Review day 1 (translations, Tables, and Counterexamples)

Monday, 17 February

TRANSLATION EXAMPLES

Translation Examples

- If ***a*** and ***b*** are both Tets, then neither is small

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- If ***a*** and ***b*** are both Tets, then neither is small
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- If ***a*** and ***b*** are both Tets, then neither is small
  - $(Tet(a) \land Tet(b)) \rightarrow \neg(Small(a) \lor Small(b))$
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- If ***a*** and ***b*** are both Tets, then neither is small
  - $(Tet(a) \land Tet(b)) \rightarrow \neg(Small(a) \lor Small(b))$
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  - $\neg(Medium(b) \lor Large(b)) \rightarrow Smaller(a,b)$
- ***a*** is a cube only if exactly one of ***a*** and ***b*** are small
  - $Cube(a) \rightarrow ((Small(a) \land \neg Small(b)) \lor (\neg Small(a) \land Small(b)))$

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- ***a*** is a cube if and only if at least one of ***a*** and ***b*** are small
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- ***a*** is a cube if and only if at least one of ***a*** and ***b*** are small
  - Cube(a) $\leftrightarrow$ (Small(a) $\vee$ Small(b))
- At least one of ***a*** and ***b*** are Tets, but not both.
  - (Tet(a) $\vee$ Tet(b)) $\wedge$ $\neg$(Tet(a) $\wedge$ Tet(b))

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- ***a*** is a cube if and only if at least one of ***a*** and ***b*** are small
  - Cube(a) ↔ (Small(a) ∨ Small(b))
- At least one of ***a*** and ***b*** are Tets, but not both.
  - (Tet(a) ∨ Tet(b)) ∧ ¬(Tet(a) ∧ Tet(b))
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  - (Tet(a) ∨ Tet(b)) ∧ ¬(Tet(a) ∧ Tet(b))
- ***a*** and ***b*** are both cubes if they name the same object
  - a=b → (Cube(a) ∧ Cube(b))

Truth Tables

- Example: joint truth table for ¬(P∧Q) and (¬P∨¬Q) This shows that the two sentences are equivalent.

P	Q	¬ (P ∧ Q)	(¬ P ∨ ¬ Q)
T	T	**F** T **T** T	**F** T **F** **F** T
T	F	**T** T **F** F	**F** T **T** **T** F
F	T	**T** F **F** T	**T** F **T** **F** T
F	F	**T** F **F** F	**T** F **T** **T** F

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Truth Tables with Conditionals

- Joint truth table for P → Q and (¬P → ¬Q)
  These are not equivalent

P	Q	P → Q	(¬P → ¬Q)
T	T	**T**	F **T** F
T	F	**F**	F **T** T
F	T	**T**	T **F** F
F	F	**T**	T **T** T

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F	T	**T**	T **F** F
F	F	**T**	T **T** T

Truth Tables with Conditionals

- Truth table for (Cube(a) $\land$ Cube(b)) $\rightarrow$ Cube(b)
  This sentence is a **Tautology**

Cube(a)	Cube(b)	(Cube(a) $\land$ Cube(b)) $\rightarrow$ Cube(b)
T	T	T **T** T
T	F	F **T** F
F	T	F **T** T
F	F	F **T** F

Truth Tables with Conditionals

- Truth table for (Cube(a) ∧ Cube(b)) → Cube(b)
  This sentence is a **<u>Tautology</u>**

Cube(a)	Cube(b)	(Cube(a) ∧ Cube(b))	→	Cube(b)
T	T	T	**T**	T
T	F	F	**T**	F
F	T	F	**T**	T
F	F	F	**T**	F

Logical and Tautological Consequence

A	B	C	A	A→B	¬B ∨ C	C
T	T	T	T	T	F **T**	T
T	T	F	T	T	F **F**	F
T	F	T	T	F	T **T**	T
T	F	F	T	F	T **T**	F
F	T	T	F	T	F **T**	T
F	T	F	F	T	F **F**	F
F	F	T	F	T	T **T**	T
F	F	F	F	T	T **T**	F

No row is T, T, T, F so YES, valid

Logical and Tautological Consequence

A	B	C	A	A→B	B ∨ C	C
T	T	T	T	T	**T**	T
T	T	F	T	T	**T**	F
T	F	T	T	F	**T**	T
T	F	F	T	F	**F**	F
F	T	T	F	T	**T**	T
F	T	F	F	T	**T**	F
F	F	T	F	T	**T**	T
F	F	F	F	T	**T**	F

Second row is T, T, T, F
So NOT valid

=1= A	=2= B	=3= C	(1) $A \lor \neg B$		(2) $(A \land \neg B) \to C$			(3) $(C \lor B) \leftrightarrow \neg A$		
T	T	T	T	F	F	F	T	T	F	F
T	T	F	T	F	F	F	T	T	F	F
T	F	T	T	T	T	T	T	T	F	F
T	F	F	T	T	T	T	F	F	T	F
F	T	T	F	F	F	F	T	T	T	T
F	T	F	F	F	F	F	T	T	T	T
F	F	T	T	T	F	T	T	T	T	T
F	F	F	T	T	F	T	T	F	F	T

Here none of these sentences are tautologies, none of the pairs are equivalent, and (3) is not a consequence of (1) and (2)