PUZZLE

$$\begin{array}{|l} P \rightarrow Q \\ \hline (P \wedge R) \rightarrow Q \end{array}$$

This argument is an example of antecedent strengthening and we know it is valid. But what about the following:

If I leave my house in the next five minutes, I will make it to the movie on time. Therefore, if I leave my house in the next five minutes and get hit by a bus, I will make it to the movie on time.

INTRODUCTION TO QUANTIFIERS

Wednesday, 12 March

TRUTH-FUNCTIONAL COMPLETENESS

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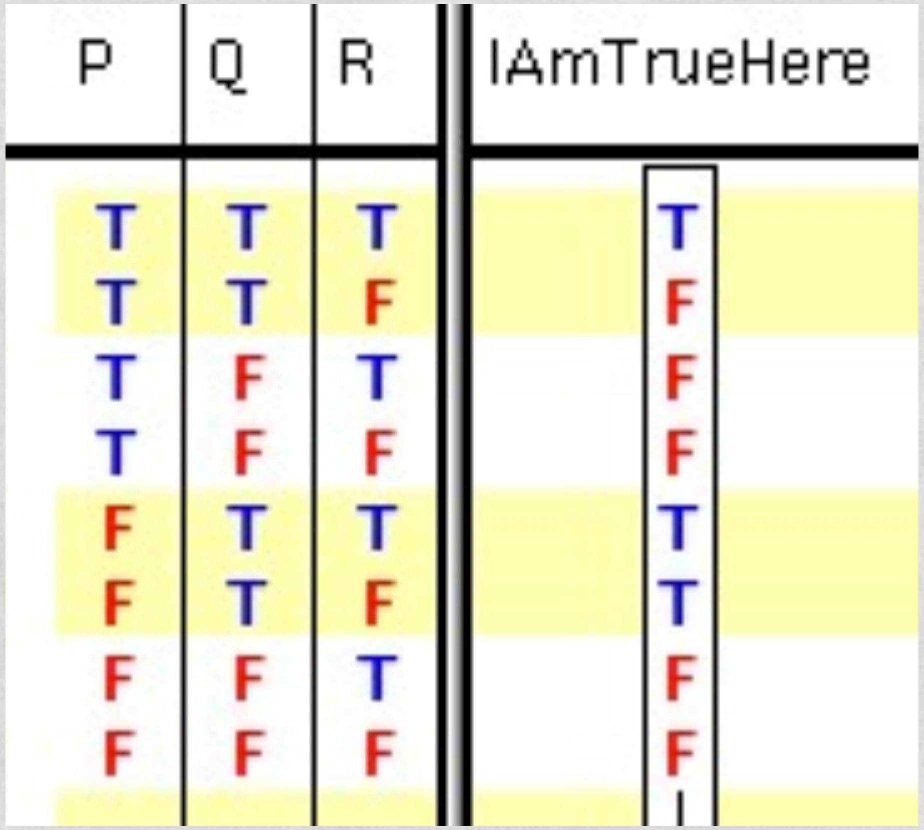
- YES. We can express ANY truth function of arbitrary size or complexity.

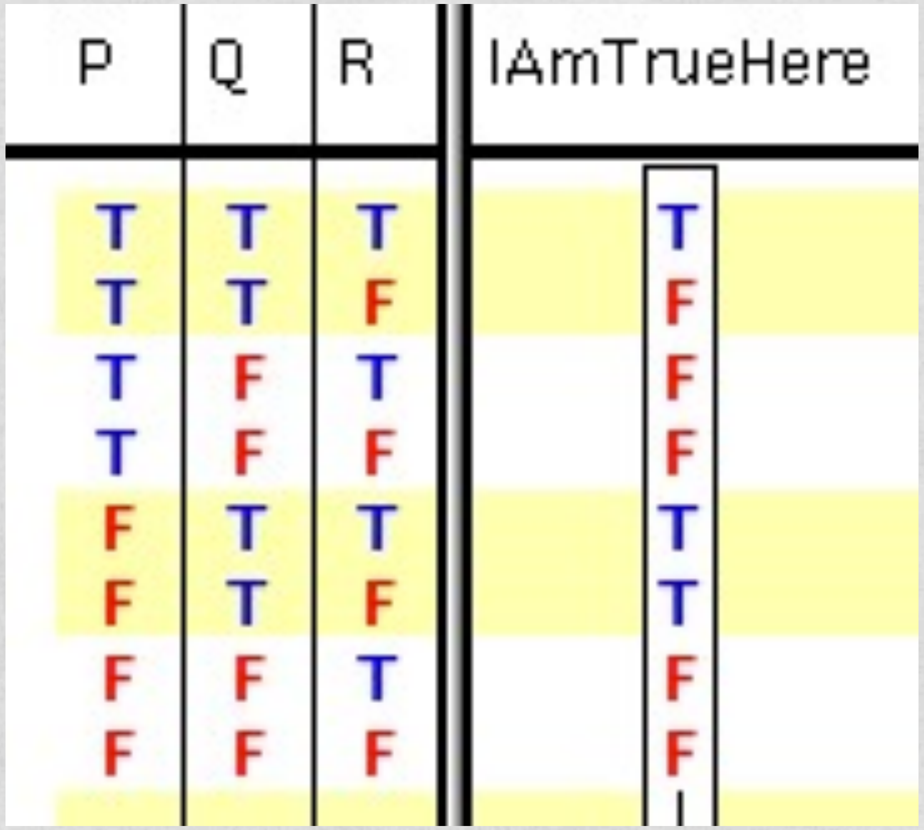
Truth-Functional Completeness



P	Q	R	IAmTrueHere
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	F
F	F	F	F

Want a sentence true in exactly these cases?

TRUTH-FUNCTIONAL COMPLETENESS



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T	F	F	F
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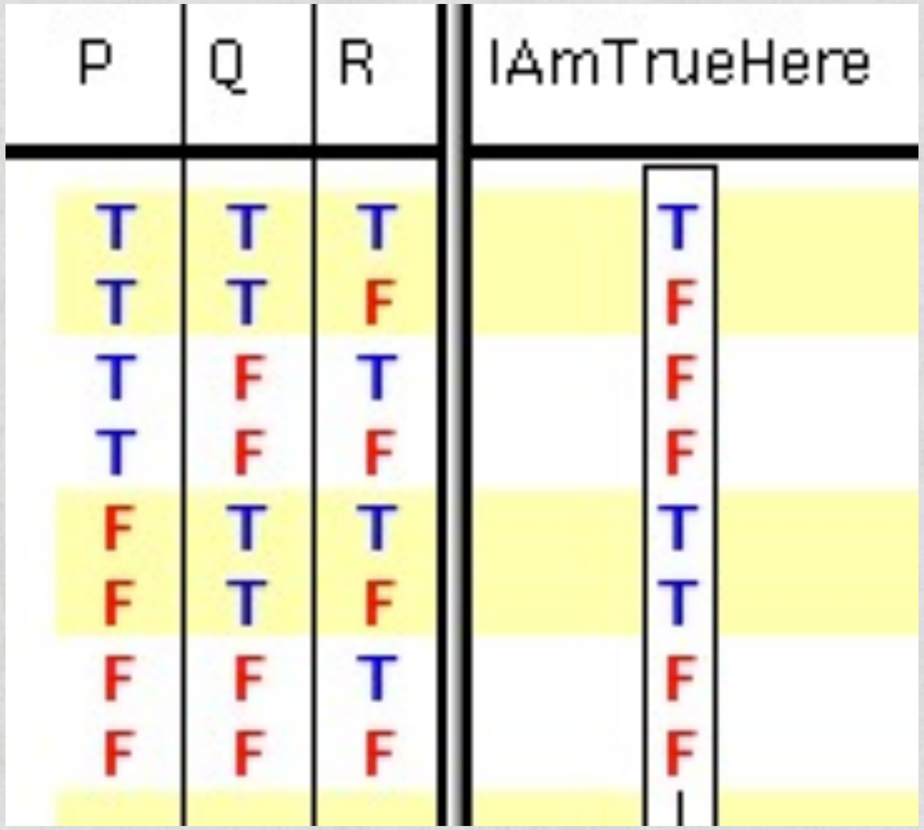
F	T	F	T
F	F	T	F
F	F	F	F

Want a sentence true
in exactly these cases?

How about:

$$(P \land Q \land R) \lor (\neg P \land Q \land R) \lor (\neg P \land Q \land \neg R)$$

TRUTH-FUNCTIONAL COMPLETENESS



P	Q	R	IAmTrueHere
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	F
F	F	F	F

Want a sentence true in exactly these cases?

How about:

$$(P\land Q\land R)\lor(\neg P\land Q\land R)\lor(\neg P\land Q\land\neg R)$$

If a sentence's truth is completely determined by the truth of its subsentences, then it is equivalent to a sentence like the above using just ¬, ∧, and ∨

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- {¬ and ∨}, {¬ and ∧}, {¬ and →}, {⊥ and →}, are also truth-functionally complete. Some combos, like {¬ and ↔} are not complete (you can't express 'A and B' with only ¬ and ↔).

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- Awesome fact: "NAND" [↑] and "NOR" [↓] each by themselves are complete.

THE POWER OF TRUTH-FUNCTIONS

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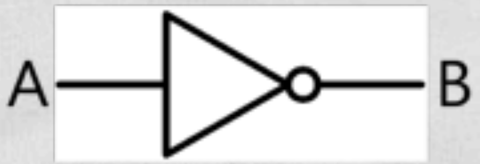
THE POWER OF TRUTH-FUNCTIONS

- You can do a lot with truth-functional logic
- For example, a logic gate is a physical structure in a digital computer made of relays or transistors used as switches or something similar
- Each logic gate implements some truth-function (takes voltage inputs of high/low and outputs high/low)
- Everything your computer does, it does with logic gates. And the only thing the gates can do is simulate doing a truth-table

7 LOGIC GATES & 4 USEFUL COMBINATIONS

Standard Logic Gates

NOT



A	B
1	0
0	1

AND

A	B	C
1	1	1
1	0	0
0	1	0
0	0	0

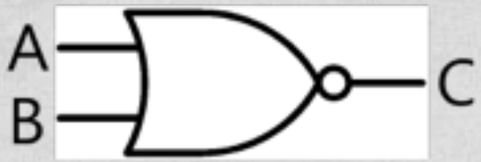
NAND

A	B	C
1	1	0
1	0	1
0	1	1
0	0	1

OR

A	B	C
1	1	1
1	0	1
0	1	1
0	0	0

NOR



A	B	C
1	1	0
1	0	0
0	1	0
0	0	1

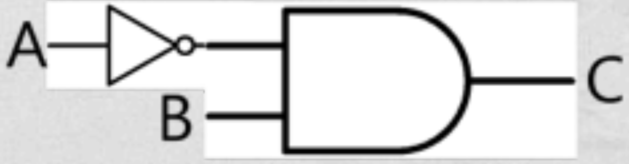
XOR

A	B	C
1	1	0
1	0	1
0	1	1
0	0	0

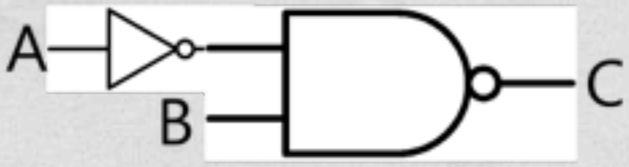
XNOR

A	B	C
1	1	1
1	0	0
0	1	0
0	0	1

Preceding NOT Gate on One Input for AND, NAND, OR, NOR Gates



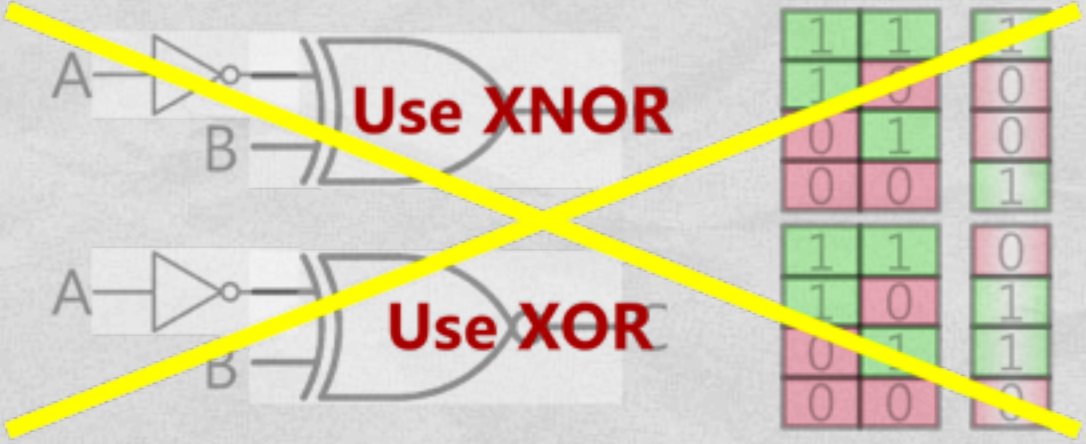
A	B	C
1	1	0
1	0	0
0	1	1
0	0	0



A	B	C
1	1	1
1	0	1
0	1	0
0	0	1

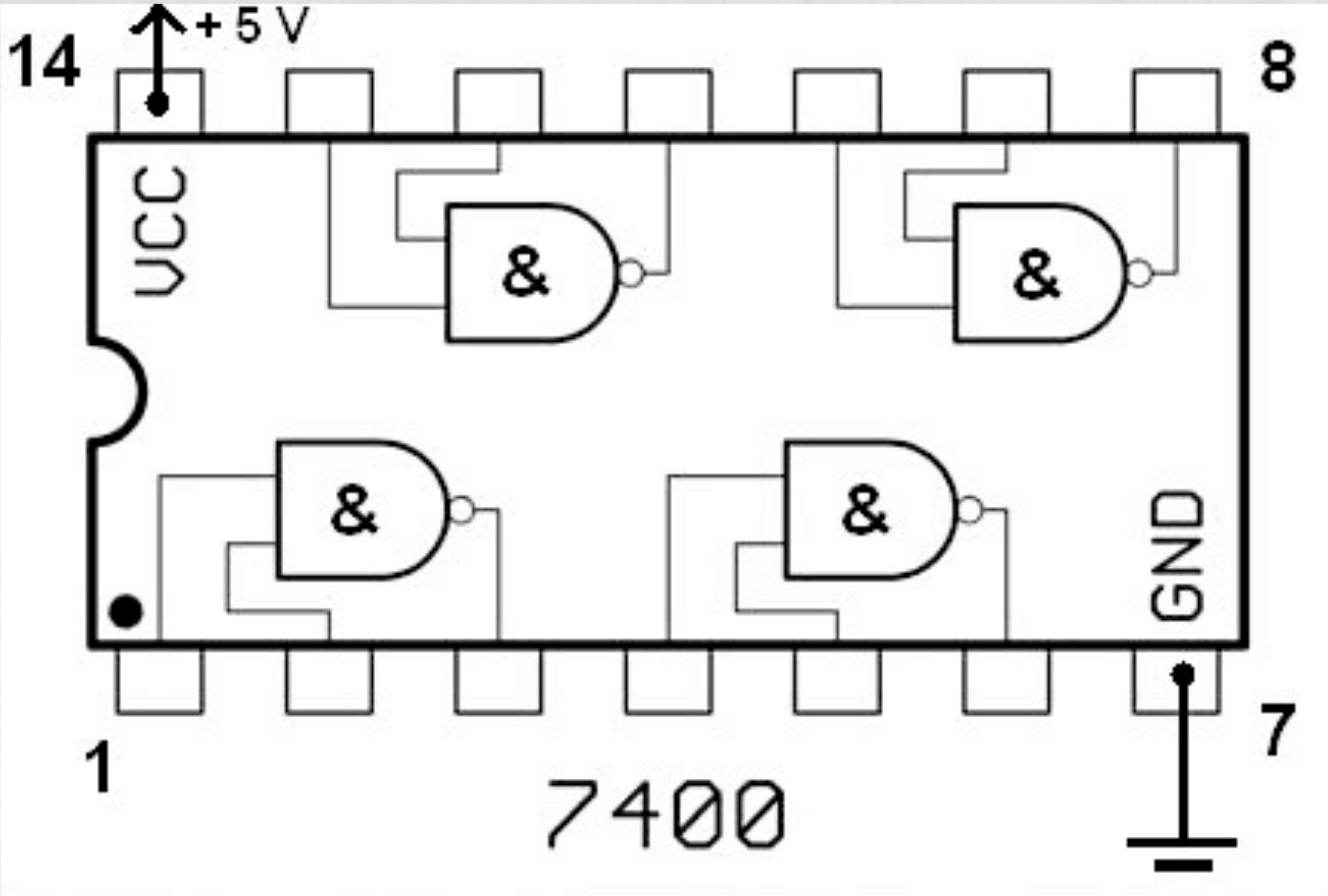
A	B	C
1	1	1
1	0	0
0	1	1
0	0	1

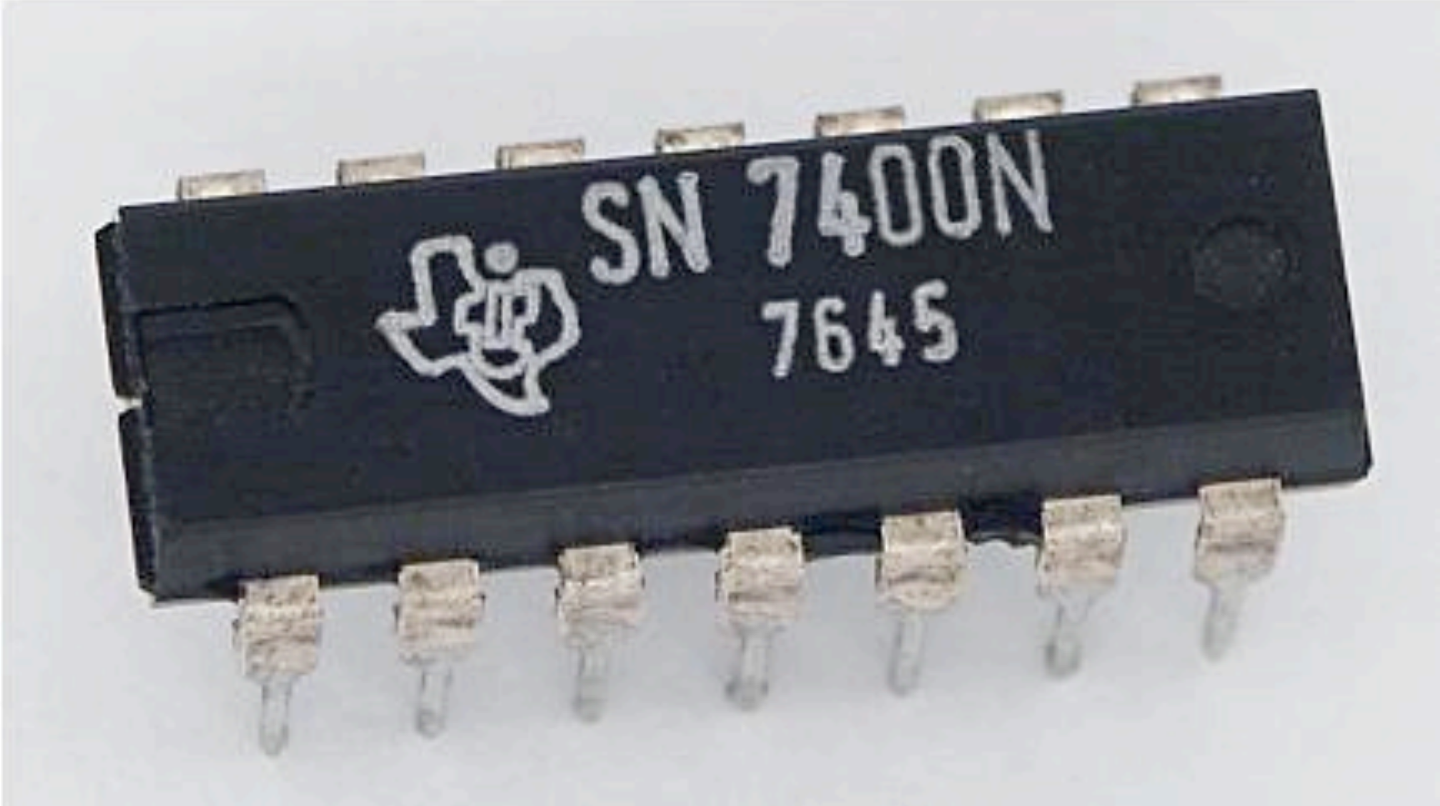
A	B	C
1	1	0
1	0	1
0	1	0
0	0	0



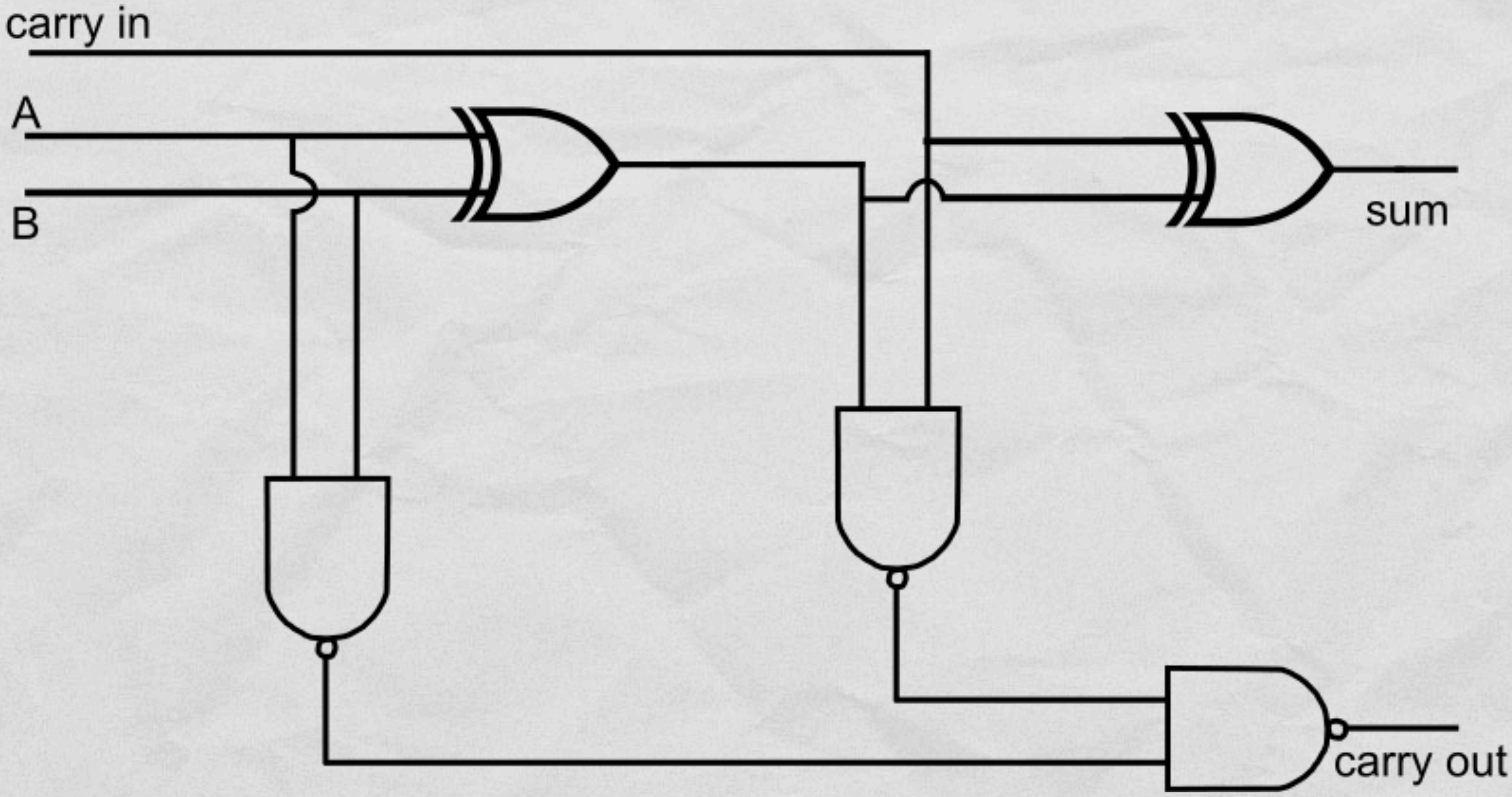
A	B	C
1	1	1
1	0	0
0	1	0
0	0	1

A	B	C
1	1	0
1	0	1
0	1	1
0	0	0





This chip is made with four NAND gates (not both)



This adds binary numbers. It uses 3 NAND gates and 2 XOR gates (16 transistors)

LIMITS OF TRUTH-FUNCTIONS

a is a cube
b is not a cube

$a \neq b$

Limits of Truth-Functions

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b is not a cube

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This is provable if you add the identity rules

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a is a cube
b is not a cube

There are at least two things

LIMITS OF TRUTH-FUNCTIONS

a is a cube
b is not a cube

$a \neq b$

This is provable if you add the identity rules

a is a cube
b is not a cube

There are at least two things

This is still not

LIMITS OF TRUTH-FUNCTIONS

All men are mortal
Socrates is a man

Socrates is mortal

All men are tall
Not every man is bald

Some tall people aren’t bald

No apples are rotten
Some fruits are rotten

Some fruits aren’t apples

For any number, there is a larger prime number

There is no largest prime number

None are truth-functionally valid
- We need a stronger logical system

QUANTIFIERS

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Two quantifier symbols:

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- ∀ means “everything” or “for all”.

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- ∃ means “something” or “there exists at least one”.

QUANTIFIERS

Two quantifier symbols:

- ∀ means "everything" or "for all".
- ∃ means "something" or "there exists at least one".
- Just these two quantifiers can be used to capture many of the quantifications we want to talk about. For example, all, every, any, none, not all of, some, some are not, at least one, at least two, exactly two, etc.

SENTENCES IN FOL

Cube(a)

True in a world if *a* is a cube in that world

SENTENCES IN FOL

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∀xCube(x)

True in a world if every object in that world is a cube

SENTENCES IN FOL

Cube(a)

True in a world if *a* is a cube in that world

$\forall x \text{Cube}(x)$

True in a world if every object in that world is a cube

For every object x, x is a cube

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True in a world if at least one object in that world is a cube

SENTENCES IN FOL

$Cube(a)$

True in a world if *a* is a cube in that world

$\exists x Cube(x)$

True in a world if at least one object in that world is a cube

For some object x, x is a cube

SENTENCES IN FOL

Cube(a)

$\exists x$Cube(x)

True in a world if *a* is a cube in that world

True in a world if at least one object in that world is a cube

For some object x, x is a cube

Cube(x) - Not true or false - not even a sentence

EXAMPLE SENTENCES

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- $\forall x$ Cube(x) - Everything is a cube

EXAMPLE SENTENCES

- ∀x Cube(x) - Everything is a cube
- ∃x Cube(x) - Something is a cube

EXAMPLE SENTENCES

- $\forall x$ Cube(x) - Everything is a cube
- $\exists x$ Cube(x) - Something is a cube
- $\forall x$(Cube(x)$\wedge$Small(x)) - Everything is a small cube

EXAMPLE SENTENCES

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EXAMPLE SENTENCES

- $\forall x\, Cube(x)$ - Everything is a cube
- $\exists x\, Cube(x)$ - Something is a cube
- $\forall x(Cube(x) \land Small(x))$ - Everything is a small cube
- $\exists x(Cube(x) \land Small(x))$ - Something is a small cube

- $\forall x(Cube(x) \lor Dodec(x))$ - Everything is a cube or a dodec

EXAMPLE SENTENCES

- $\forall x$ Cube(x) - Everything is a cube
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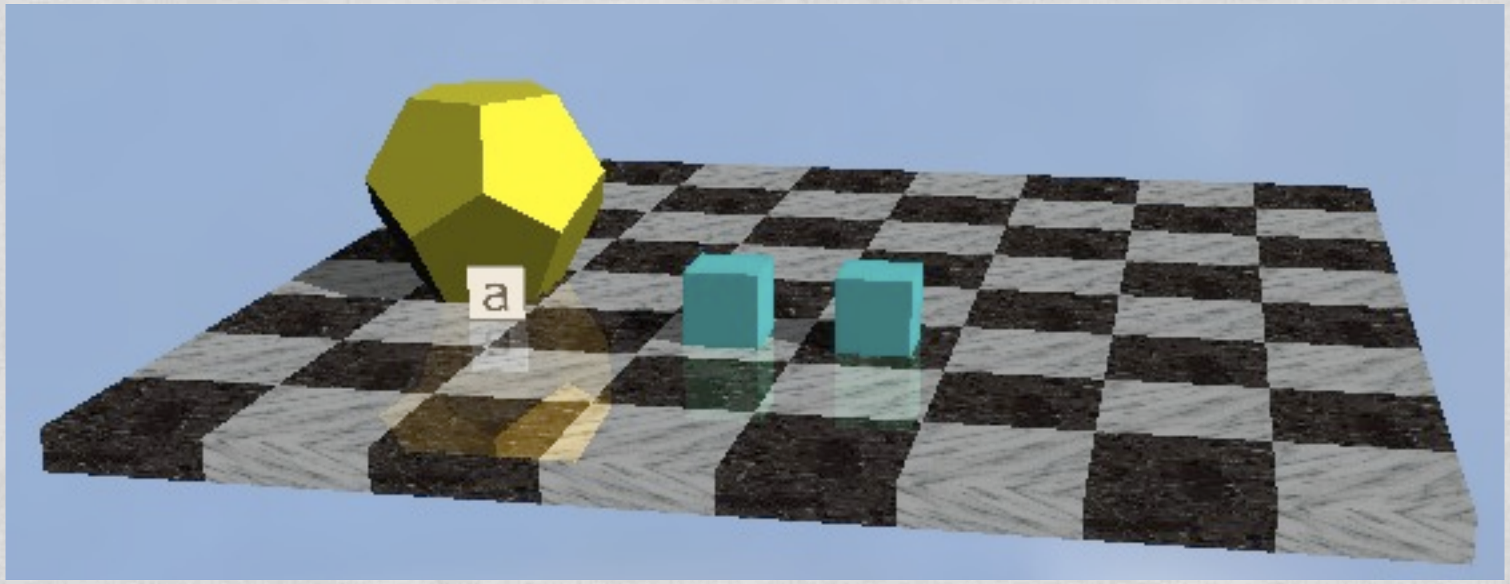
- $\forall x$(Cube(x)$\wedge$Small(x)) - Everything is a small cube
- $\exists x$(Cube(x)$\wedge$Small(x)) - Something is a small cube

- $\forall x$(Cube(x) $\vee$ Dodec(x)) - Everything is a cube or a dodec
- $\forall x$ Cube(x) $\vee$ $\forall x$ Dodec(x)- Everything is cube or everything is a dodec

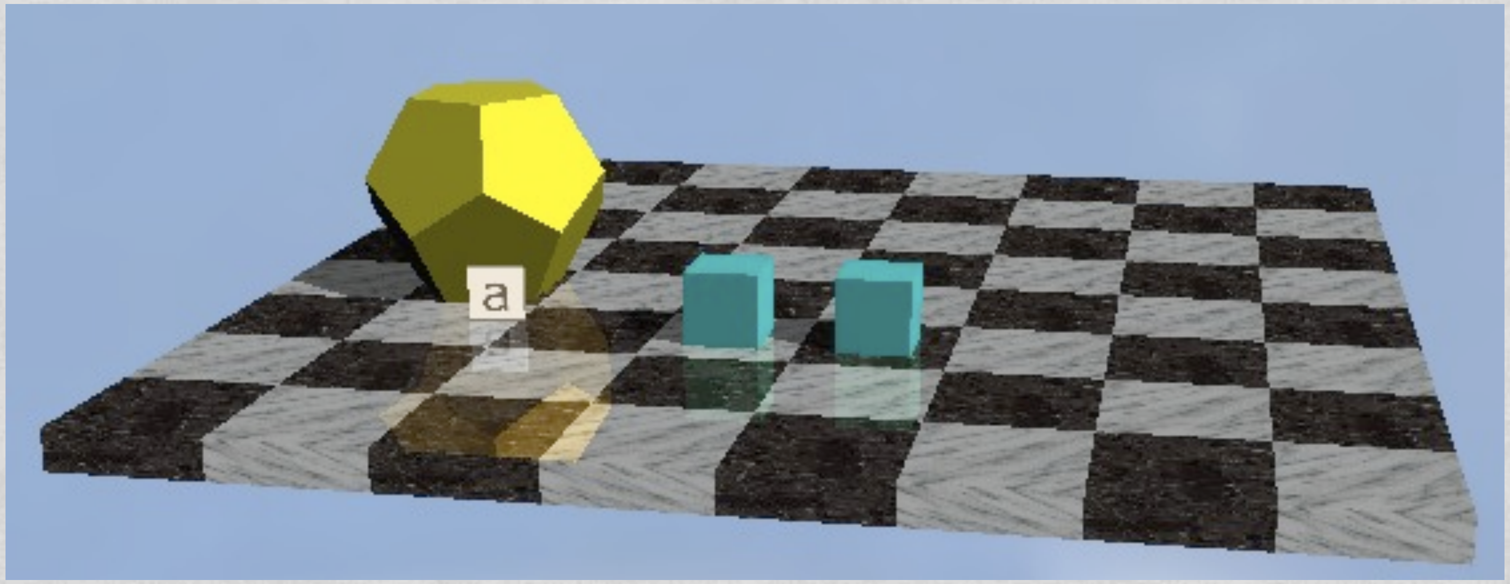
SATISFACTION

- $\forall x\ \mathrm{Cube}(x)$
- $\exists x\ \mathrm{Cube}(x)$
- $\forall x(\mathrm{Cube}(x) \land \mathrm{Small}(x))$
- $\exists x(\mathrm{Cube}(x) \land \mathrm{Small}(x))$



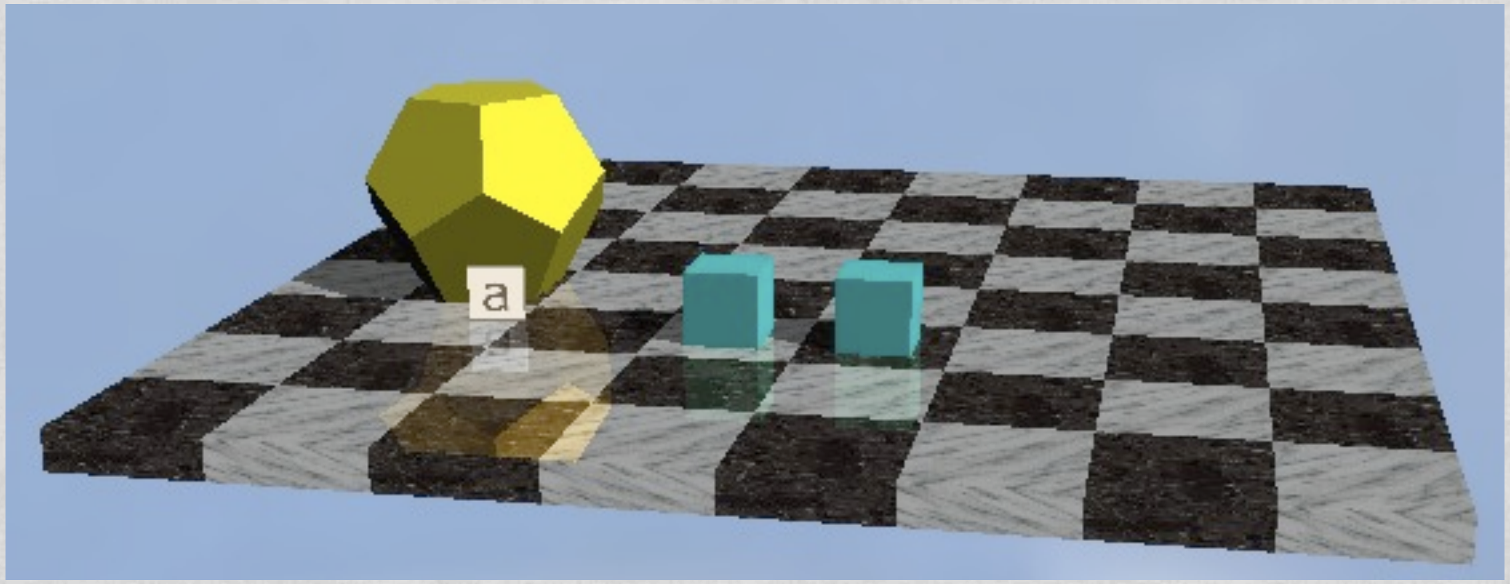
SATISFACTION

F
- $\forall x$ Cube(x)
- $\exists x$ Cube(x)
- $\forall x$(Cube(x)$\wedge$Small(x))
- $\exists x$(Cube(x)$\wedge$Small(x))



SATISFACTION

- F $\forall x$ Cube(x)
- T $\exists x$ Cube(x)
- $\forall x$(Cube(x)$\wedge$Small(x))
- $\exists x$(Cube(x)$\wedge$Small(x))



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- T $\exists x\ Cube(x)$
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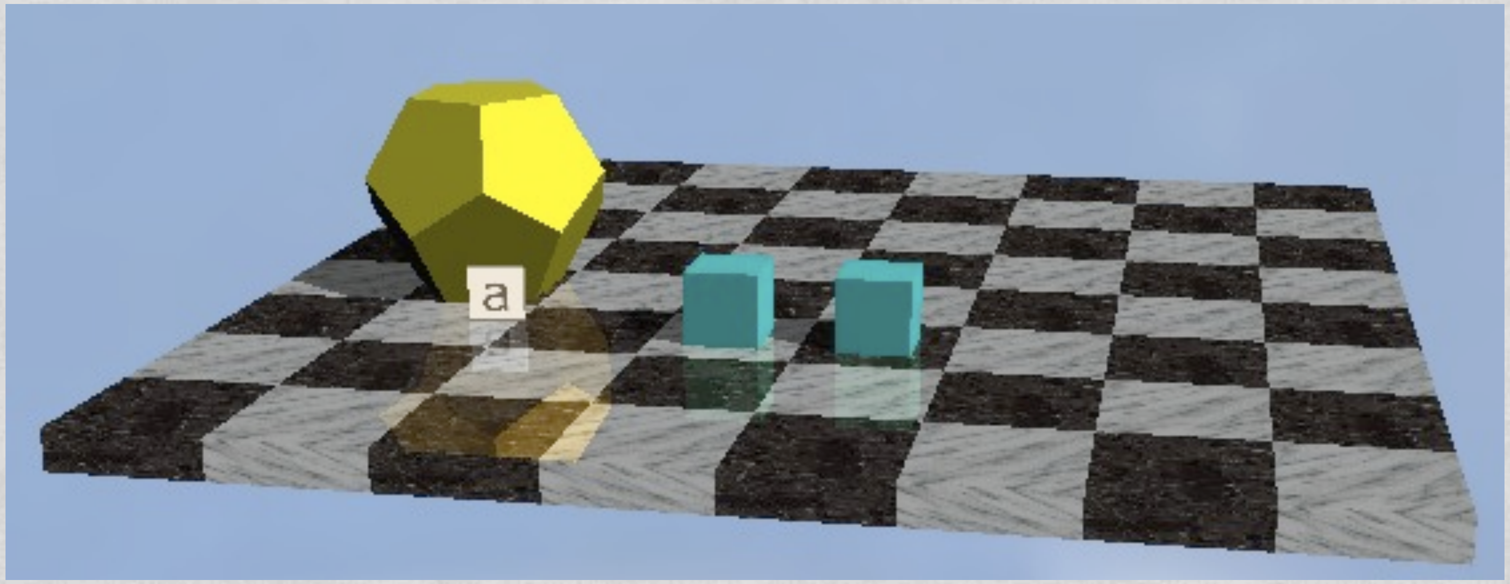

SATISFACTION

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- T $\exists x\ Cube(x)$
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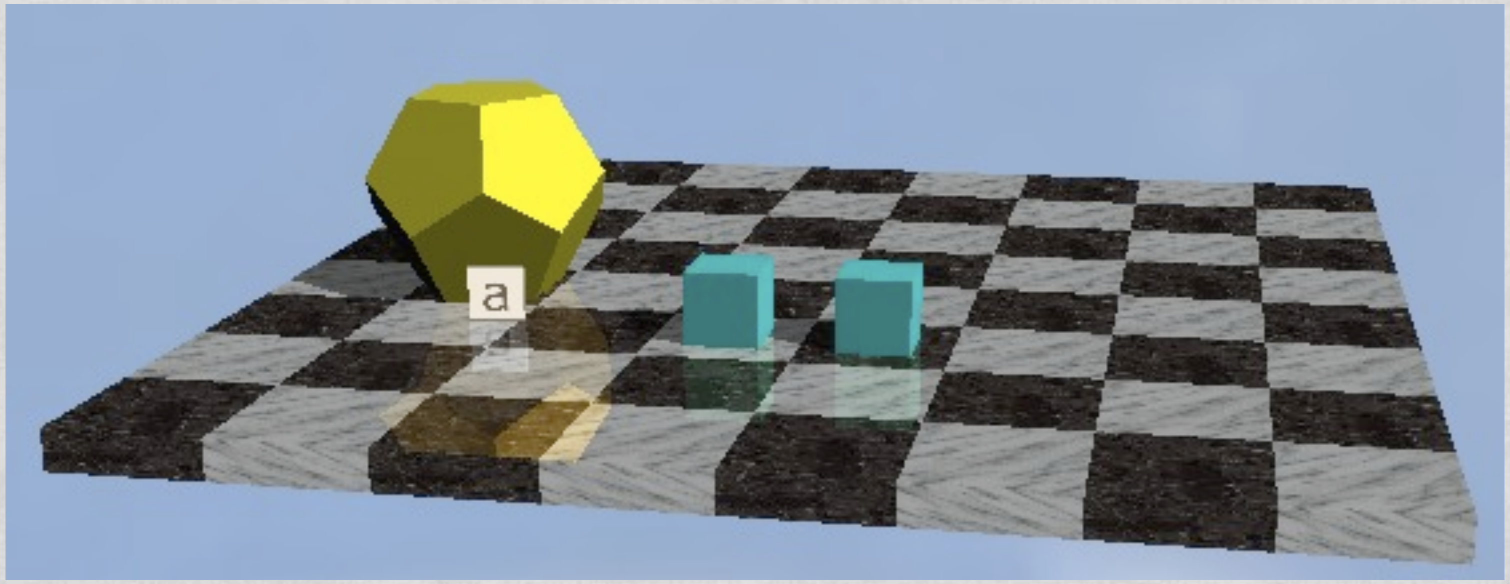
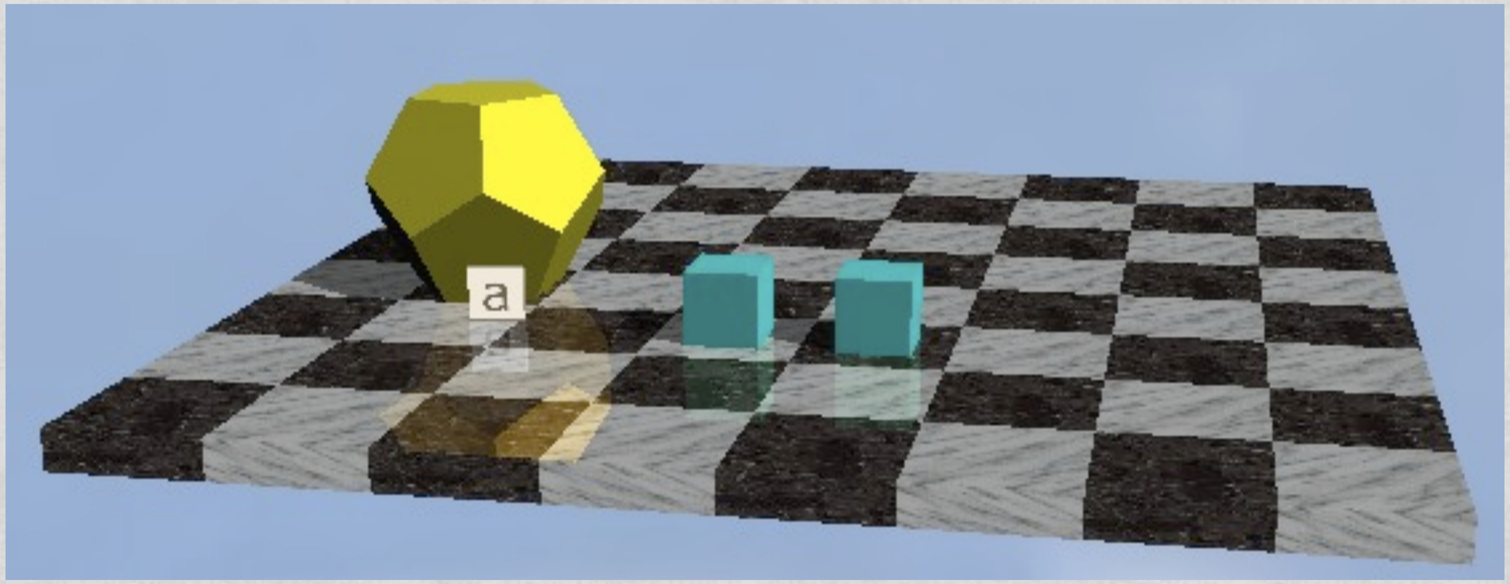
- T $\exists x (Cube(x) \land Small(x))$



SATISFACTION

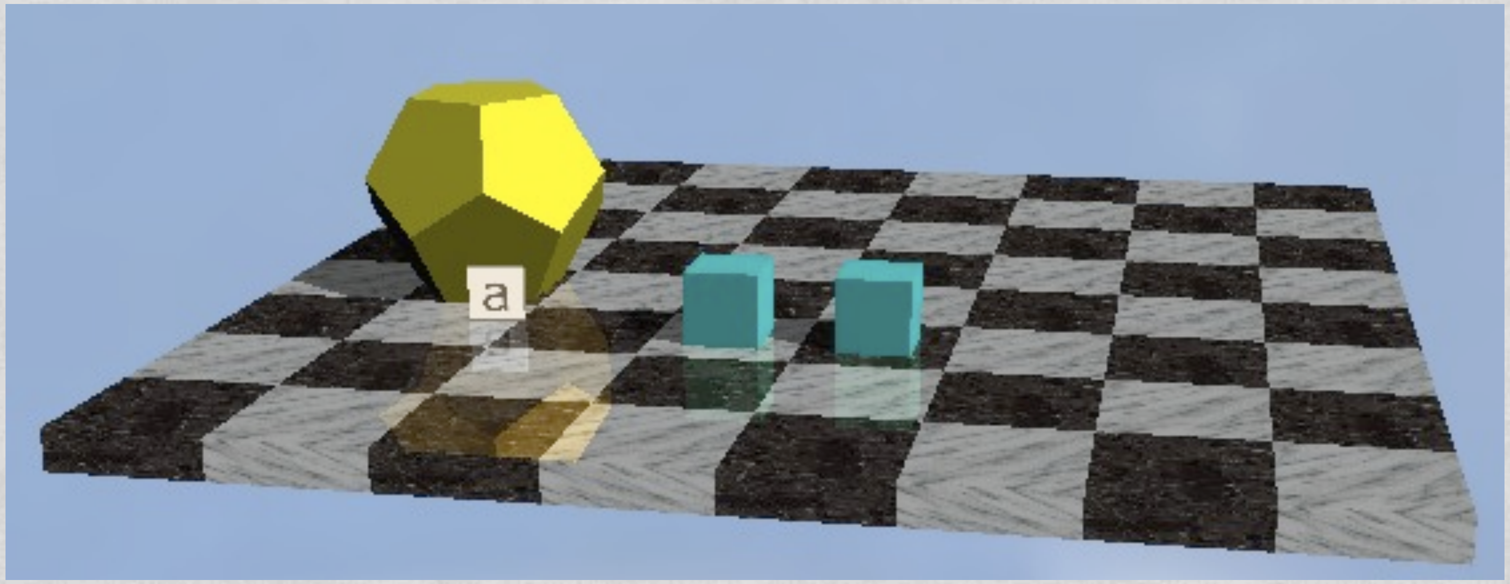
- $\forall x\,(\mathrm{Cube}(x) \lor \mathrm{Dodec}(x))$
- $\forall x\,\mathrm{Cube}(x) \lor \forall x\,\mathrm{Dodec}(x)$
- $\exists x(\mathrm{Cube}(x) \land \mathrm{Large}(x))$
- $\exists x\,\mathrm{Cube}(x) \land \exists x\,\mathrm{Large}(x)$



SATISFACTION

T

- $\forall x\ (Cube(x) \vee Dodec(x))$
- $\forall x\ Cube(x) \vee \forall x\ Dodec(x)$
- $\exists x(Cube(x) \wedge Large(x))$
- $\exists x\ Cube(x) \wedge \exists x\ Large(x)$

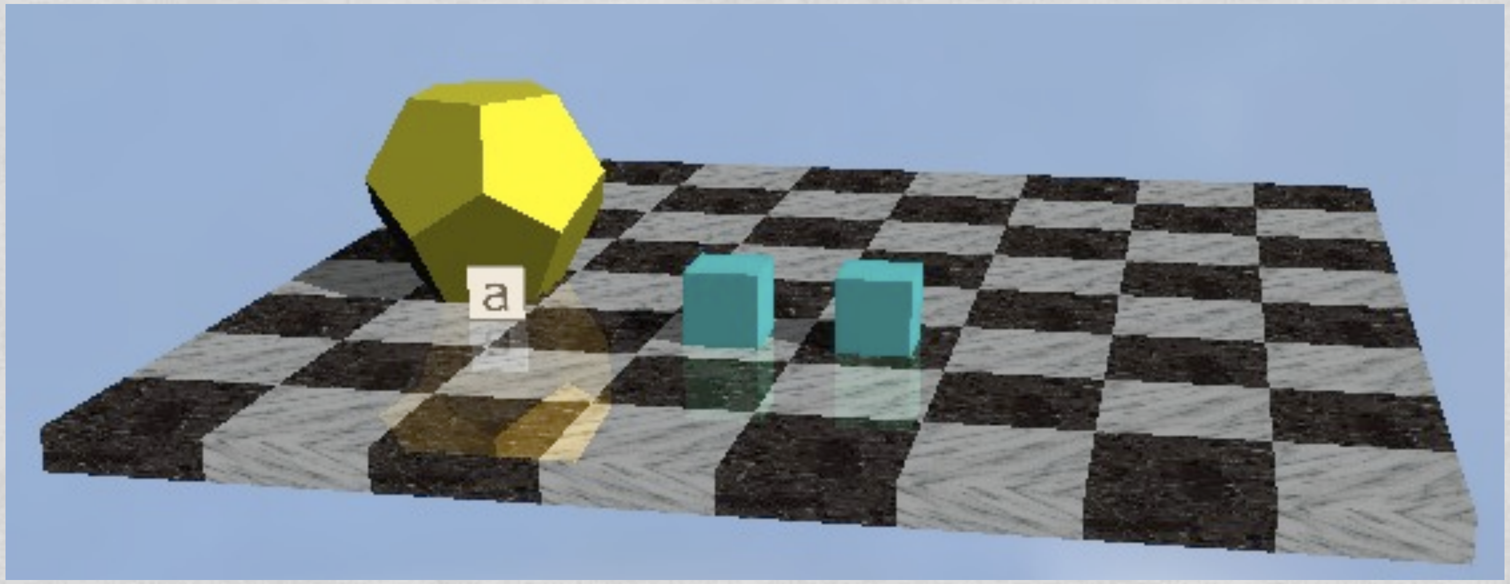


SATISFACTION

T - $\forall x\ (Cube(x) \lor Dodec(x))$

F - $\forall x\ Cube(x) \lor \forall x\ Dodec(x)$

- $\exists x(Cube(x) \land Large(x))$
- $\exists x\ Cube(x) \land \exists x\ Large(x)$



SATISFACTION

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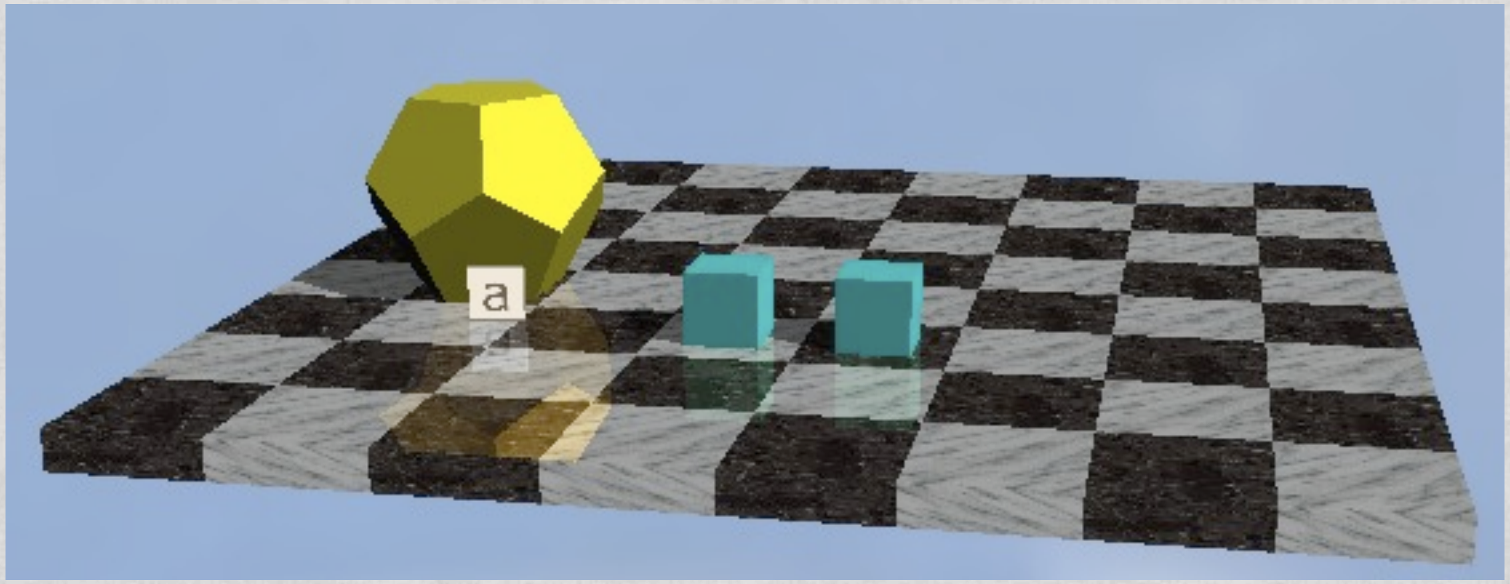
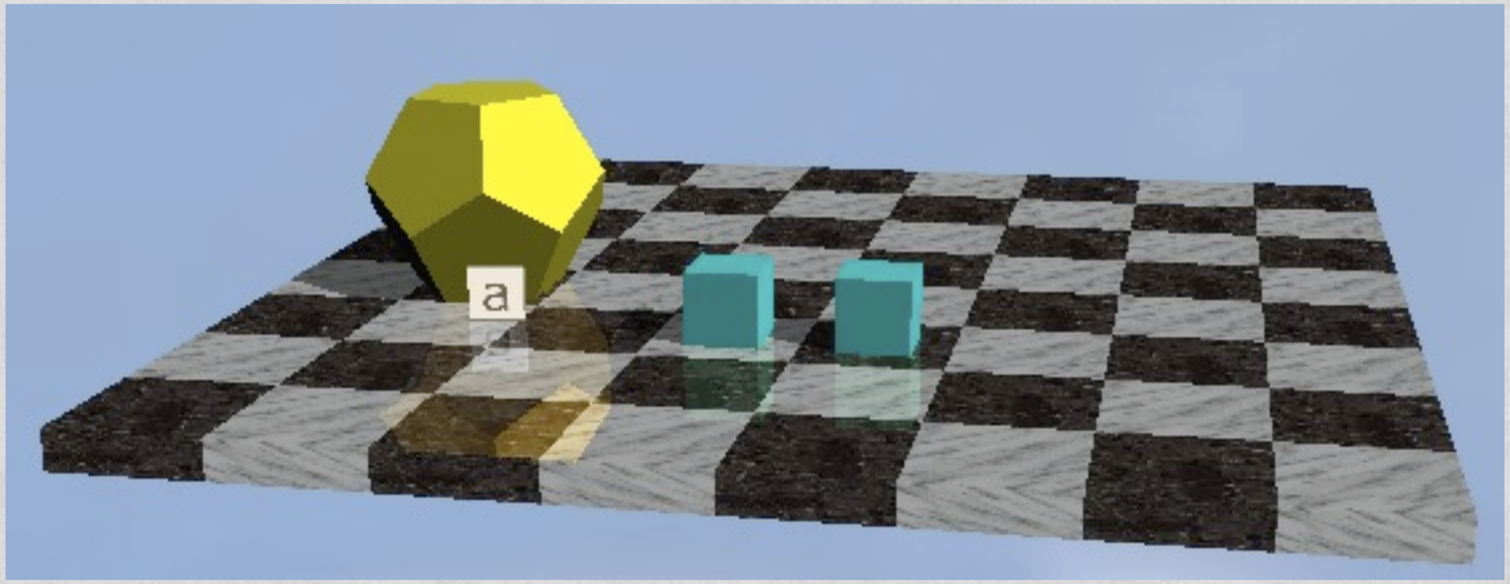
- F $\exists x(Cube(x) \land Large(x))$
- $\exists x\,Cube(x) \land \exists x\,Large(x)$



SATISFACTION

- T $\forall x\,(Cube(x) \vee Dodec(x))$
- F $\forall x\, Cube(x) \vee \forall x\, Dodec(x)$
- F $\exists x(Cube(x) \wedge Large(x))$
- T $\exists x\, Cube(x) \wedge \exists x\, Large(x)$



ARISTOTELIAN FORMS

Forms:

Examples:

ARISTOTELIAN FORMS

Forms:	Examples:
All Ps are Qs.	All mammals are animals.

ARISTOTELIAN FORMS

Forms:	Examples:
All Ps are Qs.	All mammals are animals.
Some Ps are Qs.	Some mammals live in water.

ARISTOTELIAN FORMS

Forms:	Examples:
All Ps are Qs.	All mammals are animals.
Some Ps are Qs.	Some mammals live in water.
No Ps are Qs.	No humans have wings.

Aristotelian Forms

Forms:	Examples:
All Ps are Qs.	All mammals are animals.
Some Ps are Qs.	Some mammals live in water.
No Ps are Qs.	No humans have wings.
Some Ps are not Qs.	Some birds cannot fly.

ARISTOTELIAN FORMS

All Ps are Qs

All mammals are animals

ARISTOTELIAN FORMS

All Ps are Qs

All mammals are animals

For any x, if x is a P,
then x is a Q

ARISTOTELIAN FORMS

All Ps are Qs

All mammals are animals

For any x, if x is a P,
then x is a Q

For any x, P(x) → Q(x)

ARISTOTELIAN FORMS

All Ps are Qs

All mammals are animals

For any x, if x is a P,
then x is a Q

For any x, $P(x) \rightarrow Q(x)$

$\forall x(P(x) \rightarrow Q(x))$

ARISTOTELIAN FORMS

All Ps are Qs

All mammals are animals

For any x, if x is a P,
then x is a Q

For any x, $P(x) \rightarrow Q(x)$

$\forall x(P(x) \rightarrow Q(x))$

$\forall x(\text{Mammal}(x) \rightarrow \text{Animal}(x))$

ARISTOTELIAN FORMS

Some Ps are Qs

Some mammals live in water

ARISTOTELIAN FORMS

Some Ps are Qs

Some mammals live in water

There is at least one P that is also a Q

ARISTOTELIAN FORMS

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There is at least one thing x such that x is both P and Q

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There is at least one thing x
such that $P(x) \wedge Q(x)$

ARISTOTELIAN FORMS

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Some mammals live in water

There is at least one P that is also a Q

There is at least one thing x such that x is both P and Q

There is at least one thing x such that $P(x) \wedge Q(x)$

$$\exists x(P(x) \wedge Q(x))$$

ARISTOTELIAN FORMS

Some Ps are Qs

Some mammals live in water

There is at least one P that is also a Q

There is at least one thing x such that x is both P and Q

There is at least one thing x such that $P(x) \wedge Q(x)$

$\exists x(P(x) \wedge Q(x))$

$\exists x(\mathrm{Mammal}(x) \wedge \mathrm{LiWa}(x))$

ARISTOTELIAN FORMS

No Ps are Qs

No humans have wings

ARISTOTELIAN FORMS

No Ps are Qs

No humans have wings

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ARISTOTELIAN FORMS

No Ps are Qs

No humans have wings

For any x, if x is a P,
then x is not a Q

For any x, $P(x) \rightarrow \neg Q(x)$

ARISTOTELIAN FORMS

No Ps are Qs

No humans have wings

For any x, if x is a P, then x is not a Q

For any x, $P(x) \rightarrow \neg Q(x)$

$\forall x(P(x) \rightarrow \neg Q(x))$

ARISTOTELIAN FORMS

No Ps are Qs

No humans have wings

For any x, if x is a P,
then x is not a Q

For any x, $P(x) \rightarrow \neg Q(x)$

$\forall x(P(x) \rightarrow \neg Q(x))$

$\forall x(\mathrm{Human}(x) \rightarrow \neg \mathrm{Wings}(x))$

ARISTOTELIAN FORMS

No Ps are Qs

No humans have wings

For any x, if x is a P, then x is not a Q

For any x, $P(x) \rightarrow \neg Q(x)$

$\forall x(P(x) \rightarrow \neg Q(x))$

$\forall x(\mathrm{Human}(x) \rightarrow \neg \mathrm{Wings}(x))$

$\neg \exists x(P(x) \wedge Q(x))$

$\neg \exists x(\mathrm{Human}(x) \wedge \mathrm{Wings}(x))$

ARISTOTELIAN FORMS

Some Ps are not Qs

Some birds can’t fly

ARISTOTELIAN FORMS

Some Ps are not Qs

Some birds can't fly

There is at least one P that is not a Q

ARISTOTELIAN FORMS

Some Ps are not Qs

Some birds can't fly

There is at least one P that is not a Q

There is at least one thing x such that x is P but not Q

ARISTOTELIAN FORMS

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There is at least one P that is not a Q

There is at least one thing x such that x is P but not Q

There is at least one thing x such that $P(x) \wedge \neg Q(x)$

ARISTOTELIAN FORMS

Some Ps are not Qs

Some birds can't fly

There is at least one P that is not a Q

There is at least one thing x such that x is P but not Q

There is at least one thing x such that $P(x) \wedge \neg Q(x)$

$$\exists x(P(x) \wedge \neg Q(x))$$

ARISTOTELIAN FORMS

Some Ps are not Qs

Some birds can't fly

There is at least one P that is not a Q

There is at least one thing x such that x is P but not Q

There is at least one thing x such that $P(x) \wedge \neg Q(x)$

$\exists x(P(x) \wedge \neg Q(x))$

$\exists x(Bird(x) \wedge \neg Fly(x))$

ARISTOTELIAN FORMS

Some Ps are not Qs

Some birds can't fly

There is at least one P that is not a Q

There is at least one thing x such that x is P but not Q

There is at least one thing x such that $P(x) \wedge \neg Q(x)$

$\exists x(P(x) \wedge \neg Q(x))$

$\exists x(\mathrm{Bird}(x) \wedge \neg \mathrm{Fly}(x))$

$\neg\forall x(P(x) \rightarrow Q(x))$

$\neg\forall x(\mathrm{Human}(x) \rightarrow \mathrm{Wings}(x))$

ARISTOTELIAN FORMS

Forms:

QL sentence:

ARISTOTELIAN FORMS

Forms:	QL sentence:
All Ps are Qs.	$\forall x(P(x) \rightarrow Q(x))$

ARISTOTELIAN FORMS

Forms:	QL sentence:
All Ps are Qs.	$\forall x(P(x)\rightarrow Q(x))$
Some Ps are Qs.	$\exists x(P(x)\wedge Q(x))$

ARISTOTELIAN FORMS

Forms:	QL sentence:
All Ps are Qs.	$\forall x(P(x)\rightarrow Q(x))$
Some Ps are Qs.	$\exists x(P(x)\wedge Q(x))$
No Ps are Qs.	$\forall x(P(x)\rightarrow\neg Q(x))$

ARISTOTELIAN FORMS

Forms:	QL sentence:
All Ps are Qs.	$\forall x(P(x)\rightarrow Q(x))$
Some Ps are Qs.	$\exists x(P(x)\wedge Q(x))$
No Ps are Qs.	$\forall x(P(x)\rightarrow\neg Q(x))$
Some Ps are not Qs.	$\exists x(P(x)\wedge\neg Q(x))$

COMPLEX PREDICATES

Some Ps are Qs $\exists x(P(x) \wedge Q(x))$

COMPLEX PREDICATES

Some Ps are Qs

$\exists x(P(x) \wedge Q(x))$

Some Ps that are also Rs are Qs

$\exists x([P(x) \wedge R(x)] \wedge Q(x))$

COMPLEX PREDICATES

Some Ps are Qs	$\exists x(P(x) \wedge Q(x))$
Some Ps that are also Rs are Qs	$\exists x([P(x) \wedge R(x)] \wedge Q(x))$
Some cubes are to the right of *a*	$\exists x(\mathrm{Cubes}(x) \wedge \mathrm{RightOf}(x,a))$

COMPLEX PREDICATES

Some Ps are Qs	$\exists x(P(x) \wedge Q(x))$
Some Ps that are also Rs are Qs	$\exists x([P(x) \wedge R(x)] \wedge Q(x))$
Some cubes are to the right of *a*	$\exists x(Cubes(x) \wedge RightOf(x,a))$
Some small cubes are to the right of *a*	$\exists x([Small(x) \wedge Cubes(x)] \wedge RightOf(x,a))$

COMPLEX PREDICATES

There is a large cube
to the left of *b*

COMPLEX PREDICATES

There is a large cube to the left of *b*

$\exists x(L(x) \land C(x) \land LO(x,b))$

COMPLEX PREDICATES

There is a large cube to the left of *b*

$\exists x(L(x) \wedge C(x) \wedge LO(x,b))$

There is a cube to the left of *b* which is in the same row as *c*

COMPLEX PREDICATES

There is a large cube to the left of *b*

$\exists x(L(x) \wedge C(x) \wedge LO(x,b))$

There is a cube to the left of *b* which is in the same row as *c*

$\exists x(C(x) \wedge LO(x,b) \wedge SR(x,c))$

COMPLEX PREDICATES

There is a large cube to the left of *b*

$\exists x(L(x) \wedge C(x) \wedge LO(x,b))$

There is a cube to the left of *b* which is in the same row as *c*

$\exists x(C(x) \wedge LO(x,b) \wedge SR(x,c))$

b is in the same row as a large cube

COMPLEX PREDICATES

There is a large cube to the left of *b*	$\exists x(L(x) \wedge C(x) \wedge LO(x,b))$
There is a cube to the left of *b* which is in the same row as *c*	$\exists x(C(x) \wedge LO(x,b) \wedge SR(x,c))$
b is in the same row as a large cube	$\exists x(L(x) \wedge C(x) \wedge SR(b,x))$

COMPLEX PREDICATES

All Ps are Qs $\forall x(P(x) \rightarrow Q(x))$

COMPLEX PREDICATES

All Ps are Qs

$\forall x(P(x) \rightarrow Q(x))$

All Ps that are also Rs are Qs

COMPLEX PREDICATES

All Ps are Qs

$\forall x(P(x) \rightarrow Q(x))$

All Ps that are also Rs are Qs

$\forall x([P(x) \wedge R(x)] \rightarrow Q(x))$

COMPLEX PREDICATES

All Ps are Qs

$\forall x(P(x) \rightarrow Q(x))$

All Ps that are also Rs are Qs

$\forall x([P(x) \wedge R(x)] \rightarrow Q(x))$

All cubes are to the right of *a*

COMPLEX PREDICATES

All Ps are Qs

$\forall x(P(x) \rightarrow Q(x))$

All Ps that are also Rs are Qs

$\forall x([P(x) \land R(x)] \rightarrow Q(x))$

All cubes are to the right of *a*

$\forall x(\mathrm{Cubes}(x) \rightarrow \mathrm{RightOf}(x,a))$

COMPLEX PREDICATES

All Ps are Qs	$\forall x(P(x) \rightarrow Q(x))$
All Ps that are also Rs are Qs	$\forall x([P(x) \wedge R(x)] \rightarrow Q(x))$
All cubes are to the right of *a*	$\forall x(\mathrm{Cubes}(x) \rightarrow \mathrm{RightOf}(x,a))$
All small cubes are to the right of *a*	

COMPLEX PREDICATES

All Ps are Qs	$\forall x(P(x) \rightarrow Q(x))$
All Ps that are also Rs are Qs	$\forall x([P(x) \wedge R(x)] \rightarrow Q(x))$
All cubes are to the right of *a*	$\forall x(\mathrm{Cubes}(x) \rightarrow \mathrm{RightOf}(x,a))$
All small cubes are to the right of *a*	$\forall x([\mathrm{Small}(x) \wedge \mathrm{Cubes}(x)] \rightarrow \mathrm{RightOf}(x,a))$