REVIEW DAY I (TRANSLATIONS, TABLES, AND COUNTEREXAMPLES)

Monday, 17 February

A Charles and the state of the state of the state

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a=b → (Cube(a) ∧ Cube(b))

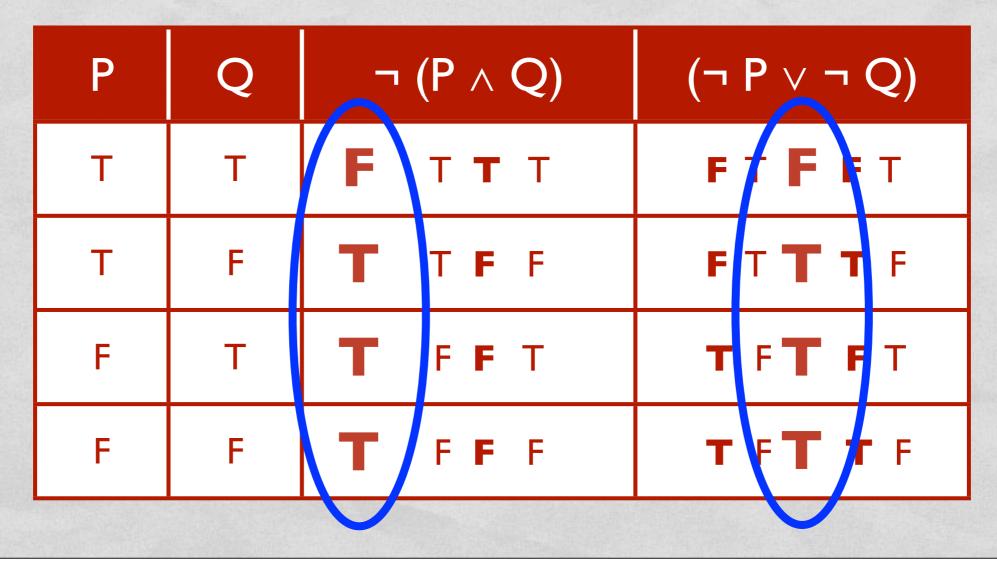
TRUTH TABLES

• Example: joint truth table for $\neg(P \land Q)$ and $(\neg P \lor \neg Q)$ This shows that the two sentences are equivalent.

Р	Q	¬ (P ∧ Q)	(¬ P ∨ ¬ Q)
Т	Т	Γ Τ Τ Τ	FTFFT
Т	F	TTFF	БТТТ
F	Т	TFFT	T F T F T
F	F	TFFF	тғттғ

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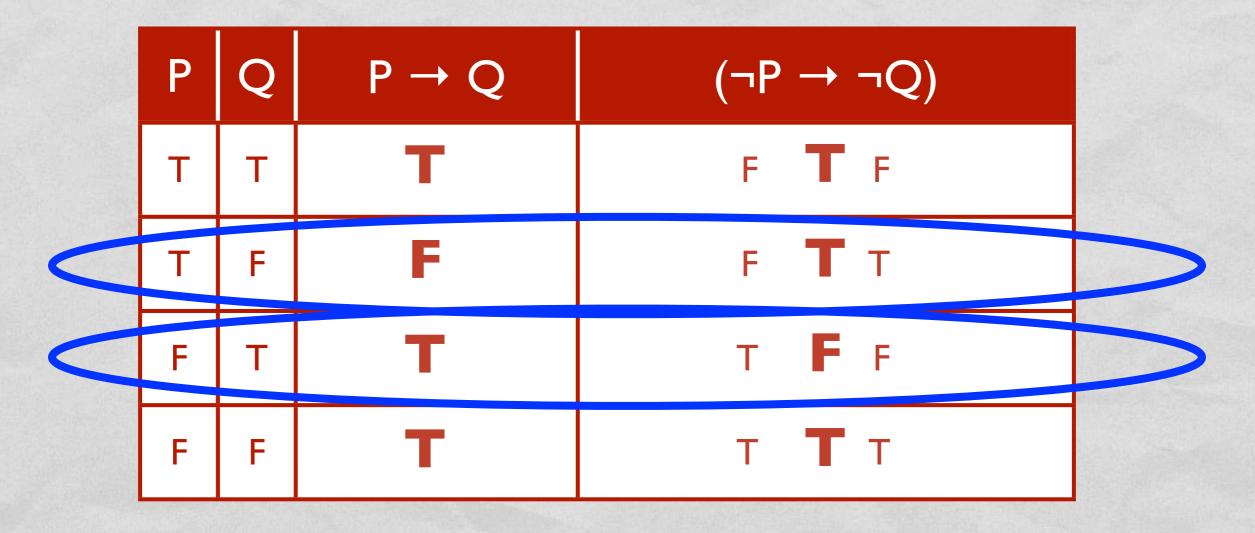
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Р	Q	$P \rightarrow Q$	(¬P → ¬Q)
т	Т	Т	F T F
т	F	F	F T T
F	т	Т	т F ғ
F	F	T	т Т Т

• Joint truth table for $P \rightarrow Q$ and $(\neg P \rightarrow \neg Q)$ These are <u>not</u> equivalent



Truth table for (Cube(a) ∧ Cube(b)) → Cube(b)
This sentence is a <u>Tautology</u>

Cube(a)	Cube(b)	$(Cube(a) \land Cube(b)) \rightarrow Cube(b)$				
т	т	Т	Тт			
Т	F	F	ΤF			
F	Т	F	Тт			
F	F	F	F			

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т	Т	Т	(т)	Т		
Т	F	F	Т	F		
F	Т	F	Т	т		
F	F	F	Т	F		

LOGICAL AND TAUTOLOGICAL CONSEQUENCE

A	В	C	Α	A→B	$\neg B \lor C$	С
Т	Т	Т	Т	Т	F T	Т
Т	Т	F	Т	Т	F F	F
Т	F	Т	Т	F	ТТ	Т
Т	F	F	Т	F	ТТ	F
F	Т	Т	F	Т	F T	Т
F	Т	F	F	Т	F F	F
F	F	Т	F	Т	ТТ	Т
F	F	F	F	Т	ТТ	F

No row is T, T, T, F so YES, valid

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A	В	C	Α	A→B	B v C	С
Т	Т	Т	Т	Т	Т	Т
Т	Т	F	Т	Т	т	F
Т	F	Т	Т	F	Т	Т
Т	F	F	Т	F	F	F
F	Т	Т	F	Т	т	Т
F	Т	F	F	Т	Т	F
F	F	Т	F	Т	Т	Т
F	F	F	F	Т	Т	F

Second row is T, T, T, F So NOT valid



Here none of these sentences are tautologies, none of the pairs are equivalent, and (3) is not a consequence of (1) and (2)