PROOFS WITH CONDITIONALS

Monday, 24 February

CONDITIONAL ELIMINATION

• \rightarrow Elimination: from P \rightarrow Q and P, we can infer Q.

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CONDITIONAL ELIMINATION

• \rightarrow Elimination: from P \rightarrow Q and P, we can infer Q. 1. P \rightarrow Q 2. P 3. Q \rightarrow Elim: 1,2

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CONDITIONAL ELIMINATION

• \rightarrow Elimination: from P \rightarrow Q and P, we can infer Q. 1. P \rightarrow Q 2. P 3. Q \rightarrow Elim: 1,2

 $1. (A \lor B) \rightarrow (C \land D)$ 2. A \times B 3. C \leftarrow D \leftarrow Elim: 1,2



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Example: |A| $|A \rightarrow B|$ $|B \rightarrow C|$ |C|



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Example: |A| $A \rightarrow B$ $B \rightarrow C$ C

I.A 2.A \rightarrow B 3.B \rightarrow C



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Example: |A| $A \rightarrow B$ $B \rightarrow C$ C

I.A $2.A \rightarrow B$ $3.B \rightarrow C$ 4. B





1.4.4

Example: |A| $A \rightarrow B$ $B \rightarrow C$ C

I.A $2.A \rightarrow B$ $3.B \rightarrow C$ 4. B 5.C

 \rightarrow Elim 1,2 \rightarrow Elim 3,4



CANADARA CANTRE ST.

Example: $| P \land Q$ $(P \lor R) \rightarrow S$ $(S \land Q) \rightarrow T$ T



 $I. P \land Q$ 2. (P \lapha R) \rightarrow S 3. (S \lapha Q) \rightarrow T



 $I. P \land Q$ $2. (P \lor R) \rightarrow S$ $3. (S \land Q) \rightarrow T$ $4. P \land Elim I$

Monday, February 24, 2014

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 $I. P \land Q$ $2. (P \lor R) \rightarrow S$ $3. (S \land Q) \rightarrow T$ $4. P \land Elim I$ $5. Q \land Elim I$



 $I.P \land Q$ $2. (P \lor R) \rightarrow S$ $3. (S \land Q) \rightarrow T$ $4. P \land Elim I$ $5. Q \land Elim I$ $6. P \lor R \lor Intro 4$



 $I. P \land Q$ $2. (P \lor R) \rightarrow S$ $3. (S \land Q) \rightarrow T$ $4. P \land Elim I$ $5. Q \land Elim I$ $6. P \lor R \lor Intro 4$ $7. S \rightarrow Elim 2,6$



 $I.P \land Q$ 2. $(P \lor R) \rightarrow S$ 3. $(S \land Q) \rightarrow T$ 4. P ∧ Elim I 5. Q ∧ Elim I 6. $P \vee R$ \vee Intro 4 7. S \rightarrow Elim 2,6 8. $S \land Q \land Intro 5,7$



 $I.P \land Q$ 2. $(P \lor R) \rightarrow S$ 3. $(S \land Q) \rightarrow T$ 4. P ∧ Elim I 5. Q ∧ Elim I 6. $P \vee R$ \vee Intro 4 7. S \rightarrow Elim 2,6 8. $S \land Q$ ∧ Intro 5,7 9. T \rightarrow Elim 3,8

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the Louis and States a contract of the

 $I.P \rightarrow Q$ 2.P 3.Q \rightarrow Elim: 1,2

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2. P	
3. Q	\rightarrow Elim: 1,2

 $I.P \rightarrow Q$ 2. ¬P 3. ¬Q



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 $I.P \rightarrow Q$ 2.P 3.Q \rightarrow Elim: 1,2 $I.P \rightarrow Q$ 2. ¬P 3. ¬Q

INVALID

 $I.P \rightarrow Q$ 2. Q 3. P INVALID

- $1. P \rightarrow Q$ 2. P $3. Q \rightarrow Elim: 1,2$
- $I.P \rightarrow Q$ 2. Q 3. P INVALID

 $I.P \rightarrow Q$ 2. ¬P 3. ¬Q INVALID $I.P \rightarrow Q$ $2. \neg Q$ 3. $\neg P$ VALID, but not $\rightarrow E$

 $1.P \rightarrow Q$ 2.P 3.Q \rightarrow Elim: 1,2 $I.P \rightarrow Q$ 2. ¬Q 3. ¬P VALID, but not →E

Modus Ponens

Modus Tollens

A Marine Landerson Marine Strate



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 $I.P \rightarrow Q$ $2. \neg Q$

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A Charles I and the second of the state



 $I.P \rightarrow Q$ $2. \neg Q$ $3.P \quad \text{for } \text{Intro}$ $\neg P \quad \neg \text{Intro}$

And states the second states and a state of the

Example: $I.P \rightarrow Q$ $2. \neg Q$ $3. \neg P$

 $I.P \rightarrow Q$ $2. \neg Q$ 4. Q $\neg P$

for \neg Intro \rightarrow Elim 1,3

¬ Intro

COLUMN STATES A PARTY OF THE

Example: $I.P \rightarrow Q$ $2. \neg Q$ $3. \neg P$

 $I.P \rightarrow Q$ $2. \neg Q$ 3.P 4.Q $5. \bot$ $\neg P$

for \neg Intro \rightarrow Elim 1,3 \bot Intro 2,4 \neg Intro

Example: $I.P \rightarrow Q$ $2. \neg Q$ $3. \neg P$

 $I.P \rightarrow Q$ $2. \neg Q$ 3.P 4.Q $5. \bot$ $6. \neg P$

for \neg Intro \rightarrow Elim 1,3 \perp Intro 2,4 \neg Intro 3-5

BICONDITIONAL ELIMINATION

• \leftrightarrow Elimination: from P \leftrightarrow Q and P/Q, we can infer Q/P.

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BICONDITIONAL ELIMINATION

• \leftrightarrow Elimination: from P \leftrightarrow Q and P/Q, we can infer Q/P.

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 $1. P \leftrightarrow Q$ 2. P $3. Q \qquad \leftrightarrow Elim: 1,2$

BICONDITIONAL ELIMINATION

• \leftrightarrow Elimination: from P \leftrightarrow Q and P/Q, we can infer Q/P.

 $1. P \leftrightarrow Q$ 2. P $3. Q \qquad \leftrightarrow Elim: 1,2$

$$\begin{array}{ccc}
I. P \leftrightarrow Q \\
2. Q \\
3. P \qquad \leftrightarrow \text{Elim: } 1,2
\end{array}$$



the same state should all the states and the states of the

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Example: |A| $A \leftrightarrow B$ $\neg C \leftrightarrow B$ $\neg C$



1.4.4

Example: |A| $A \leftrightarrow B$ $\neg C \leftrightarrow B$ $\neg C$

I.A 2.A \leftrightarrow B 3. \neg C \leftrightarrow B



1.4.

Example: |A| $A \leftrightarrow B$ $\neg C \leftrightarrow B$ $\neg C$

I.A 2.A \leftrightarrow B 3. \neg C \leftrightarrow B 4. B

↔ Elim 1,2



Example: |A| $A \leftrightarrow B$ $\neg C \leftrightarrow B$ $\neg C$

I.A 2.A \leftrightarrow B 3. \neg C \leftrightarrow B 4. B \leftrightarrow 5. \neg C \leftrightarrow \leftrightarrow

 $\leftrightarrow \text{Elim 1,2} \\ \leftrightarrow \text{Elim 3,4}$

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Example: $P \lor Q$ $P \rightarrow R$ $Q \rightarrow R$ R

Example: $P \lor Q$ $P \rightarrow R$ $Q \rightarrow R$ R $I.P \lor Q$ 2. P \rightarrow R 3. Q \rightarrow R

Example: $P \lor Q$ $P \rightarrow R$ $Q \rightarrow R$ R

 $I.P \lor Q$ 2. $P \rightarrow R$ $3.Q \rightarrow R$ 4. P for vElim R for vElim Q R R ∨Elim 1, 4-...

Example: $P \lor Q$ $P \rightarrow R$ $Q \rightarrow R$ R

 $I.P \vee Q$ 2. $P \rightarrow R$ $3.Q \rightarrow R$ **4**. P for vElim 5. R \rightarrow Elim 2,4 6.Q for ∨Elim 7. R \rightarrow Elim 3,6 8. R ∨Elim 1, 4-5, 6-7

6-7

Example:	$ I. P \lor Q 2 P \rightarrow R$	
$P \lor Q$	$3.Q \rightarrow R$	
$P \rightarrow R$ $Q \rightarrow R$	4. P	for ∨Elim
R	5. R	→ Elim 2,4
rom P, Proof from	6. Q	for ∨Elim
P to R	7. R	→ Elim 3,6
	8. R	∨Elim 1, 4-5,

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The conditional P→R says that if P is true, then R is also true

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 A subproof from P to R shows that if P were true, then R would also be true.

- The conditional P→R says that if P is true, then R is also true
- A subproof from P to R shows that if P were true, then R would also be true.
- In \mathcal{F} , the disjunction elimination rule (\lor Elim case argument) uses a disjunction and two subproofs. In many other systems (and on Wikipedia for example) you need the disjunction and two conditionals

VElim (alternate version)

Example: $I \cdot P \lor Q$ $2 \cdot P \rightarrow R$ $3 \cdot Q \rightarrow R$ $4 \cdot R \lor Elim 1, 2, 3$

VElim (alternate version)

Example: $I.P \lor Q$ $2.P \rightarrow R$ $3.Q \rightarrow R$ $4.R \lor Elim 1, 2, 3$

The missing subproofs just are the conditionals

- The conditional P→R says that if P is true, then R is also true
- A subproof from P to R shows that if P were true, then R would also be true.
- The Introduction rule for the conditional says that to prove that a conditional P→R is true, assume P and show that it does lead to R

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● <u>→ Introduction</u>

From a subproof from P to Q, we can infer $P \rightarrow Q$.

● → Introduction

From a subproof from P to Q, we can infer $P \rightarrow Q$.

| I.P... j.Q k.P \rightarrow Q \rightarrow Intro: I-j

● → Introduction

From a subproof from P to Q, we can infer $P \rightarrow Q$.



This rule is often known as Conditional Proof

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 $I. P \rightarrow Q$ $2. Q \rightarrow R$

 $P \rightarrow R$

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 $I.P \rightarrow Q$ $2.Q \rightarrow R$ 3. P R $P \rightarrow R$ →Intro



Example:

 $P \rightarrow Q$ $Q \rightarrow R$ $P \rightarrow R$

 $I.P \rightarrow Q$ $2.Q \rightarrow R$ 3.P 4.Q R R $P \rightarrow R$

for →Intro → Elim 1,3

→Intro

Example:

 $P \rightarrow Q$ $Q \rightarrow R$ $P \rightarrow R$

 $I.P \rightarrow Q$ $2.Q \rightarrow R$ 3.P 4.Q 5.R $P \rightarrow R$

for \rightarrow Intro \rightarrow Elim 1,3 \rightarrow Elim 2,4 \rightarrow Intro

Example:

 $P \rightarrow Q$ $Q \rightarrow R$ $P \rightarrow R$

 $I.P \rightarrow Q$ $2.Q \rightarrow R$ 3.P 4.Q 5.R $6.P \rightarrow R$

for \rightarrow Intro \rightarrow Elim 1,3 \rightarrow Elim 2,4 \rightarrow Intro 3-5





ANTRI MAL MATRI

- Example:
 - $P \rightarrow (Q \rightarrow R)$ $(P \land Q) \rightarrow R$

Example:

 $P \rightarrow (Q \rightarrow R)$ $(P \land Q) \rightarrow R$

 $I.P \rightarrow (Q \rightarrow R)$

 $(P \land Q) \rightarrow R$

Example:

 $P \rightarrow (Q \rightarrow R)$ $(P \land Q) \rightarrow R$

 $I. P \rightarrow (Q \rightarrow R)$ $2. P \land Q \qquad \text{for } \rightarrow \text{Intro}$

| R $(P \land Q) \rightarrow R \rightarrow Intro$

Example:

 $P \rightarrow (Q \rightarrow R)$ $(P \land Q) \rightarrow R$

 $I.P \rightarrow (Q \rightarrow R)$ $2.P \land Q \quad \text{for } \rightarrow \text{Intro}$ $3.P \quad \land \text{Elim 2}$ R

 $(P \land Q) \rightarrow R \rightarrow Intro$

Example:

 $P \rightarrow (Q \rightarrow R)$ $(P \land Q) \rightarrow R$

 $I.P \rightarrow (Q \rightarrow R)$ 2. $P \land Q$ for \rightarrow Intro 3. P 4. Q ∧ Elim 2 \wedge Elim 2 R $(P \land Q) \rightarrow R \rightarrow Intro$

Example:

 $P \rightarrow (Q \rightarrow R)$ $(P \land Q) \rightarrow R$

 $I.P \rightarrow (Q \rightarrow R)$ 2. $P \land Q$ for \rightarrow Intro 4. Q \wedge Elim 2 5. Q \rightarrow R \rightarrow Elim 1,3 | R $(P \land Q) \rightarrow R \rightarrow Intro$

Example:

 $P \rightarrow (Q \rightarrow R)$ -- $(P \land Q) \rightarrow R$

 $I.P \rightarrow (Q \rightarrow R)$ 2. $P \land Q$ for \rightarrow Intro 4. Q ^ Elim 2 5. $Q \rightarrow R \rightarrow \text{Elim 1,3}$ $\begin{array}{ccc} & 6. & R & \rightarrow & \text{Elim } 4,5 \\ \hline 7. & (P \land Q) \rightarrow & R & \rightarrow & \text{Intro } 2-6 \end{array}$