

You meet two people on the island of knights and knaves.

A says "Either I am a knight or B is a knight." B says "A is a knave."

Who is what?

# ARGUMENT BY CASES

Friday, 7 February

COLORADOR CANTEL & THE

Sunday, February 9, 2014

 In a proof you assume a set of premises, and work step by step to the desired conclusion (if the conclusion is a logical consequence of the premises!)

- In a proof you assume a set of premises, and work step by step to the desired conclusion (if the conclusion is a logical consequence of the premises!)
- Each step is justified by invoking a rule that is part of our formal system of deduction.

- In a proof you assume a set of premises, and work step by step to the desired conclusion (if the conclusion is a logical consequence of the premises!)
- Each step is justified by invoking a rule that is part of our formal system of deduction.
- In this class, we have been using Fitch but there are other systems of proof (deductive systems).

## FORMAL PROOF RULES FOR A

A Introduction
 From P and Q, we can infer PAQ.

•  $\land$  Elimination From P $\land$ Q, we can infer P.

Sunday, February 9, 2014

### Formal Proof Rules for $\land$

•  $\land$  Introduction From P and Q, we can infer P $\land$ Q. 1. P 2. Q 3. P  $\land$  Q  $\land$  Intro: 1,2

A Elimination
 From PAQ, we can infer P.

### Formal Proof Rules for $\land$

•  $\land$  Introduction From P and Q, we can infer P $\land$ Q. 1. P 2. Q 3. P  $\land$  Q  $\land$  Intro: 1,2

A Elimination
 From PAQ, we can infer P.

## FORMAL PROOF RULES FOR A

•  $\land$  Introduction From P and Q, we can infer P $\land$ Q. 1. P 2. Q 3. P  $\land$  Q  $\land$  Intro: 1,2

•  $\land$  Elimination From P $\land$ Q, we can infer P.  $1.P \land Q$  $2.P \land$  Elim: I



La Calendar and Marsh

1. H.



A Destroy Land and Des a second a Des to

Disjunction (v) Introduction



#### **Disjunction** (v) Introduction

 Intuitively, if you know that A is true, then you can conclude that either A or B (or both).



#### **Disjunction** (V) Introduction

- Intuitively, if you know that A is true, then you can conclude that either A or B (or both).
- Ex: If Alice will be at the party, then it is true that either Alice or Bill will be there.



#### **Disjunction** (V) Introduction

- Intuitively, if you know that A is true, then you can conclude that either A or B (or both).
- Ex: If Alice will be at the party, then it is true that either Alice or Bill will be there.
- In general, from P we can infer 'P or Q'.

State Block Martin State

V Introduction
 From P, we can infer PVQ.

Another example:

ALL AND DE ALL ANTICIAN DE DA

•  $\vee$  Introduction From P, we can infer P $\vee$ Q. 1.P $2.P \vee Q$   $\vee$  Intro: I

Another example:

ALL AND DE ALL ANTICIAN DE DA

•  $\vee$  Introduction From P, we can infer P $\vee$ Q. 1.P $2.P \vee Q$   $\vee$  Intro: I

Another example:

Control Stort of Martin State

v Introduction
 From P, we can infer P∨Q.
 I.P
 2.P ∨ Q
 v Intro: I

Another example:

I.P2.P∨((Q↔R)→¬S) ∨ Intro: I

A Cherry Land and Straight and a Cherry Straight

 Intuitively, if you know that A or B is the case, and that C follows from A and C also follows from B, then you know that C is the case.

 Intuitively, if you know that A or B is the case, and that C follows from A and C also follows from B, then you know that C is the case.

 Example: I will either go to the bank on Monday or Tuesday. So either way, I will have some money to buy lunch on Wednesday.

C. L. C. M. Barris Marsh

Disjunction (v) Elimination

**Disjunction** (V) Elimination

In general, proof by cases (disjunction elimination) is when you start with a disjunction and show for each disjunct that, if you assume its truth, some sentence S follows.

**Disjunction** (V) Elimination

 In general, proof by cases (disjunction elimination) is when you start with a disjunction and show for each disjunct that, if you assume its truth, some sentence S follows.

Note: you don't need to know which disjunct is true.

A Cherry Land and Straight and a Cherry Straight

#### Disjunction Elimination formalizes proof by cases.

Constanting and Strengton

Disjunction Elimination formalizes proof by cases.

CANADAR MARTIN AND IN

 In order to use proof by cases, we need to be able to make assumptions in our proof.

- Disjunction Elimination formalizes proof by cases.
- In order to use proof by cases, we need to be able to make assumptions in our proof.
- To show that certain things follow from a set of assumptions, we use subproofs.

- Disjunction Elimination formalizes proof by cases.
- In order to use proof by cases, we need to be able to make assumptions in our proof.
- To show that certain things follow from a set of assumptions, we use subproofs.

BUT we can only make assumptions within a subproof.

A Cherry Land and Straight and a Cherry Straight

 V Elimination
 If R follows from P, and if R follows from Q, then from PVQ, we can infer R.

 V Elimination
 If R follows from P, and if R follows from Q, then from
 PVQ, we can infer R.

$I.P \vee Q$	
2. P	
 j. R k. Q	??
k.Q	
 m. R	??
n. R	∨Elim: I,2-j,k-m

A State of the state of the state of the state

<ul> <li>V Elimination</li> <li>If R follows from P, and if R</li> </ul>	I.P ∨ Q 2.P	
follows from Q, then from $P \lor Q$ , we can infer R.	 j. R	??
Scope Lines	k.Q	
	 m. R	??
	n. R	∨Elim: I,2-j,k-m

A State of the state of the state of the state

• v Elimination	$I.P \lor Q$	
If R follows from P, and if R follows from Q, then from	2. P	
$P \lor Q$ , we can infer R.	 j. R	??
Scope Lines	k.Q	
Scope Lines indicate assumptions	 m. R	??
that don't necessarily follow from earlier assumptions	n. R	∨Elim: I,2-j,k-m

Stand Block owned a Const

Example:  $(A \land B) \lor (A \land C)$ A

AND STALL AND A STALL

Example:  $(A \land B) \lor (A \land C)$ A

### $I.(A \land B) \lor (A \land C)$

And Balanta ANTRI

Example:  $(A \land B) \lor (A \land C)$ A

 $\frac{I.(A \land B) \lor (A \land C)}{2. A \land B}$ 

And Black ANTRS

Example:  $(A \land B) \lor (A \land C)$ A I.  $(A \land B) \lor (A \land C)$ 2.  $A \land B$ 3.  $A \land C$ 

Example:  $(A \land B) \lor (A \land C)$ A I.  $(A \land B) \lor (A \land C)$ 2.  $A \land B$ 3.  $A \land Elim: 2$ 

**4.** A ∧ C

#### Sunday, February 9, 2014

Example:  $(A \land B) \lor (A \land C)$ A I.  $(A \land B) \lor (A \land C)$ 2.  $A \land B$ 3.  $A \land A$ 3.  $A \land A$ 4.  $A \land C$ 5.  $A \land A$ 

Example:  $(A \land B) \lor (A \land C)$ A

I.  $(A \land B) \lor (A \land C)$ 2. A ∧ B 3. A  $\wedge$  Elim: 2 **4.** A ∧ C 5. A 6.A  $\wedge$  Elim: 4 ∨ Elim: 1,2-3,4-5

L. Martina and Party

```
Example:
Cube(a) ∨ Dodec(a)
¬Tet(a)
```

C. S. Martine Contra Paral

Example: Cube(a) ∨ Dodec(a) ¬Tet(a)

#### I. Cube(a) ∨ Dodec(a)

the second state where a second second

Example: Cube(a) ∨ Dodec(a) ¬Tet(a)

I. Cube(a)  $\lor$  Dodec(a)2. Cube(a)3.  $\neg$ Tet(a)Ana Con 2

in a service and states and states in the service of the service o

Example: Cube(a) ∨ Dodec(a) ¬Tet(a)

I. Cube(a) ∨ Dodec(a)
2. Cube(a)
3. ¬Tet(a) Ana Con 2
4. Dodec(a)
5. ¬Tet(a) Ana Con 4

ALL & LATAN AND BELLE

Example: Cube(a) ∨ Dodec(a) ¬Tet(a)

I. Cube(a) ∨ Dodec(a) 2. Cube(a) 3. ¬Tet(a) Ana Con 2 4. Dodec(a) 5. ¬Tet(a) Ana Con 4
6. ¬Tet(a) ∨ Elim: 1,2-3,4-5

# $\wedge, \vee$ DISTRIBUTION RULES

- Distribution rules:
- $P \land (Q \lor R) \Leftrightarrow (P \land Q) \lor (P \land R)$
- $P \lor (Q \land R) \Leftrightarrow (P \lor Q) \land (P \lor R)$