PUZZLE

You meet two people on the island of knights and knaves.

A says "Either I am a knight or B is a knight."

B says "A is a knave."

Who is what?

Argument by Cases

Friday, 7 February

Constructing a Formal Proof

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- In a proof you assume a set of premises, and work step by step to the desired conclusion (if the conclusion is a logical consequence of the premises!)
- Each step is justified by invoking a rule that is part of our formal system of deduction.
- In this class, we have been using Fitch but there are other systems of proof (deductive systems).

Formal Proof Rules for ∧

- ∧ Introduction
  From P and Q, we can infer $P \wedge Q$.

- ∧ Elimination
  From $P \wedge Q$, we can infer P.

Formal Proof Rules for ∧

- ∧ Introduction

  From P and Q, we can infer P∧Q.

  1. P
  2. Q

  ---

  3. P ∧ Q ∧ Intro: 1,2

- ∧ Elimination

  From P∧Q, we can infer P.

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- ∧ Elimination

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  1. P ∧ Q

  ---

  2. P ∧ Elim: 1

PROOFS

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Disjunction (∨) Introduction

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<u>Disjunction (∨) Introduction</u>

- Intuitively, if you know that A is true, then you can conclude that either A or B (or both).
- Ex: If Alice will be at the party, then it is true that either Alice or Bill will be there.
- In general, from P we can infer 'P or Q'.

FORMAL PROOF RULES (∨)

- ∨ Introduction
  From P, we can infer P∨Q.

- Another example:

Formal Proof Rules (∨)

- ∨ Introduction
  From P, we can infer P∨Q.

1. P	
2. P ∨ Q	∨ Intro: 1

- Another example:

Formal Proof Rules (∨)

- ∨ Introduction
  From P, we can infer P∨Q.

1. P	
2. P ∨ Q	∨ Intro: 1

- Another example:

Formal Proof Rules (∨)

- ∨ Introduction
  From P, we can infer P∨Q.

  | 1. P
  |---
  | 2. P ∨ Q   ∨ Intro: 1

- Another example:

  | 1. P
  |---
  | 2. P∨((Q↔R)→¬S)   ∨ Intro: 1

Proof by Cases

Proof by Cases

- Intuitively, if you know that A or B is the case, and that C follows from A and C also follows from B, then you know that C is the case.

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- Example: I will either go to the bank on Monday or Tuesday. So either way, I will have some money to buy lunch on Wednesday.

Proof by Cases

Disjunction (∨) Elimination

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Disjunction (∨) Elimination

- In general, proof by cases (disjunction elimination) is when you start with a disjunction and show for each disjunct that, if you assume its truth, some sentence S follows.

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Disjunction (∨) Elimination

- In general, proof by cases (disjunction elimination) is when you start with a disjunction and show for each disjunct that, if you assume its truth, some sentence S follows.
- Note: you don't need to know which disjunct is true.

Proof by Cases

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- To show that certain things follow from a set of assumptions, we use subproofs.

PROOF BY CASES

- Disjunction Elimination formalizes proof by cases.
- In order to use proof by cases, we need to be able to make assumptions in our proof.
- To show that certain things follow from a set of assumptions, we use subproofs.
- BUT we can only make assumptions within a subproof.

Proof by Cases

Proof by Cases

- ∨ Elimination

  If R follows from P, and if R follows from Q, then from $P \vee Q$, we can infer R.

PROOF BY CASES

- ∨ Elimination

  If R follows from P, and if R follows from Q, then from P∨Q, we can infer R.

1. P ∨ Q

    2. P

    …

    j. R   ??

    k. Q

    …

    m. R   ??

n. R   ∨Elim: 1,2-j,k-m

Proof by Cases

- ∨ Elimination

  If R follows from P, and if R follows from Q, then from P∨Q, we can infer R.

Scope Lines

1. P ∨ Q
2. P

...

j. R ??

k. Q

...

m. R ??

n. R ∨Elim: 1,2-j,k-m

Proof by Cases

- ∨ Elimination

  If R follows from P, and if R follows from Q, then from P∨Q, we can infer R.

1. P ∨ Q
2. P

…

j. R ??

k. Q

…

m. R ??

n. R ∨Elim: 1,2-j,k-m

Scope Lines

Scope Lines indicate assumptions that don't necessarily follow from earlier assumptions

Proof by Cases

Example:

$$\begin{array}{|l} (A \land B) \lor (A \land C) \\ \hline A \end{array}$$

PROOF BY CASES

Example:

$(A \wedge B) \vee (A \wedge C)$

A

1. $(A \wedge B) \vee (A \wedge C)$

Proof by Cases

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$(A \wedge B) \vee (A \wedge C)$

A

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2. $A \wedge B$

Proof by Cases

Example:

$(A \wedge B) \vee (A \wedge C)$

A

1. $(A \wedge B) \vee (A \wedge C)$

2. $A \wedge B$

3. A $\wedge$ Elim: 2

Proof by Cases

Example:

$(A \wedge B) \vee (A \wedge C)$

A

1. $(A \wedge B) \vee (A \wedge C)$

2. $A \wedge B$

3. A $\wedge$ Elim: 2

4. $A \wedge C$

Proof by Cases

Example:

$(A \wedge B) \vee (A \wedge C)$

A

1. $(A \wedge B) \vee (A \wedge C)$

2. $A \wedge B$

3. A $\quad\wedge$ Elim: 2

4. $A \wedge C$

5. A $\quad\wedge$ Elim: 4

Proof by Cases

Example:

$(A \wedge B) \vee (A \wedge C)$

A

1. $(A \wedge B) \vee (A \wedge C)$

   2. $A \wedge B$

   3. A — $\wedge$ Elim: 2

   4. $A \wedge C$

   5. A — $\wedge$ Elim: 4

6. A — $\vee$ Elim: 1,2-3,4-5

PROOF BY CASES

Example:

Cube(a) ∨ Dodec(a)

¬Tet(a)

PROOF BY CASES

Example:

Cube(a) ∨ Dodec(a)

¬Tet(a)

1. Cube(a) ∨ Dodec(a)

PROOF BY CASES

Example:

Cube(a) ∨ Dodec(a)

¬Tet(a)

1. Cube(a) ∨ Dodec(a)

2. Cube(a)

3. ¬Tet(a)   Ana Con 2

Proof by Cases

Example:

Cube(a) ∨ Dodec(a)

¬Tet(a)

1. Cube(a) ∨ Dodec(a)

---

2. Cube(a)

---

3. ¬Tet(a) Ana Con 2

4. Dodec(a)

---

5. ¬Tet(a) Ana Con 4

Proof by Cases

Example:

Cube(a) ∨ Dodec(a)

¬Tet(a)

1.	Cube(a) ∨ Dodec(a)	
2.	Cube(a)	
3.	¬Tet(a)	Ana Con 2
4.	Dodec(a)	
5.	¬Tet(a)	Ana Con 4
6.	¬Tet(a)	∨ Elim: 1,2-3,4-5

$\wedge$, $\vee$ Distribution Rules

- Distribution rules:
- $P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$
- $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$