Puzzle

On the island of knights and knaves, I meet A and B. I heard A make a muffled sound, but I couldn't make out the words. I asked B, "What did A say?" B says, "A said exactly one of us is a knight."

What is B?

TESTING VALIDITY

Friday, 7 March

Invalid Arguments

$(S \land P) \leftrightarrow Q$

$T \land S$

$(T \rightarrow P) \rightarrow \neg Q$

Invalid Arguments

$(S \wedge P) \leftrightarrow Q$

$T \wedge S$

$(T \rightarrow P) \rightarrow \neg Q$

1. $(S \wedge P) \leftrightarrow Q$
2. $T \wedge S$

$(T \rightarrow P) \rightarrow \neg Q$

Invalid Arguments

$(S \wedge P) \leftrightarrow Q$

$T \wedge S$

$(T \to P) \to \neg Q$

1. $(S \wedge P) \leftrightarrow Q$
2. $T \wedge S$

---

3. $T \to P$ — for $\to$Intro

$\neg Q$

$(T \to P) \to \neg Q$

Invalid Arguments

$(S \wedge P) \leftrightarrow Q$
$T \wedge S$
$(T \rightarrow P) \rightarrow \neg Q$

1. $(S \wedge P) \leftrightarrow Q$
2. $T \wedge S$
3. $T \rightarrow P$ — for $\rightarrow$Intro
4. T — $\wedge$Elim 2

$\neg Q$

$(T \rightarrow P) \rightarrow \neg Q$

Invalid Arguments

$(S \wedge P) \leftrightarrow Q$

$T \wedge S$

$(T \rightarrow P) \rightarrow \neg Q$

1. $(S \wedge P) \leftrightarrow Q$
2. $T \wedge S$
3. $T \rightarrow P$ — for $\rightarrow$Intro
4. T — $\wedge$Elim 2
5. S — $\wedge$Elim 2

$\neg Q$

$(T \rightarrow P) \rightarrow \neg Q$

Invalid Arguments

$(S \wedge P) \leftrightarrow Q$
$T \wedge S$

$(T \rightarrow P) \rightarrow \neg Q$

1.	$(S \wedge P) \leftrightarrow Q$	
2.	$T \wedge S$	
3.	$T \rightarrow P$	for $\rightarrow$Intro
4.	T	$\wedge$Elim 2
5.	S	$\wedge$Elim 2
6.	P	$\rightarrow$Elim 3,4
	$\neg Q$	
	$(T \rightarrow P) \rightarrow \neg Q$	

Invalid Arguments

$(S \wedge P) \leftrightarrow Q$

$T \wedge S$

$(T \rightarrow P) \rightarrow \neg Q$

1. $(S \wedge P) \leftrightarrow Q$
2. $T \wedge S$
3. $T \rightarrow P$ — for $\rightarrow$Intro
4. T — $\wedge$Elim 2
5. S — $\wedge$Elim 2
6. P — $\rightarrow$Elim 3,4
7. $S \wedge P$ — $\wedge$Intro 5,6

$\neg Q$

$(T \rightarrow P) \rightarrow \neg Q$

Invalid Arguments

$(S \wedge P) \leftrightarrow Q$	
$T \wedge S$	
$(T \rightarrow P) \rightarrow \neg Q$	

1. $(S \wedge P) \leftrightarrow Q$	
2. $T \wedge S$	
3. $T \rightarrow P$	for $\rightarrow$Intro
4. T	$\wedge$Elim 2
5. S	$\wedge$Elim 2
6. P	$\rightarrow$Elim 3,4
7. $S \wedge P$	$\wedge$Intro 5,6
8. Q	$\leftrightarrow$Elim 1,7
$\neg Q$	
$(T \rightarrow P) \rightarrow \neg Q$	

Invalid Arguments

$(S \land P) \leftrightarrow Q$
$T \land S$

$(T \rightarrow P) \rightarrow \neg Q$

How could you get $\neg Q$??

1. $(S \land P) \leftrightarrow Q$	
2. $T \land S$	
3. $T \rightarrow P$	for $\rightarrow$Intro
4. T	$\land$Elim 2
5. S	$\land$Elim 2
6. P	$\rightarrow$Elim 3,4
7. $S \land P$	$\land$Intro 5,6
8. Q	$\leftrightarrow$Elim 1,7
$\neg Q$	
$(T \rightarrow P) \rightarrow \neg Q$	

Invalid Arguments

1. $(S \wedge P) \leftrightarrow Q$
2. $T \wedge S$

3. $T \rightarrow P$

4. T
5. S
6. P
7. $S \wedge P$
8. Q

$\neg Q$

$(T \rightarrow P) \rightarrow \neg Q$

Invalid Arguments

1. $(S \wedge P) \leftrightarrow Q$
2. $T \wedge S$

3. $T \rightarrow P$

4. T
5. S
6. P
7. $S \wedge P$
8. Q

$\neg Q$

$(T \rightarrow P) \rightarrow \neg Q$

A counterexample makes all of the premises true and the conclusion false.

Invalid Arguments

1. $(S \wedge P) \leftrightarrow Q$
2. $T \wedge S$

3. $T \rightarrow P$

4. T
5. S
6. P
7. $S \wedge P$
8. Q

$\neg Q$

$(T \rightarrow P) \rightarrow \neg Q$

A counterexample makes all of the premises true and the conclusion false.

T, S, P, and Q all true makes all the premises true

Invalid Arguments

1. $(S \wedge P) \leftrightarrow Q$
2. $T \wedge S$

3. $T \rightarrow P$

4. T
5. S
6. P
7. $S \wedge P$
8. Q

$\neg Q$

$(T \rightarrow P) \rightarrow \neg Q$

A counterexample makes all of the premises true and the conclusion false.

T, S, P, and Q all true makes all the premises true

and the conclusion false

The Hard Way

=1=	=2=	=3=	=4=	(1)		(2)	(3)		
S	P	Q	T	(S∧P)	↔ Q	T∧S	(T →	P) →	¬Q
T	T	T	T	T	T	T	T	F	F
T	T	T	F	T	T	F	T	F	F
T	T	F	T	T	F	T	T	T	T
T	T	F	F	T	F	F	T	T	T
T	F	T	T	F	F	T	F	T	F
T	F	T	F	F	F	F	T	F	F
T	F	F	T	F	T	T	F	T	T
T	F	F	F	F	T	F	T	T	T
F	T	T	T	F	F	F	T	F	F
F	T	T	F	F	F	F	T	F	F
F	T	F	T	F	T	F	T	T	T
F	T	F	F	F	T	F	T	T	T
F	F	T	T	F	F	F	F	T	F
F	F	T	F	F	F	F	T	F	F
F	F	F	T	F	T	F	F	T	T
F	F	F	F	F	T	F	T	T	T

The Hard Way

=1=	=2=	=3=	=4=	(1)		(2)	(3)		
S	P	Q	T	(S∧P) ↔ Q		T∧S	(T → P) → ¬Q		
T	T	T	T	T	T	T	T	F	F
T	T	T	F	T	T	F	T	F	F
T	T	F	T	T	F	T	T	T	T
T	T	F	F	T	F	F	T	T	T
T	F	T	T	F	F	T	F	T	F
T	F	T	F	F	F	F	T	F	F
T	F	F	T	F	T	T	F	T	T
T	F	F	F	F	T	F	T	T	T
F	T	T	T	F	F	F	T	F	F
F	T	T	F	F	F	F	T	F	F
F	T	F	T	F	T	F	T	T	T
F	T	F	F	F	T	F	T	T	T
F	F	T	T	F	F	F	F	T	F
F	F	T	F	F	F	F	T	F	F
F	F	F	T	F	T	F	F	T	T
F	F	F	F	F	T	F	T	T	T

Notice all premises true, conclusion false

The Short Table Method

The Short Table Method

- To show that a conclusion is a tautological consequence of the premises, producing a proof in $\mathcal{F}_{\mathrm{T}}$ suffices.

The Short Table Method

- To show that a conclusion is a tautological consequence of the premises, producing a proof in $\mathcal{F}_{\mathrm{T}}$ suffices.
- To show that a conclusion is not a tautological consequence of the premises, a truth value assignment (TVA) that makes all of the premises true and the conclusion false at the same time suffices.

The Short Table Method

- To show that a conclusion is a tautological consequence of the premises, producing a proof in $\mathcal{F}_T$ suffices.
- To show that a conclusion is not a tautological consequence of the premises, a truth value assignment (TVA) that makes all of the premises true and the conclusion false at the same time suffices.
- One way of detecting consequence is to assume it is not a consequence and then try to produce such a row. Either you will succeed or see why it is impossible.

The Short Table Method

$P \rightarrow Q$

$P \vee R$

R

Tautologically Valid or not?

THE SHORT TABLE METHOD

$P \rightarrow Q$

$P \vee R$

R

Tautologically Valid or not?

1) If there is a counterexample, it must make the premises true and the conclusion false. Therefore it makes R False.

The Short Table Method

$P \rightarrow Q$

$P \vee R$

R

Tautologically Valid or not?

1) If there is a counterexample, it must make the premises true and the conclusion false. Therefore it makes R False.

2) Since it makes $P \vee R$ True (2nd premise) and R false, it makes P True.

The Short Table Method

$P \rightarrow Q$

$P \vee R$

R

Tautologically Valid or not?

1) If there is a counterexample, it must make the premises true and the conclusion false. Therefore it makes R False.

2) Since it makes $P \vee R$ True (2nd premise) and R false, it makes P True.

3) By premise 1, $P \rightarrow Q$ is True and since this assignment makes P True, it must make Q True.

The Short Table Method

$P \rightarrow Q$

$P \vee R$

R

Tautologically Valid or not?

1) If there is a counterexample, it must make the premises true and the conclusion false. Therefore it makes R False.

2) Since it makes $P \vee R$ True (2nd premise) and R false, it makes P True.

3) By premise 1, $P \rightarrow Q$ is True and since this assignment makes P True, it must make Q True.

Counterexample: R: F, P: T, Q: T

Short Table Method

$P \rightarrow Q$

$P \vee R$

R

Tautologically Valid or not?

Short Table Method

P → Q
P ∨ R
R

Tautologically Valid or not?

If not valid, some row of the truth table looks like this:

P	Q	R	P → Q	P ∨ R	R
			T	T	F

Short Table Method

P Q R	P → Q	P ∨ R	R
	T	T	F

Short Table Method

P Q R	$P \rightarrow Q$	$P \vee R$	R
	T	T	F

Since R false, R: F

Short Table Method

P	Q	R	$P \rightarrow Q$	$P \vee R$	R
		F	T	T	F

Since R false, R: F

Short Table Method

P	Q	R	P → Q	P ∨ R	R
		F	T	T	F

Since R false, R: F

Since P ∨ R true and ¬R, P: T

Short Table Method

P	Q	R	P → Q	P ∨ R	R
T		F	T	T	F

Since R false, R: F

Since P ∨ R true and ¬R, P: T

Short Table Method

P	Q	R	$P \rightarrow Q$	$P \vee R$	R
T		F	T	T	F

Since R false, R: F

Since $P \vee R$ true and $\neg R$, P: T

Since $P \rightarrow Q$ true and P, Q: T

Short Table Method

P	Q	R	$P \rightarrow Q$	$P \vee R$	R
T	T	F	T	T	F

Since R false, R: F

Since $P \vee R$ true and $\neg R$, P:T

Since $P \rightarrow Q$ true and P, Q:T

Short Table Method

P	Q	R	P → Q	P ∨ R	R
T	T	F	T	T	F

Since R false, R: F

Since P ∨ R true and ¬R, P:T

Since P → Q true and P, Q:T

Since this row is on the truth table,
the argument is **invalid**

Short Table Method

$A \rightarrow B$

$A \vee C$

$B \vee D$

Tautologically Valid or not?

Short Table Method

$A \rightarrow B$

$A \vee C$

$B \vee D$

Tautologically Valid or not?

If not valid, some row of the truth table looks like this:

A	B	C	D	$A \rightarrow B$	$A \vee C$	$B \vee D$
				T	T	F

Short Table Method

A B C D	$A \rightarrow B$	$A \vee C$	$B \vee D$
	T	T	F

Short Table Method

A B C D	A → B	A ∨ C	B ∨ D
	T	T	F

Since B∨D false, B: F and D: F

Short Table Method

A	B	C	D	$A \rightarrow B$	$A \vee C$	$B \vee D$
	F		F	T	T	F

Since $B \vee D$ false, B: F and D: F

Short Table Method

A	B	C	D	A → B	A ∨ C	B ∨ D
	F		F	T	T	F

Since B∨D false, B: F and D: F

Since A→B true and ¬B, A: F

Short Table Method

A	B	C	D	$A \rightarrow B$	$A \vee C$	$B \vee D$
F	F		F	T	T	F

Since $B \vee D$ false, B: F and D: F

Since $A \rightarrow B$ true and $\neg B$, A: F

Short Table Method

A	B	C	D	A → B	A ∨ C	B ∨ D
F	F		F	T	T	F

Since B∨D false, B: F and D: F

Since A→B true and ¬B, A: F

Since A∨C true and ¬A, C:T

Short Table Method

A	B	C	D	A → B	A ∨ C	B ∨ D
F	F	T	F	T	T	F

Since B∨D false, B: F and D: F

Since A→B true and ¬B, A: F

Since A∨C true and ¬A, C:T

Short Table Method

A	B	C	D	A → B	A ∨ C	B ∨ D
F	F	T	F	T	T	F

Since B∨D false, B: F and D: F

Since A→B true and ¬B, A: F

Since A∨C true and ¬A, C:T

Since this row is correct, the argument is invalid

Short Table Method

P	Q	R	S	$\neg P \to Q$	$(R \land S) \lor \neg R$	$Q \to \neg R$
				T	T	F

Short Table Method

P Q R S	$\neg P \rightarrow Q$	$(R \wedge S) \vee \neg R$	$Q \rightarrow \neg R$
	T	T	F

Since $Q \rightarrow \neg R$ false, Q: T and $\neg R$: F so R: T

Short Table Method

P	Q	R	S	$\neg P$	$\rightarrow$	Q	$(R$	$\land$	$S)$	$\lor$	$\neg R$	Q	$\rightarrow$	$\neg R$
	T	T			T					T	F		F	F

Since $Q \rightarrow \neg R$ false, Q: T and $\neg R$: F so R: T

Short Table Method

P	Q	R	S	¬P → Q	(R ∧ S) ∨ ¬R	Q → ¬R
	T	T		T	T F	F F

Since Q→¬R false, Q:T and ¬R: F so R:T

Since (R∧S)∨¬R true and R, R∧S:T so R:T and S:T

Short Table Method

P	Q	R	S	¬P	→	Q	(R	∧	S)	∨	¬R	Q	→	¬R
	T	T	T		T			T		T	F		F	F

Since Q→¬R false, Q:T and ¬R: F so R:T

Since (R∧S)∨¬R true and R, R∧S:T so R:T and S:T

Short Table Method

P	Q	R	S	¬P	→	Q	(R	∧	S)	∨	¬R	Q	→	¬R
	T	T	T		T			T		T	F		F	F

Since Q→¬R false, Q:T and ¬R: F so R:T

Since (R∧S)∨¬R true and R, R∧S:T so R:T and S:T

Since ¬P→Q and Q, we know what about ¬P?

Short Table Method

P	Q	R	S	¬P	→	Q	(R	∧	S)	∨	¬R	Q	→	¬R
	T	T	T		T			T		T	F		F	F

Since Q→¬R false, Q:T and ¬R: F so R:T

Since (R∧S)∨¬R true and R, R∧S:T so R:T and S:T

Since ¬P→Q and Q, we know what about ¬P?

It doesn't matter what P is

Short Table Method

P	Q	R	S	¬P → Q	(R ∧ S) ∨ ¬R	Q → ¬R
T/F	T	T	T	T	T T F	F F

Since Q→¬R false, Q:T and ¬R: F so R:T

Since (R∧S)∨¬R true and R, R∧S:T so R:T and S:T

Since ¬P→Q and Q, we know what about ¬P?

It doesn't matter what P is

Short Table Method

P	Q	R	S	$\neg P$	$\to$	Q	$(R$	$\wedge$	$S)$	$\vee$	$\neg R$	Q	$\to$	$\neg R$
T/F	T	T	T		T			T		T	F		F	F

Since $Q \to \neg R$ false, Q:T and $\neg R$: F so R:T

Since $(R \wedge S) \vee \neg R$ true and R, $R \wedge S$:T so R:T and S:T

Since $\neg P \to Q$ and Q, we know what about $\neg P$?

It doesn't matter what P is

Since this row is correct, the argument is invalid

The Short Table Method

$A \leftrightarrow B$

$A \rightarrow \neg B$

$\neg B$

Tautologically Valid or not?

The Short Table Method

$A \leftrightarrow B$

$A \rightarrow \neg B$

$\neg B$

Tautologically Valid or not?

1) If there is a counterexample, it must make the premises true and the conclusion false. Therefore it makes ¬B False so it must make B True.

The Short Table Method

$A \leftrightarrow B$

$A \rightarrow \neg B$

$\neg B$

Tautologically Valid or not?

1) If there is a counterexample, it must make the premises true and the conclusion false. Therefore it makes ¬B False so it must make B True.

2) Since B is true, then by premise 1, A is True.

The Short Table Method

$A \leftrightarrow B$

$A \rightarrow \neg B$

$\neg B$

Tautologically Valid or not?

1) If there is a counterexample, it must make the premises true and the conclusion false. Therefore it makes ¬B False so it must make B True.

2) Since B is true, then by premise 1, A is True.

3) But since this assignment makes A true and also A→ ¬B it true (premise 2), it must make ¬B True and so B False.

THE SHORT TABLE METHOD

$A \leftrightarrow B$

$A \rightarrow \neg B$

$\neg B$

Tautologically Valid or not?

1) If there is a counterexample, it must make the premises true and the conclusion false. Therefore it makes ¬B False so it must make B True.

2) Since B is true, then by premise 1, A is True.

3) But since this assignment makes A true and also A→ ¬B it true (premise 2), it must make ¬B True and so B False.

4) But now we have B is True and B is False. So there can be no such assignment. So the argument is valid.

The Short Table Method

$A \leftrightarrow B$

$A \rightarrow \neg B$

$\neg B$

Tautologically Valid or not?

1) If there is a counterexample, it must make the premises true and the conclusion false. Therefore it makes ¬B False so it must make B True.

2) Since B is true, then by premise 1, A is True.

3) But since this assignment makes A true and also A→ ¬B it true (premise 2), it must make ¬B True and so B False.

4) But now we have B is True and B is False. So there can be no such assignment. So the argument is valid.

Alleged counterexample must have A:T, B:T to get premise 1 True and conc False, but then premise 2 is false.

Short Table Method

A	B	$A \leftrightarrow B$	$A \rightarrow \neg B$	$\neg B$
		T	T	F

Short Table Method

A	B	$A \leftrightarrow B$	$A \rightarrow \neg B$	$\neg B$
		T	T	F

Since $\neg B$ is false, B:T

Short Table Method

A	B	A ↔ B	A → ¬B	¬B
	T	T	T	F

Since ¬B is false, B:T

Short Table Method

A	B	A ↔ B	A → ¬B	¬B
	T	T	T	F

Since ¬B is false, B:T

Since A ↔ B true and B, A:T

Short Table Method

A	B	A ↔ B	A → ¬B	¬B
T	T	T	T	F

Since ¬B is false, B:T

Since A ↔ B true and B, A:T

Short Table Method

A	B	A ↔ B	A → ¬B	¬B
T	T	T	T	F

Since ¬B is false, B:T

Since A ↔ B true and B, A:T

But this row is **NOT** correct. - Look at premise 2

Short Table Method

A	B	A ↔ B	A → ¬B	¬B
T	T	T	T	F

Since ¬B is false, B:T

Since A ↔ B true and B, A:T

But this row is **NOT** correct. - Look at premise 2

There is no way to make a TTF row,
so the argument is Valid

The Short Table Method

$A \wedge B$

$(A \wedge C) \leftrightarrow D$

$B \wedge (D \vee \neg C)$

Tautologically Valid or not?

The Short Table Method

$A \wedge B$

$(A \wedge C) \leftrightarrow D$

$B \wedge (D \vee \neg C)$

Tautologically Valid or not?

If there is a counterexample, it must make the premises true and the conclusion false. Therefore it makes $A \wedge B$ True. So it makes A True and B True. Since it makes $B \wedge (D \vee \neg C)$ False, it must make either B False or $D \vee \neg C$ False. But B is true, so $D \vee \neg C$ must be False. This means that D is False and C is True. But now we have A and C both True and D False which makes premise 2 False. So there can be no such assignment.

The Short Table Method

$A \wedge B$

$(A \wedge C) \leftrightarrow D$

$B \wedge (D \vee \neg C)$

Tautologically Valid or not?

If there is a counterexample, it must make the premises true and the conclusion false. Therefore it makes $A \wedge B$ True. So it makes A True and B True. Since it makes $B \wedge (D \vee \neg C)$ False, it must make either B False or $D \vee \neg C$ False. But B is true, so $D \vee \neg C$ must be False. This means that D is False and C is True. But now we have A and C both True and D False which makes premise 2 False. So there can be no such assignment.

Alleged counterexample must have A:T, B:T, D: F, C: F to get premise 1 True and conc False, but then premise 2 is also false.

Inconsistency and Logical Consequence

Inconsistency and Logical Consequence

- Notice in all these cases, we are trying to get an assignment where all the premises are true and the conclusion is false. If we do it, the argument is invalid. If this is impossible, then the argument is valid.

Inconsistency and Logical Consequence

- Notice in all these cases, we are trying to get an assignment where all the premises are true and the conclusion is false. If we do it, the argument is invalid. If this is impossible, then the argument is valid.

- $\{P_1, P_2, ... P_n, \neg C\}$ is inconsistent (can't all be true at the same time)

  iff

  $\{P_1, P_2, ... P_n\}$ logically entails C

Inconsistency and Logical Consequence

$\{P_1, P_2\}$ logically entails C

Inconsistency and Logical Consequence

$\{P_1, P_2\}$ logically entails C

$=_{def}$ If P_1, P_2 were both true, then C would be true as well

Inconsistency and Logical Consequence

$\{P_1, P_2\}$ logically entails C

$=_{def}$ If P_1, P_2 were both true, then C would be true as well

= No possible way for P_1 and P_2 to be true and C false

Inconsistency and Logical Consequence

$\{P_1, P_2\}$ logically entails C

$=_{def}$ If P_1, P_2 were both true, then C would be true as well

= No possible way for P_1 and P_2 to be true and C false

= No way for P_1 and P_2 and $\neg C$ to be true

Inconsistency and Logical Consequence

$\{P_1, P_2\}$ logically entails C

$=_{def}$ If P_1, P_2 were both true, then C would be true as well

= No possible way for P_1 and P_2 to be true and C false

= No way for P_1 and P_2 and $\neg C$ to be true

$=_{def}$ $\{P_1, P_2, \neg C\}$ is inconsistent

Provability in a Formal System

- If it is possible to prove C from $\{P_1, P_2, ... P_n\}$ using just the truth functional rules we say that:

$$\{P_1, P_2, ... P_n\} \vdash (\text{in } \mathcal{F}_T)\ C$$

Provability in a Formal System

- If it is possible to prove C from $\{P_1, P_2, ... P_n\}$ using just the truth functional rules we say that:

$$\{P_1, P_2, ... P_n\} \vdash (\text{in } \mathcal{F}_T)\ C$$

We also say that $P_1, P_2, ... P_n \vdash C$ is a valid sequent

Connecting Proofs to Truth

Connecting Proofs to Truth

- If we think of subproofs as conditionals, all of the rules represent valid arguments.

Connecting Proofs to Truth

- If we think of subproofs as conditionals, all of the rules represent valid arguments.
- Therefore each of the rules we use is Truth-Preserving. If the assumptions we make are true, then each new line would be true as well.

CONNECTING PROOFS TO TRUTH

- If we think of subproofs as conditionals, all of the rules represent valid arguments.
- Therefore each of the rules we use is Truth-Preserving. If the assumptions we make are true, then each new line would be true as well.

SOUNDNESS THEOREM (for $\mathcal{F}_T$):

If $\{P_1, P_2, \ldots. P_n\} \vdash$ (in $\mathcal{F}_T$) C then
$\{P_1, P_2, \ldots. P_n\}$ tf-entails C

Soundness Theorem

<u>SOUNDNESS THEOREM</u> (for $\mathcal{F}_{\mathrm{T}}$):

If $\{P_1, P_2, ... P_n\} \vdash$(in $\mathcal{F}_{\mathrm{T}}$) C then
$\{P_1, P_2, ... P_n\}$ tf-entails C

Soundness Theorem

SOUNDNESS THEOREM (for $\mathcal{F}_T$):

If $\{P_1, P_2, ... P_n\} \vdash$(in $\mathcal{F}_T$) C then
$\{P_1, P_2, ... P_n\}$ tf-entails C

Think Contrapositively

If $\{P_1, P_2, ... P_n\}$ DOES NOT tf-entail C then
$\{P_1, P_2, ... P_n\} \nvdash$(in $\mathcal{F}_T$) C

Soundness Theorem

SOUNDNESS THEOREM (for $\mathcal{F}_T$):

If $\{P_1, P_2, ... P_n\} \vdash$(in $\mathcal{F}_T$) C then
$\{P_1, P_2, ... P_n\}$ tf-entails C

Think Contrapositively

If $\{P_1, P_2, ... P_n\}$ DOES NOT tf-entail C then
$\{P_1, P_2, ... P_n\} \nvdash$(in $\mathcal{F}_T$) C

Therefore a falsifying assignment
shows that you can't do a proof

Proofs and Counterexamples

Proofs and Counterexamples

- Valid arguments have no counterexamples - so if you find a counterexample, the argument is definitely invalid. So by Soundness, you can't do a proof (in $\mathcal{F}_{\mathrm{T}}$).

Proofs and Counterexamples

- Valid arguments have no counterexamples - so if you find a counterexample, the argument is definitely invalid. So by Soundness, you can't do a proof (in $\mathcal{F}_{\mathrm{T}}$).
- You can't both do a proof and find a counterexample. But maybe you could do neither? Is it possible that there is an argument that you can't produce a counterexample for and that you can't do a proof for?

PROOFS AND COUNTEREXAMPLES

- Valid arguments have no counterexamples - so if you find a counterexample, the argument is definitely invalid. So by Soundness, you can't do a proof (in $\mathcal{F}_T$).
- You can't both do a proof and find a counterexample. But maybe you could do neither? Is it possible that there is an argument that you can't produce a counterexample for and that you can't do a proof for?
- NO if the argument is t-f valid. (Yes/(maybe?) in general)

Completeness Theorem

Completeness Theorem

- As a matter of fact, the converse of soundness is true - if an argument is tf-valid, then you can do a proof in $\mathcal{F}_T$.

COMPLETENESS THEOREM

- As a matter of fact, the converse of soundness is true - if an argument is tf-valid, then you can do a proof in $\mathcal{F}_{\mathrm{T}}$.
- This is much harder to prove [take 3310 or read chapter 17]. But you can just assume it is true.

Completeness Theorem

- As a matter of fact, the converse of soundness is true - if an argument is tf-valid, then you can do a proof in $\mathcal{F}_{\mathrm{T}}$.
- This is much harder to prove [take 3310 or read chapter 17]. But you can just assume it is true.
- Since $\mathcal{F}_{\mathrm{T}}$ is sound and complete, you can prove all and only the tf-valid arguments. Many other systems of natural deduction have this same quality.