes are to the right of *a*

$\forall x(Cubes(x) \rightarrow RightOf(x,a))$

All small cubes are to the right of *a*

$\forall x([Small(x) \land Cubes(x)] \rightarrow RightOf(x,a))$