Lady or the Tiger?

(puzzle book by Raymund Smullyan)

There are two doors. Behind each is a lady or a tiger (not both). Each door has a statement on it. If a lady is in room 1, that door's statement is true, otherwise it is false. If a lady is in room 2, that door's statement is false, otherwise it is true. - Try to figure out which door has the lady (to save her!)

<u>Door 1</u>

It makes no difference which door you pick

<u>Door 2</u>

There is a lady in the other room

Practice using Taut Con

Monday, 3 March

Pushing Negations Inside

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DeMorgan's Laws

$\neg(P \vee Q) \Leftrightarrow (\neg P \wedge \neg Q)$

$\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$

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Negated Conditional

$\neg(P \rightarrow Q) \Leftrightarrow (P \wedge \neg Q)$

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With repeated applications of these rules, we can convert any sentence with main connective ¬ into something with a different main connective.

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Negated Biconditional

$\neg(P \leftrightarrow Q) \Leftrightarrow (\neg P \leftrightarrow Q)$

With repeated applications of these rules, we can convert any sentence with main connective ¬ into something with a different main connective.

Or get rid of any particular connectives that we don't like

The Taut Con Rule

- If some sentence C really is a tautological consequence of some other sentences P_1, P_2, ... the rule Taut Con allows you to infer C from P_1, P_2, ...

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Example

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2. $\neg Q$

---

3. $\neg P$ Taut Con 1,2

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(Modus Tollens)

Example

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2. $\neg Q$

---

3. P Taut Con 1,2

(Disjunctive Syllogism)

The Taut Con Rule (in our class)

- On <u>future</u> homeworks (homeworks 6-9) I will allow you to use Taut Con for any step I consider to be sufficiently obvious. It will be useful to use for the following cases:

1. $P \rightarrow Q$
2. $\neg Q$

---

3. $\neg P$ Modus Tollens

1. $P \vee Q$
2. $\neg Q$

---

3. P DS

1. $P \leftrightarrow Q$
2. $\neg P$

---

3. $\neg Q$ Biconditional

The Taut Con Rule (in our class)

- Or to replace one sentence with something equivalent to it

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- Or to replace one sentence with something equivalent to it

DeMorgan's Laws

$\neg(P \vee Q) \Leftrightarrow (\neg P \wedge \neg Q)$

$\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$

Conditionals

$\neg(P \rightarrow Q) \Leftrightarrow (P \wedge \neg Q)$

$(P \rightarrow Q) \Leftrightarrow (\neg P \vee Q)$

The Taut Con Rule (in our class)

- Or to replace one sentence with something equivalent to it

DeMorgan's Laws

$\neg(P \vee Q) \Leftrightarrow (\neg P \wedge \neg Q)$

$\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$

Biconditionals

$\neg(P \leftrightarrow Q) \Leftrightarrow (\neg P \leftrightarrow Q)$

$(P \leftrightarrow Q) \Leftrightarrow (\neg P \leftrightarrow \neg Q)$

Conditionals

$\neg(P \rightarrow Q) \Leftrightarrow (P \wedge \neg Q)$

$(P \rightarrow Q) \Leftrightarrow (\neg P \vee Q)$

Problem using Taut Con

$S \vee P$

$S \rightarrow R$

$\neg R \rightarrow P$

Problem using Taut Con

$S \vee P$

$S \rightarrow R$

$\neg R \rightarrow P$

1. $S \vee P$
2. $S \rightarrow R$

$\neg R \rightarrow P$

Problem using Taut Con

S ∨ P

S→R

¬R→P

1. S ∨ P
2. S→R

3. ¬R — for →Intro

P

¬R→P — →Intro

Problem using Taut Con

S ∨ P

S→R

¬R→P

1. S ∨ P
2. S→R

3. ¬R for →Intro

4. ¬S Taut Con 2,3 (MT)

P

¬R→P →Intro

Problem using Taut Con

S ∨ P

S→R

¬R→P

1. S ∨ P
2. S→R

3. ¬R for →Intro

4. ¬S Taut Con 2,3 (MT)

5. P Taut Con 1,4 (DS)

¬R→P →Intro

Problem using Taut Con

S ∨ P

S→R

¬R→P

1. S ∨ P
2. S→R

3. ¬R for →Intro

4. ¬S Taut Con 2,3 (MT)

5. P Taut Con 1,4 (DS)

6. ¬R→P →Intro 3-5

Double ¬Intro (6.30)

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1. ¬(¬Cube(a) ∧ Cube(b))
2. ¬(¬Cube(b) ∨ Cube(c))

Cube(a)

Double ¬Intro (6.30)

1. ¬(¬Cube(a) ∧ Cube(b))
2. ¬(¬Cube(b) ∨ Cube(c))

   3. ¬Cube(a) for ¬ Intro

      Cube(a) ¬ Intro

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   - 3. ¬Cube(a) — for ¬ Intro
     - 4. Cube(b) — for ¬ Intro
     - 5. ¬Cube(a) ∧ Cube(b) ∧ Intro 3,4

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     - 4. Cube(b) — for ¬ Intro
     - 5. ¬Cube(a) ∧ Cube(b) — ∧ Intro 3,4
     - 6. ⊥ — ⊥ Intro 1,5
   - Cube(a) — ¬ Intro

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   - 3. ¬Cube(a) — for ¬ Intro
     - 4. Cube(b) — for ¬ Intro
     - 5. ¬Cube(a) ∧ Cube(b) — ∧ Intro 3,4
     - 6. ⊥ — ⊥ Intro 1,5
   - 7. ¬Cube(b) — ¬ Intro 4-6

Cube(a) — ¬ Intro

Double ¬Intro (6.30)

1. ¬(¬Cube(a) ∧ Cube(b))
2. ¬(¬Cube(b) ∨ Cube(c))
   - 3. ¬Cube(a) — for ¬ Intro
     - 4. Cube(b) — for ¬ Intro
       - 5. ¬Cube(a) ∧ Cube(b) — ∧ Intro 3,4
       - 6. ⊥ — ⊥ Intro 1,5
     - 7. ¬Cube(b) — ¬ Intro 4-6
     - 8. ¬Cube(b) ∨ Cube(c) — ∨ Intro 7
   - Cube(a) — ¬ Intro

Double ¬Intro (6.30)

1. ¬(¬Cube(a) ∧ Cube(b))
2. ¬(¬Cube(b) ∨ Cube(c))
   - 3. ¬Cube(a) — for ¬ Intro
     - 4. Cube(b) — for ¬ Intro
       - 5. ¬Cube(a) ∧ Cube(b) — ∧ Intro 3,4
       - 6. ⊥ — ⊥ Intro 1,5
     - 7. ¬Cube(b) — ¬ Intro 4-6
     - 8. ¬Cube(b) ∨ Cube(c) — ∨ Intro 7
     - 9. ⊥ — ⊥ Intro 2,8
   - Cube(a) — ¬ Intro

Double ¬Intro (6.30)

1. ¬(¬Cube(a) ∧ Cube(b))
2. ¬(¬Cube(b) ∨ Cube(c))
   - 3. ¬Cube(a) — for ¬ Intro
     - 4. Cube(b) — for ¬ Intro
       - 5. ¬Cube(a) ∧ Cube(b) — ∧ Intro 3,4
       - 6. ⊥ — ⊥ Intro 1,5
     - 7. ¬Cube(b) — ¬ Intro 4-6
     - 8. ¬Cube(b) ∨ Cube(c) — ∨ Intro 7
     - 9. ⊥ — ⊥ Intro 2,8
   - 10. Cube(a) — ¬ Intro 3-8

6.30 with Taut Con

1. ¬(¬Cube(a) ∧ Cube(b))
2. ¬(¬Cube(b) ∨ Cube(c))

Cube(a)

6.30 with Taut Con

1. ¬(¬Cube(a) ∧ Cube(b))
2. ¬(¬Cube(b) ∨ Cube(c))

---

3. Cube(a) ∨ ¬Cube(b)) Taut Con 1 (DeM)

Cube(a)

6.30 with Taut Con

1. ¬(¬Cube(a) ∧ Cube(b))
2. ¬(¬Cube(b) ∨ Cube(c))

---

3. Cube(a) ∨ ¬Cube(b)) Taut Con 1 (DeM)

4. Cube(b) ∧ ¬Cube(c) Taut Con 2 (DeM)

Cube(a)

6.30 with Taut Con

1. ¬(¬Cube(a) ∧ Cube(b))
2. ¬(¬Cube(b) ∨ Cube(c))

---

3. Cube(a) ∨ ¬Cube(b)) Taut Con 1 (DeM)
4. Cube(b) ∧ ¬Cube(c) Taut Con 2 (DeM)
5. Cube(b) ∧ Elim 4

Cube(a)

6.30 with Taut Con

1. ¬(¬Cube(a) ∧ Cube(b))
2. ¬(¬Cube(b) ∨ Cube(c))

---

3. Cube(a) ∨ ¬Cube(b)) Taut Con 1 (DeM)
4. Cube(b) ∧ ¬Cube(c) Taut Con 2 (DeM)
5. Cube(b) ∧ Elim 4

6. Cube(a) Taut Con 3,5 (DS)

Problem using Taut Con

$(P \rightarrow Q) \rightarrow P$

P

Problem using Taut Con

$(P \rightarrow Q) \rightarrow P$

P

1. $(P \rightarrow Q) \rightarrow P$

P

Problem using Taut Con

(P→Q) → P

P

1. (P→Q) → P

2. ¬P for ¬ Intro

P

Problem using Taut Con

(P→Q) → P

P

1. (P→Q) → P

2. ¬P for ¬ Intro

3. ¬(P→Q) TautCon 1,2 (MT)

P

Problem using Taut Con

(P→Q) → P

P

1. (P→Q) → P

2. ¬P for ¬ Intro

3. ¬(P→Q) TautCon 1,2 (MT)

4. P∧ ¬Q) TautCon 3 (NegCon)

P

Problem using Taut Con

(P→Q) → P

P

1. (P→Q) → P

2. ¬P for ¬ Intro

3. ¬(P→Q) TautCon 1,2 (MT)

4. P∧ ¬Q) TautCon 3 (NegCon)

5. P ∧ Elim 4

P

Problem using Taut Con

(P→Q) → P

P

1. (P→Q) → P

2. ¬P for ¬ Intro

3. ¬(P→Q) TautCon 1,2 (MT)

4. P∧ ¬Q) TautCon 3 (NegCon)

5. P ∧ Elim 4

6. ⊥ ⊥ Intro 2,5

P

Problem using Taut Con

(P→Q) → P

P

1. (P→Q) → P

2. ¬P for ¬ Intro

3. ¬(P→Q) TautCon 1,2 (MT)

4. P∧ ¬Q) TautCon 3 (NegCon)

5. P ∧ Elim 4

6. ⊥ ⊥ Intro 2,5

7. P ¬ Intro 2-6

Problem using Taut Con

$S \vee (P \leftrightarrow Q)$

$S \rightarrow R$

$P \vee (Q \rightarrow R)$

Problem using Taut Con

$S \vee (P \leftrightarrow Q)$

$S \rightarrow R$

$P \vee (Q \rightarrow R)$

1. $S \vee (P \leftrightarrow Q)$
2. $S \rightarrow R$

$P \vee (Q \rightarrow R)$

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2. $S \rightarrow R$

$P \vee (Q \rightarrow R)$

1. $S \vee (P \leftrightarrow Q)$
2. $S \rightarrow R$

   3. $\neg(P \vee (Q \rightarrow R))$ for ¬I

$P \vee (Q \rightarrow R)$

1. $S \vee (P \leftrightarrow Q)$
2. $S \rightarrow R$

   3. $\neg(P \vee (Q \rightarrow R))$ for ¬I

      4. $\neg P \wedge \neg(Q \rightarrow R)$ DeMorgans 3

$P \vee (Q \rightarrow R)$

1. $S \vee (P \leftrightarrow Q)$
2. $S \rightarrow R$

   3. $\neg(P \vee (Q \rightarrow R))$ for ¬I

      4. $\neg P \wedge \neg(Q \rightarrow R)$ DeMorgans 3
      5. $\neg P$ ∧Elim 4

$P \vee (Q \rightarrow R)$

1. $S \vee (P \leftrightarrow Q)$
2. $S \rightarrow R$

   3. $\neg(P \vee (Q \rightarrow R))$ for ¬I

      4. $\neg P \wedge \neg(Q \rightarrow R)$ DeMorgans 3
      5. $\neg P$ ∧Elim 4
      6. $\neg(Q \rightarrow R)$ ∧Elim 4

$P \vee (Q \rightarrow R)$

1. S ∨ (P↔Q)
2. S→R

3. ¬(P ∨ (Q→R)) for ¬I

4. ¬P ∧ ¬(Q→R) DeMorgans 3
5. ¬P ∧Elim 4
6. ¬(Q→R) ∧Elim 4
7. Q∧¬R NegCon 6

P ∨ (Q→R)

1. $S \vee (P \leftrightarrow Q)$
2. $S \rightarrow R$

---

3. $\neg(P \vee (Q \rightarrow R))$ for ¬I

---

4. $\neg P \wedge \neg(Q \rightarrow R)$ DeMorgans 3
5. $\neg P$ ∧Elim 4
6. $\neg(Q \rightarrow R)$ ∧Elim 4
7. $Q \wedge \neg R$ NegCon 6
8. Q ∧Elim 7

$P \vee (Q \rightarrow R)$

1. $S \vee (P \leftrightarrow Q)$
2. $S \rightarrow R$

3. $\neg(P \vee (Q \rightarrow R))$	for ¬I	
4. $\neg P \wedge \neg(Q \rightarrow R)$	DeMorgans 3	
5. $\neg P$	∧Elim 4	
6. $\neg(Q \rightarrow R)$	∧Elim 4	
7. $Q \wedge \neg R$	NegCon 6	
8. Q	∧Elim 7	
9. $\neg R$	∧Elim 7	

$P \vee (Q \rightarrow R)$

1. $S \vee (P \leftrightarrow Q)$
2. $S \rightarrow R$

---

3. $\neg(P \vee (Q \rightarrow R))$ for ¬I

---

4. $\neg P \wedge \neg(Q \rightarrow R)$ DeMorgans 3
5. $\neg P$ ∧Elim 4
6. $\neg(Q \rightarrow R)$ ∧Elim 4
7. $Q \wedge \neg R$ NegCon 6
8. Q ∧Elim 7
9. $\neg R$ ∧Elim 7
10. $\neg S$ Modus Tollens 2,9

$P \vee (Q \rightarrow R)$

1. $S \vee (P \leftrightarrow Q)$
2. $S \rightarrow R$

---

3. $\neg(P \vee (Q \rightarrow R))$ for ¬I

---

4. $\neg P \wedge \neg(Q \rightarrow R)$ DeMorgans 3
5. $\neg P$ ∧Elim 4
6. $\neg(Q \rightarrow R)$ ∧Elim 4
7. $Q \wedge \neg R$ NegCon 6
8. Q ∧Elim 7
9. $\neg R$ ∧Elim 7
10. $\neg S$ Modus Tollens 2,9
11. $P \leftrightarrow Q$ Disjunctive Syllogism 1,10

$P \vee (Q \rightarrow R)$

1. $S \vee (P \leftrightarrow Q)$
2. $S \rightarrow R$

---

3. $\neg(P \vee (Q \rightarrow R))$ for ¬I

---

4. $\neg P \wedge \neg(Q \rightarrow R)$ DeMorgans 3
5. $\neg P$ ∧Elim 4
6. $\neg(Q \rightarrow R)$ ∧Elim 4
7. $Q \wedge \neg R$ NegCon 6
8. Q ∧Elim 7
9. $\neg R$ ∧Elim 7
10. $\neg S$ Modus Tollens 2,9
11. $P \leftrightarrow Q$ Disjunctive Syllogism 1,10
12. P ↔Elim 8,11

$P \vee (Q \rightarrow R)$

1. S ∨ (P↔Q)
2. S→R

---

3. ¬(P ∨ (Q→R)) for ¬I

---

4. ¬P ∧ ¬(Q→R) DeMorgans 3
5. ¬P ∧Elim 4
6. ¬(Q→R) ∧Elim 4
7. Q∧¬R NegCon 6
8. Q ∧Elim 7
9. ¬R ∧Elim 7
10. ¬S Modus Tollens 2,9
11. P↔Q Disjunctive Syllogism 1,10
12. P ↔Elim 8,11
13. ⊥ ⊥ Intro 5,12

P ∨ (Q→R)

Red text for Fitch

1. $S \vee (P \leftrightarrow Q)$			
2. $S \rightarrow R$			
3. $\neg(P \vee (Q \rightarrow R))$	for ¬I		
4. $\neg P \wedge \neg(Q \rightarrow R)$	Taut Con 3	DeMorgans 3	
5. $\neg P$	∧Elim 4		
6. $\neg(Q \rightarrow R)$	∧Elim 4		
7. $Q \wedge \neg R$	Taut Con 6	NegCon 6	
8. Q	∧Elim 7		
9. $\neg R$	∧Elim 7		
10. $\neg S$	Taut Con 2,9	Modus Tollens 2,9	
11. $P \leftrightarrow Q$	Taut Con 1,10	Disjunctive Syllogism	1,10
12. P	↔Elim 11,12		
13. $\perp$	$\perp$ Intro 5,12		
14. $P \vee (Q \rightarrow R)$	¬Intro 3-13		

Harder Proofs (even with Taut Con)

$$\frac{P \leftrightarrow (Q \leftrightarrow R)}{(P \leftrightarrow Q) \leftrightarrow R}$$

$$\frac{}{(P \leftrightarrow Q) \vee (P \leftrightarrow R) \vee (Q \leftrightarrow R)}$$

Harder Proofs (even with Taut Con)

$P \leftrightarrow (Q \leftrightarrow R)$

$\vdash$

$(P \leftrightarrow Q) \leftrightarrow R$

$\vdash$

$(P \leftrightarrow Q) \vee (P \leftrightarrow R) \vee (Q \leftrightarrow R)$

True because there are only two truth values

Harder Proofs (even with Taut Con)

$P \leftrightarrow (Q \leftrightarrow R)$

$\vdash$

$(P \leftrightarrow Q) \leftrightarrow R$

Premise and conclusion are each true iff one or three of P,Q,R are true

$\vdash$

$(P \leftrightarrow Q) \vee (P \leftrightarrow R) \vee (Q \leftrightarrow R)$

True because there are only two truth values